SOLUTION TO "BARS OF SOAP"

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(Thanks to Derek Kisman for a programming assist.)

When a solver comes to Hunt HQ, they are presented with a collection of six items, each with a label attached. The puzzle involves the UPC codes of these items. The list of objects, labels, and UPC codes, are included later in this solution.

This puzzle is part of the object round (the yellow normal round), so its answer is an object. The puzzle is similar in spirit to Mastermind: the six given objects are clues to a missing seventh object, the answer to the puzzle. The answer object is to be identified by determining its UPC code.

The text accompanying the puzzle ("ten-dimensional space") should make it clear than we want you to use the main 10-digit part of the UPC code, rather than the expanded 12-digit code which includes the leading and checksum digits (often in smaller print) to the left and right of the main code. Moreover, the notation of the labels should be suggestive of L^p-norms. (At least, it should suggest this to a math major. Hopefully, you took a hint from the course 18 reference in the text of the puzzle and sent a math major to visit us at HQ.)

Indeed, in this puzzle UPC codes should be regarded as vectors in $\mathbb{R}^{10}$, with each digit as a separate coordinate. The label on an object gives you the L^p-distance between the object's UPC code (regarded as a vector) and the solution UPC code (similarly regarded as a vector), for some value of p. In other words, if the solution UPC code is $s_0 s_1 s_2 ... s_9$, and if an object's UPC code is $o_0 o_1 o_2 ... o_9$, then the object's label could read

$$|| \cdot ||_p = \left(\sum_{k=0}^{9} |s_k - o_k|^p \right)^{1/p}.$$

(On the actual labels, we've rounded the numbers off five digits after the decimal point, so instead of giving equalities we give approximate equalities.)

As a toy example, if UPC codes were four digits long, and if the solution UPC code were 1234, then an object with UPC code 5521 would have L^2 distance $(|1 - 5|^2 + |2 - 5|^2 + |3 - 2|^2 + |4 - 1|^2)^{1/2} = 35^{1/2}$ from the solution code. The label would then read $|| \cdot ||_2 \approx 5.91608$.

Here are the objects and labels:

Label for Carmex cold sore balm (UPC code 83078 11215):
$$|| \cdot ||_2 \approx 13.52775$$

Label for Bee Playing Cards (UPC code 73854 00092):
$$|| \cdot ||_1 = 39$$

Label for Oral-B toothbrush (UPC code 00410 84100):
$$|| \cdot ||_4 \approx 7.99560$$

Label for Right Guard (UPC code 47400 21116):
$$|| \cdot ||_3 \approx 5.57205$$

Label for Bic Mechanical Pencils (UPC code 70330 91187):
$$|| \cdot ||_3 \approx 9.22087$$

Label for Listerine (UPC code 12547 70128):
$$|| \cdot ||_2 \approx 13.49074$$

Given the above information, it is then a logic puzzle to determine the solution UPC code: 37000 33353. (If you solved the logic part by hand, we'll be very impressed. We couldn't do it. Solving it by computer seems to be the way to go.)

Now the task is to find the object with UPC code 37000 33353. A few years ago this would have been a pain, but now there are searchable UPC databases on the web (e.g. http://www.upcdatabase.com): it is a bottle of Pert shampoo, of a variety available at CVS. This bottle is the solution to the puzzle. (Use PERT for the yellow normal metapuzzle, and use the bottle for the yellow supermeta.)