Diversity Analysis of Multi-antenna UWB Impulse Radio Systems with Correlated Propagation Channels

Junsheng Liu, Mohammad Ghavami and Xiaoli Chu
UWB Communications Group
Division of Engineering
King's College London
Strand, WC2R 2LS
London, UK
Email: junsheng.liu, mohammad.ghavami, xiaoli.chu @kcl.ac.uk

Ben Allen and Wasim Malik
Department of Engineering Science
University of Oxford
Parks Road
Oxford OX1 3PJ
UK
Email: ben.allen, wasim.malik @eng.ox.ac.uk

***Abstract*— Providing diversity is one of the two possible objectives (the other one is providing multiplexing) of a multi-input multi-output (MIMO) system. For Ultra wide-band (UWB) impulse radio systems using rake receivers, there are already richly inherent multipath diversities. However, there may be UWB applications making use of more than one UWB transmit antenna and/or receive antenna, which leads to UWB MIMO systems. The difference between the diversity analyse of a UWB MIMO system from that of a narrow band system lies in the different choice of propagation channel model for the branches linking each transmit antenna and receive antenna. Similar to the narrow band system, the existence of the correlation between the branches can severely degrade the diversity performance, and complicate the performance analyse. In this paper, the wide-band propagation channel for the UWB MIMO system, with pre-defined correlation model for the branches, is broken down into several uncorrelated elementary narrow-band propagation channels with equivalent diversity performance. Based on this, a novel method for evaluating the diversity performance of a MIMO UWB impulse radio system is presented.**

I. INTRODUCTION

UWB systems in the form of impulse radio can provide a stable level of received signal power due to the highly resolvable multi-path diversity [1]. This requires a rake receiver that can combine an adequate number of multi-path components [2]. However, due to the limited number of fingers that can be incorporated in practice, the rake recevier may not collect enough signal energy from the multi-path components. Applying multi-antennas can help reduced the required number of rake fingers and [3] be one of the solutions to compensate this shortage of the UWB impulse system. The consideration is how to balance the number transceive antennas and the number of fingers built in the rake receivers.

Diversity order has been interpreted in [5] as the slope of symbol error probability (SEP) curve when the signal-to-noise ratio (SNR) approaches infinity. There is a tradeoff between the diversity order and the multiplexing [5]; maximum diversity performance is achieved when the system does not provide any multiplexing. However, diversity order can not be used to evaluated the deteriorated diversity performance due to the impact of the correlation between the branches linking different transceive antennas because, as we will show later in this paper, the curves of the SEP of MIMO systems undergoing any degree of correlations (except the case that the correlation is equal to 1) have the same slops in the high-SNR region.

This paper focuses on the diversity performance of the MIMO UWB impulse radio systems with correlated propagation branches. The maximum diversity performance is of interest here and is regarded as the diversity performance of the system. We will derive a measure of the diversity performance for a MIMO UWB impulse radio system with arbitrarily correlated branches, build up a link between this measure and the correlations, and give the expression of the measure for two pre-defined correlation model.

Several assumptions have been made as follows. Firstly, this paper focuses on the UWB indoor wireless propagation environment, so it is reasonable to assume that the propagation channels are time invariant for a large number of symbols and spatially stationary in a sufficient small area [7]. Secondly, it is assumed that full channel information and perfect synchronisation are achieved on the receiver side. Thirdly, the channel magnitude is Nakagami-m distributed [8]. Finally, anti-podal modulation scheme, that is, binary phase shift keying (BPSK), is applied on the transmitted pulses when analysing the symbol error probability (SEP) performance of the system.

The rest of this paper is organised as follows: in Section II, the matrix structure is given to describe the MIMO propagation channel for the UWB impulse radio system; in Section III, virtual branches technique is used to break down the correlated propagation channels into uncorrelated channel components; a measure of evaluating the diversity performance is derived Section IV; a link between this measure and the correlation between the branches is presented in Section V for two special structures of the covariance matrix of the channels; conclusion and future work appears in Section VI.

II. CHANNEL MATRIX OF THE UWB IMPULSE RADIO MIMO SYSTEM

Suppose that N_t transmit antennas and N_r receive antennas are employed in the UWB MIMO communication system. The MIMO channel matrix can be represented by a three dimensional matrix as follows

$$\mathbf{H}(\tau) = \begin{pmatrix} H_{11}(\tau) & H_{12}(\tau) & \cdots & H_{1N_t}(\tau) \\ H_{21}(\tau) & H_{22}(\tau) & \cdots & H_{2N_t}(\tau) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N_r1}(\tau) & H_{N_r2}(\tau) & \cdots & H_{N_rN_t}(\tau) \end{pmatrix} \quad (1)$$

where $H_{pq}(\tau)$ itself is a tapped-delay line channel model

$$H_{pq}(\tau) = \sum_{i=0}^{N_{pq}-1} a_{pq,i}\delta(\tau - i\cdot\Delta\tau), \quad 1 \le p \le N_r, \ 1 \le q \le N_t \quad (2)$$

$\Delta\tau$ is the minimum resolved time bin-width, which is approximately the reciprocal of the bandwidth occupied by the transmitted signal; N_{pq} is the total number of the resolved time bins, which can be calculated as $N_{pq} = \tau_e/\Delta\tau$, where τ_e is the channel excess delay. For the first arriving component, the delay is zero. $a_{pq,i}$ is set to be zero when there is no multi-path component appearing in the i^{th} time bin.

Assume that the transmitted signal is $s(t)$. The received signal, $r_{pq}(t)$, corresponding to the sub-channel connecting the q^{th} transmit antenna and p^{th} receive antenna is given as

$$r_{pq}(t) = s(t) * H_{pq}(t) \quad (3)$$

where $*$ denotes convolution.

A sequence of matched filters with different delays are employed in the rake receiver for the selected paths. With full channel information and perfect synchronisation at the receiver, maximum ratio combining (MRC) is applied on each finger with finger weight $a^*_{pq,i}$, where $*$ indicates complex conjugate here. The signal energy at the output of the rake receiver, γ_{pq}, is thus the combination of the energy born on the selected paths as follows

$$\begin{aligned} \gamma_{pq} &= \sum_{i \in I_R} |a_{pq,i}|^2 \cdot \int_{-\infty}^{\infty} |s(\tau)|^2 d\tau \\ &= \sum_{i \in I_R} |a_{pq,i}|^2 \end{aligned} \quad (4)$$

where the pulse energy has been normalised to 1, i.e., $\int_{-\infty}^{\infty} |s(\tau)|^2 d\tau = 1$. I_R is the aggregate set of the indexes of the chosen paths, depending on the type of the rake receiver, selective rake (S-rake) or partial rake (P-rake) [10], [11].

Because $a_{pq,i}$ is Nakagami-m distributed, γ_{pq} has a Gamma distribution [9] given below:

$$p_{\gamma_{pq}}(\gamma_{pq}) = \frac{m_{pq}^{m_{pq}} \gamma^{m_{pq}-1}}{\overline{\gamma}^{m_{pq}} \Gamma(m_{pq})} \exp\left(-\frac{m_{pq}\gamma}{\overline{\gamma}}\right) \quad (5)$$

where $\Gamma(m_{pq})$ is the gamma function with the m-parameter m_{pq} associated with the Nakagami-m propagation channel between the p^{th} transmit antenna and the q^{th} receive antenna, and $\overline{\gamma} = E\{\gamma\}$ is the expectation of γ over the fading channel.

Note that in order to apply virtual-branch de-construction [15] on the correlated propagation channels, $2m_{qp}$ has to be constrained to be an integer in the rest of this paper.

For convenience of analysis, γ_{pq} are organised in an $N_r \times N_t$ matrix similar to the channel matrix for the narrow band MIMO system

$$\mathbf{H}_\gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N_t} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N_r1} & \gamma_{N_r2} & \cdots & \gamma_{N_rN_t} \end{bmatrix} \quad (6)$$

III. VIRTUAL BRANCHES OF THE RAKE OUTPUT MATRIX

When MRC is applied at the receiver, the signal-to-noise ratio at the input to the decision block, $\mathrm{SNR_r}$, is the sum of SNRs of the selected paths from all transmit-receive antenna links. Assuming that all the channels share the same noise power spectral density N_0, $\mathrm{SNR_r}$ can be written as

$$\begin{aligned} \mathrm{SNR_r} &= \sum_{p=1}^{N_t}\sum_{q=1}^{N_r} \frac{E_b\gamma_{pq}}{N_0} \\ &= \mathrm{SNR_t} \cdot \gamma \end{aligned} \quad (7)$$

where E_b is the signal energy per bit, $\mathrm{SNR_t} = \frac{E_b}{N_0}$ is the signal-to-noise ratio at the transmitter side, and $\gamma = \sum_{p=1}^{N_t}\sum_{q=1}^{N_r}\gamma_{pq}$.

Generally speaking, since the different transmit-receive antenna links are correlated, γ_{pq} are correlated with each other, which makes it difficult to apply the moment generation function (MGF) [12] directly to the integral form of bit error probability (BEP) in order to derive a closed form expression. Fortunately, by using the Karhunen-Loeve (KL) expansion [13],[14], the rake output matrix in equation (6) can be projected onto a set of virtual branches that have independent elements [15].

A. Virtual Branches Representation of Matrix $\mathbf{H}_\gamma$

First, $\mathbf{H}_\gamma$ in equation (6) needs to be vectorised as

$$\begin{aligned} \vec{h}_\gamma &= \mathrm{vec}\{\mathbf{H}_\gamma\} \\ &= [\gamma_{11}\ \gamma_{12}\ \cdots\ \gamma_{1N_t}\ \gamma_{21}\ \gamma_{22}\ \cdots \gamma_{N_rN_t}]^T \end{aligned} \quad (8)$$

where $\gamma_{pq} = \vec{r}_{pq}^{\,T} \cdot \vec{r}_{pq}$, and $\vec{h}_\gamma$ is an $N_rN_t \times 1$ vector. $\vec{r}_{pq}$ is a $2m_{pq} \times 1$ Gaussian distributed vector with zero-mean and covariance matrix $\mathrm{diag}\{\frac{\overline{\gamma}_{pq}}{2m_{pq}}\}$, where $\mathrm{diag}\{X\}$ is a diagonal matrix with X being on the diagonal. The total collected signal energy can now be written as

$$\gamma = \vec{r}^{\,T} \cdot \vec{r} \quad (9)$$

where

$$\vec{r} = [\vec{r}_{11}^{\,T}\vec{r}_{12}^{\,T}...\vec{r}_{21}^{\,T}\vec{r}_{22}^{\,T}...\vec{r}_{N_rN_t}^{\,T}]^T = [\vec{r}_1^{\,T}\vec{r}_2^{\,T}...\vec{r}_{N_r\cdot N_t}^{\,T}]^T \quad (10)$$

Note that the 2-dimensional indexes of $\vec{r}_{pq}$ have been changed to 1-dimensional indexes in the above equation for analytical convenience.

Since $\vec{r}$ is zero mean, it can be KL expanded to

$$\vec{r} = \sum_{i=0}^{N-1} \nu_i \vec{\varphi}_i \tag{11}$$

where N is the total number of non-zero eigenvalues[1] of the covariance matrix $\mathbf{C}_{\vec{r}} = E\{\vec{r}\vec{r}^T\}$, $\vec{\varphi}_i$ are orthonormal vectors, and ν_i are independent Gaussian distributed KL coefficients with zero means and covariances λ_i, i.e., $\nu_i \sim N(0, \lambda_i)$, where λ_i is an arbitrary non-zero eigenvalue of the covariance matrix $\mathbf{C}_{\vec{r}}$.

Substituting (11) into (9) results in

$$\begin{aligned} \gamma &= \left(\sum_{i=0}^{N-1} \nu_i \vec{\varphi}_i \right)^T \left(\sum_{i=0}^{N-1} \nu_i \vec{\varphi}_i \right) \\ &= \sum_{i=0}^{N-1} \nu_i^2 \end{aligned} \tag{12}$$

where ν_i^2 has a Gamma distribution with $m_{pq} = 1/2$ and $\overline{\nu_i^2} = \lambda_i$.

In this paper, $\bar{\gamma}$ is normalised to 1, that is

$$\bar{\gamma} = \sum_{i=0}^{N-1} \lambda_i = \mathrm{tr}\{\mathbf{C}_{\vec{r}}\} = E\{\vec{r}^T \vec{r}\} = 1 \tag{13}$$

where $\mathrm{tr}\{\cdot\}$ is the trace of the matrix.

B. Structure of the Covariance Matrix

An expression for the correlation coefficients between the received energies from different diversity branches, i.e., $H_{ij}(t)$ in (1), has been given in [15]. The derivation of the correlation coefficient is given here. However, the expression of the correlation coefficient is slightly different from the result given in [15].

$\mathbf{C}_{\vec{r}}$ has the following form

$$\mathbf{C}_{\vec{r}} = \{\mathbf{C}_{i,j}\} \qquad i,j \in \{1, 2, \cdots, N_r N_t\} \tag{14}$$

$\mathbf{C}_{\vec{r}}$ is composed of matrices $\mathbf{C}_{i,j}$, and each $\mathbf{C}_{i,j}$ is a $2m_i \times 2m_j$ diagonal matrix:

$$\mathbf{C}_{i,j} = \mathrm{diag}\{\rho_{i,j} \cdot \sigma_i \sigma_j\} \tag{15}$$

where $\sigma_i^2 = \mathrm{E}\{\vec{r}_i(k)^2\} = \frac{\bar{\gamma}_i}{2m_i}$, and $\rho_{i,j} = \frac{\mathrm{E}\{\vec{r}_i(k) \cdot \vec{r}_j(k)\}}{\sigma_i \sigma_j}$. $\vec{r}_i(k)$ is the k^{th} element of $\vec{r}_i$ in (10), and k is any integer in $[1, \min\{2m_i, 2m_j\}]$.

Under the assumption that the indoor wireless propagation environment is spatially stationary, the statistical parameters describing the propagation channels should be the same for different realizations of the channels. Thus,

$$\sigma_i^2 = \sigma_j^2 = \frac{\bar{\gamma}}{2mN_rN_t} \tag{16}$$

where $m_i = m_j = m$. $\mathbf{C}_{\vec{r}}$ can thus be rewritten as:

$$\mathbf{C}_{\vec{r}} = \{\mathrm{diag}\{\rho_{i,j}\}\} \cdot \frac{\bar{\gamma}}{2mN_rN_t} \tag{17}$$

[1]Eigenvalues with multiplicities of more than 1 are counted as different eigenvalues.

where $\mathrm{diag}\{\rho_{i,j}\}$ are all $2m \times 2m$ diagonal matrices

The correlation coefficient of the received energies from different transmit-receive antenna pairs is defined as

$$\rho_{\gamma_i,\gamma_j} = \frac{E\{\gamma_i\gamma_j\} - \overline{\gamma_i} \cdot \overline{\gamma_j}}{\sigma_{\gamma_i}\sigma_{\gamma_j}} \tag{18}$$

$\sigma_{\gamma_i}^2 = E\{(\gamma_i - \overline{\gamma_i})^2\}$ is the variance of γ_i

Making use of the expression of the 4^{th} order moment of a Gaussian real vector, $[a\ b\ c\ d]^T$, with zero mean vector given below [18]

$$E\{abcd\} = E\{ab\}E\{cd\} + E\{ac\}E\{bd\} + E\{ad\}E\{bc\} \tag{19}$$

we have

$$E\{\gamma_i\gamma_j\} = \overline{\gamma_i} \cdot \overline{\gamma_j} + \frac{\rho_{i,j}^2 \overline{\gamma_i} \cdot \overline{\gamma_j}}{\max(m_i, m_j)} \tag{20}$$

and

$$\sigma_{\gamma_i}^2 = \frac{\overline{\gamma_i}^2}{m_i} \tag{21}$$

And finally equation (18) can be rewritten as[2]

$$\rho_{\gamma_i,\gamma_j} = \frac{\sqrt{m_i m_j}}{\max(m_i, m_j)} \rho_{i,j}^2 \tag{22}$$

Thus the correlation coefficients, ρ_{γ_i,γ_j}, between the diversity branches are linked to the correlation coefficients, $\rho_{i,j}$, between the Gaussian distributed components of $\vec{r}_i$.

C. Simplification of the Covariance Matrix

It can be seen in the previous sub-section that the covariance matrix, $\mathbf{C}_{\vec{r}}$, is a sparse matrix, since the correlation between different paths in the same transmit-receive antenna link is nearly zero, as stated earlier. For propagation channels with the same m-parameter, $\mathbf{C}_{\vec{r}}$ can be simplified to an $N_rN_t \times N_rN_t$ matrix $\mathbf{C}_r = \{\rho_{i,j}\}$ containing $\rho_{i,j} \cdot \sigma_i\sigma_j$ as the $(i,j)^{\text{th}}$ element.

Theorem 1: λ_i is one of the eigenvalues of a Hermitian matrix, $\mathbf{C}$, with a multiplicity of μ_i. Let $\mathbf{C}_l = \mathbf{C} \otimes \mathbf{I}_{l \times l}$, where $\otimes$ indicates the matrix Kronecker production, and $\mathbf{I}_{l\times l}$ is an $l \times l$ identity matrix. Then λ_i are the eigenvalues of $\mathbf{C}_l$ with a multiplicity of $\mu_i l$.

The proof is given in appendix I.

With the help of the above theorem, we only need to consider the spread of the eigenvalues of $\mathbf{C}_r = \{\rho_{i,j}\}$ with a size of $N_rN_t \times N_rN_t$, instead of $\mathbf{C}_{\vec{r}}$ with a size of $2N_rN_tm \times 2N_rN_tm$. Note that $\rho_{\gamma_i,\gamma_j} = \rho_{i,j}^2$ because $m_i = m_j$. The eigenvalues of $\mathbf{C}_{\vec{r}}$ in (17) are given the the eigenvalues of $\mathbf{C}_{\vec{r}}$ multiplied with $1/(2mN_rN_t)$, with a multiplicity of $2m$.

IV. EVALUATION OF DIVERSITY PERFORMANCE

Based on an upper bound of the SEP performance, a measure of the diversity performance of the UWB impulse radio MIMO system with correlated propagation channels is derived in this section.

[2]It is slightly different from the corresponding equation, equation (2), in [15]. However, the author believes that the derivation in this paper is correct.

A. Average SEP of the system

For anti-podal modulation BPSK scheme, the average SEP given as

$$\begin{aligned} P_e &= E\{P_{ins}(\mathrm{SNR_r})\} \\ &= \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=0}^{N-1}\left(\frac{\sin^2\theta}{\sin^2\theta+2\lambda_i\cdot\mathrm{SNR_t}}\right)^{1/2}d\theta \end{aligned} \tag{23}$$

where $P_{ins}(\mathrm{SNR}_r)$ is the instant SEP, λ_i and N are the non-zero eigenvalues and the rank of the sparse covariance matrix $\mathbf{C}_{\vec{r}}$, respectively. Note that under the assumption that all transmit-receiver antenna links have the same m-parameter m, N is the product of the rank of the covariance matrix $\mathbf{C}_r$ and $2m$.

B. Upper Bound on SEP

Since λ_i is non-zero, an upper bound on the SEP can be given as

$$\begin{aligned} P_e &< \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=0}^{N-1}\left(\frac{\sin^2\theta}{2\lambda_i\cdot\mathrm{SNR_t}}\right)^{1/2}d\theta \\ &= K\cdot{\mathrm{SNR_t}}^{-N/2}\cdot d_\lambda \end{aligned} \tag{24}$$

where

$$K=\pi^{-1}2^{-N/2}\int_0^{\pi/2}\sin^N\theta d\theta \tag{25}$$

$$d_\lambda=\left(\prod_{i=0}^{N-1}\lambda_i\right)^{-1/2} \tag{26}$$

and

$$\int_0^{\pi/2}\sin^N\theta d\theta=\begin{cases}\frac{(N-1)!!}{N!!}\cdot\frac{\pi}{2}, & N \text{ is even}\\ \frac{(N-1)!!}{N!!}, & N \text{ is odd}\end{cases} \tag{27}$$

$(N)!!$ is given as

$$N!!=\begin{cases}N\cdot(N-2)\cdots 1, & N \text{ is odd}\\ N\cdot(N-2)\cdots 2, & N \text{ is even}\end{cases} \tag{28}$$

The tightness of the upper bound can be seen in Figure 1. Note that if any λ_i approaches 0, it is required that $\mathrm{SNR_t}$ grows large enough to ensure the tightness.

C. A Measure for the Diversity Performance of UWB Impulse Radio MIMO System

Now we derive a measure for the diversity performance of UWB impulse radio MIMO system. According to the arithmetic-geometric inequality [19], d_λ satisfies the following inequality

$$d_\lambda=\left(\prod_{i=0}^{N-1}\lambda_i\right)^{-1/2}\geq\left(\frac{1}{N}\sum_{i=0}^{N-1}\lambda_i\right)^{-N/2}=\left(\frac{1}{N}\right)^{-N/2} \tag{29}$$

with equality iff λ_i are all of the same value. When N is fixed and $\mathrm{SNR_t}\to\infty$, the logarithmed ratio of the P_e with MIMO correlated propagation channels to that with uncorrelated MIMO propagation channels, P_λ, can be expressed as

$$P_\lambda=-\frac{1}{2}\sum_{i=0}^{N-1}\log_{10}\lambda_i+\frac{N}{2}\log_{10}\frac{1}{N} \tag{30}$$

The parameter P_λ is the order of the difference between the SEP performances with zero correlation and with non-zero correlation when $\mathrm{SNR_t}\to\infty$. The degraded SEP due to the abscence of the correlation is $10P_\lambda$ dB.

When $\mathrm{SNR_t}\to\infty$, the increased $\mathrm{SNR_t}$ to maintain the same SEP performance as if there is no correlation between the MIMO propagation channels can be given as

$$\mathrm{SNR_s}=\frac{P_\lambda}{10\tan\theta}=-\frac{1}{20N}\sum_{i=0}^{N-1}\log_{10}\lambda_i+\frac{1}{20}\log_{10}\frac{1}{N} \tag{31}$$

where $\tan\theta=N$.

V. The Spread of the Eigenvalues of the Covariance Matrix

As shown in the previous section, the measure of the diversity performance of the UWB MIMO system, P_λ, is a function of the eigenvalues of the covariance matrix $\mathbf{C}_{\vec{r}}$, which is simplified to $\mathbf{C}_r$ in this paper. In this section, two different mathematical models for the matrix $\mathbf{C_r}$ that are of practical interest are presented. Eigenvalues of $\mathbf{C_r}$ for these two special cases are analysed, thus a link is built up between the diversity performance, P_λ, and the correlation coefficient, ρ_{γ_i,γ_j}.

A. Same $\rho_{i,j}$ for All the Transmit-Receive Antenna Links

The MIMO propagation channels experience similar correlation when the antennas are closely placed [16]. Let $\rho_{i,j}=\rho$ for $i,j=1\cdots N_rN_t$ and $i\neq j$, then $\mathbf{C}_r$ becomes a circulant matrix as follows

$$\mathbf{C}_{cir}=\begin{bmatrix}1&\rho&\cdots&\rho\\ \rho&1&\cdots&\rho\\ \vdots&\vdots&\ddots&\vdots\\ \rho&\rho&\cdots&1\end{bmatrix} \tag{32}$$

The eigenvalues of a circulant matrix can be given by the discrete Fourier transform (DFT) of its first row [17]. Thus, the N_rN_t eigenvalues of $\mathbf{C}_{cir}$ in (32) are given as

$$\lambda_i=\begin{cases}[1+\rho(N_rN_t-1)], & i=0\\ (1-\rho), & i=1,2...N_rN_t-1\end{cases} \tag{33}$$

and $\frac{\lambda_i}{2mN_rN_t}$ is the eigenvalues of $\mathbf{C}_{\vec{r}}$ in (14) with a multiplicity of $2m$, according to theorem 1. Substituting $\frac{\lambda_i}{2mN_rN_t}$ into (30)

$$P_\lambda=-m\log_{10}[1+\rho(N_tN_r-1)]-m(N_tN_r-1)\log_{10}(1-\rho) \tag{34}$$

where $\rho=\sqrt{\rho_{\gamma_i,\gamma_j}}$, $i,j=1,2...N_rN_t, i\neq j$.

As shown by the measurement results in [20], the m-parameter of the Gamma pdf in (5) range from 10 to 39, depending on the number of rake fingers used and the propagation environment. Thus the size of the covariance matrix

$\mathbf{C}_{\tilde{\mathbf{r}}}$ can range from several tens to several hundreds, depending on the number of the transceiver antennas. Figure 1 shows the SEP and its upper bound as a function of the transmit SNR_t. The m-parameter of the channel is set to be 20. $N_r \cdot N_t$ is set to be 2, which means that it can be 2 transmitters and 1 receiver, or 1 transmitter and 2 receivers. It can be seen that the existence of the correlation between the transmit-receive antenna links can cause a parallel shift of the upper bound of the SEP. Both P_λ and SNR_s can be used to described this shift, and their relation is given by (31).

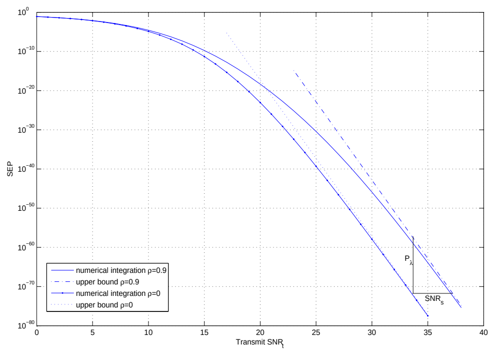

Fig. 1. SEP and its bounds as a function of the transmit SNR. m=20.

Figure 2 shows P_λ as a function of the correlation coefficient, $\rho_{\gamma_i,\gamma_j} = \rho_{i,j}^2$, between the energies of different channels. It can be seen that the existence of the correlation between the transmit-receive antennas links can bring huge difference on the SEP performance, which also depends on the m-parameter and the value of$N_r N_t$.

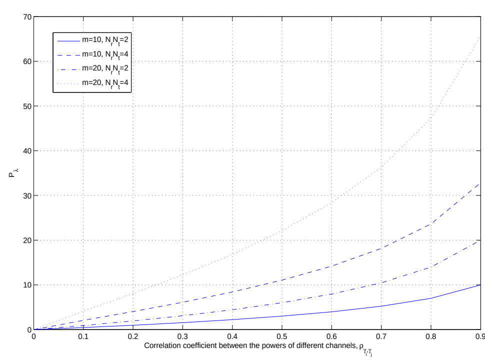

Fig. 2. P_λ as a function of the correlation coefficient, ρ_{γ_i,γ_j}, between the powers of different channels.

B. $\rho_{i,j}$ obey the Bessel function

In this subsection, the correlation coefficients $\rho_{i,j}$ are modeled by a Bessel function of the first kind with 0^{th} order [19]

$$\rho(d_{i,j}) = J_0(2\pi d_{i,j}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\cdot 2\pi d_{i,j}\cdot \sin\theta} d\theta \tag{36}$$

where $d_{i,j}$is the parameter of $\rho(d_{i,j})$ between the i^{th} and j^{th} branch. Note that because $J_0(2\pi x)$ is symmetric about the ordinate, that is, $\rho(x) = \rho(-x)$. The correlation coefficient, ρ_{γ_i,γ_j}, of the energies from different transmit-receive antenna links can be given as

$$\rho_{\gamma_i,\gamma_j} = J_0^2(2\pi d_{i,j}) \tag{37}$$

Since the positions of the antennas are discrete in space, it is reasonable to discretisize $d_{i,j}$ in (37) as follows

$$t_k = \rho(k \cdot d_{\nu 0}) \quad k \in \mathbb{N} \tag{38}$$

where $d_{\nu 0}$ is the resolution of $d_{i,j}$.

Further assume that $d_{i,j}$ between the k^{th} and the l^{th} branch is given by $|k-l|d_{\nu 0}$. Thus the covariance matrix $\mathbf{C}_r$ becomes a Toeplitz matrix as followed:

$$\mathbf{C}_{toe}(k,l) = t_{l-k} \tag{39}$$

with eigenvalues asymptotically given by [17][3]

$$\lambda_{n,l} = g(f_s l/n) \tag{40}$$

where $n = N_t N_r$, $l = 0, 1, \cdots, N_t N_r - 1$, $f_s = 1/d_{\nu 0}$, and $g(f)$ is the Fourier transform of the sampled Bessel function $t_k = \rho(k \cdot d_{\nu 0})$, with the expression given by

$$g(f) = \frac{1}{\pi d_{\nu 0}} \sum_{k=-\infty}^{\infty} \frac{u(f - kf_s + 1) - u(f - kf_s - 1)}{\sqrt{1 - (f - kf_s)^2}} \tag{41}$$

where $u(x)$ is the step function given as:

$$u(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{42}$$

The expression of P_λ as a function of $d_{\nu 0}$ can be given as equation (35) at the top of next page, where $\lceil x \rceil$ indicates the smallest integer no less than x.

VI. Conclusion and Future Work

The virtual-branch technique is applied in this paper to deconstruct the MIMO propagation channel for the MIMO UWB impulse radio system. A measure, P_λ, based on the upper bound on the SEP performance, is proposed to evaluate the diversity performance of the UWB impulse radio system equipped with different numbers of transceiver antennas and undergoing different degrees of correlations. The general expression of P_λ is a function of the non-zero eigenvalues of the covariance matrix of the MIMO channels. The exact expressions of P_λ are given for two special channel covariance matrix structures.

The result of this paper will serve as one of the elements to build up the propagation channel models for the MIMO UWB impulse radio system with an arbitrary correlation.

Acknowledgements

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[3]In [17], the inverse Fourier kernel used is $\exp[-jd\tau]$ rather than $\exp[jd\tau]$. However, this would not change the properties of the Toeplitz matrix applied in this paper.

$$P_\lambda(d_{\nu 0}) = \begin{cases} m \sum_{l=1}^{\lceil d_{\nu 0}\cdot N_r N_t\rceil -1} \log_{10}\left(1 - \frac{l^2}{d_{\nu 0}^2 N_r^2 N_t^2}\right) + m(2\lceil d_{\nu 0}\cdot N_r N_t\rceil - 1)\log_{10}\frac{\pi d_{\nu 0} N_t N_r}{2\lceil d_{\nu 0}\cdot N_r N_t\rceil - 1} & \text{for } \frac{1}{2d_{\nu 0}} > 1 \\ -m \sum_{l=0}^{N_r N_t - 1} \log_{10}\left(\sum_{k=-\infty}^{\infty} \frac{u(\frac{l}{d_{\nu 0}\cdot N_r N_t} - \frac{k}{d_{\nu 0}} + 1) - u(\frac{l}{d_{\nu 0}\cdot N_r N_t} - \frac{k}{d_{\nu 0}} - 1)}{\sqrt{1 - (\frac{l}{d_{\nu 0}\cdot N_r N_t} - \frac{k}{d_{\nu 0}})^2}}\right) + m N_t N_r \log_{10}(\pi d_{\nu 0}) & \text{for } \frac{1}{2d_{\nu 0}} \le 1 \end{cases} \tag{35}$$

APPENDIX I

The proof of the theorem 1 is given here. The eigenvectors of a Hermitian matrix are orthogonal. Let $\vec{v}_i$ be one of the μ_i eigenvectors corresponding to λ_i.

$$\mathbf{C}\cdot\vec{v}_i = \lambda_i\cdot\vec{v}_i \tag{43}$$

Based on vector $\vec{v}_i$, we construct a matrix $\mathbf{V}_i$:

$$\mathbf{V}_i = \vec{v}_i \otimes \mathbf{I}_{l\times l} \tag{44}$$

where $\otimes$ indicates the Kronecker product. It is obvious that

$$\mathbf{V}_i^H\cdot\mathbf{V}_k = 0 \quad \text{for} \quad i \neq k \tag{45}$$

Then we have

$$\begin{aligned} \mathbf{C}_l\cdot\mathbf{V}_i &= (\mathbf{C}\otimes\mathbf{I}_{l\times l})(\vec{v}_i\otimes\mathbf{I}_{l\times l}) \\ &= (\mathbf{C}\cdot\vec{v}_i)\otimes(\mathbf{I}_{l\times l}\cdot\mathbf{I}_{l\times l}) \\ &= \lambda_i\cdot\vec{v}_i\otimes\mathbf{I}_{l\times l} \\ &= \lambda_i\cdot\mathbf{V}_i \end{aligned} \tag{46}$$

It is easy to show that because $\mathbf{C}$ is Hermitian, $\mathbf{C}_l$ is also Hermitian. Based on matrix $\mathbf{V}_i$, we now construct the orthogonal eigenvectors, $\vec{v}_{i,j}$, for $\mathbf{C}_l$. Let $\vec{p}_j$ be an $l\times 1$ vector with the j^{th} entry being 1, and all the other entries being 0s. Construct $\vec{v}_{i,j}$ as

$$\vec{v}_{i,j} = \mathbf{V}_i\cdot\vec{p}_j \tag{47}$$

Based on equation (45), it can be shown that

$$\vec{v}_{i,p}^H\vec{v}_{k,q} = 0 \quad \text{for} \quad p\neq q \quad \text{or} \quad i\neq k \tag{48}$$

Multiplying both sides of equation (46) by $\vec{p}_j$ results in

$$\mathbf{C}_l\cdot\vec{v}_{i,j} = \lambda_i\cdot\vec{v}_{i,j} \tag{49}$$

where $1\le i\le\mu_i$, and $1\le j\le l$.

Thus we have λ_i being one of the eigenvalues of $\mathbf{C}_l$ with a multiplicity of $\mu_i l$

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