Concatenated RS-Convolutional Codes for Ultrawideband Multiband-OFDM

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***Abstract* — In this paper, we investigate the performance of concatenated Reed Solomon and convolutional (RS-convolutional) codes for ultrawideband (UWB) multiband orthogonal frequency division multiplexing (MB-OFDM) systems. We show that RS-convolutional codes perform better than single codes at high energy-per-bit to noise ratio (E_b/N_o). We also show that using a bit interleaver between the two concatenated codes degrades the system performance. Our simulations indicate that 10-bit and 11-bit RS-convolutional codes outperform single codes at high E_b/N_o, with gains of more than 4 dB. We further demonstrate that 11-bit RS-convolutional codes outperform turbo codes at high E_b/N_o by more than 2 dB in various UWB channels.**

***Index Terms* — Channel coding, concatenated coding, convolutional codes, interleaved coding, MB-OFDM, Reed-Solomon codes, ultrawideband.**

I. Introduction

There is a demand for indoor wireless short-range services such as home video networking which require high data rate, low cost, low power consumption and high quality of service systems. Ultrawideband (UWB) multiband orthogonal frequency division multiplexing (MB-OFDM), a proposal within IEEE 802.15.3 physical layer offers promising results with data rates as high as 480 Mbps [1]. The UWB scheme in [1] utilizes the 3.1 GHz to 10.6 GHz unlicensed band allocated by the Federal Federal Communications Commission (FCC) in the United States to UWB radio systems [2].

The error rate performance of MB-OFDM under various channel conditions has been presented in [1, 3, 4]. In the MB-OFDM scheme, channel coding is restricted to convolutional codes only. It is well known, however, that the concatenation of two or more codes leads to more robust channel codes than single codes [5, 6]. Usually, the concatenation involves block codes (outer codes) and convolutional codes (inner) [7]. In [8], the bit error rate (BER) performance of concatenated RS-convolutional codes with 6, 7 and 8 bits for MB-OFDM was presented. The authors showed that single codes perform better than concatenated codes.

In this paper, we investigate the performance of 8-bit, 10-bit and 11-bit RS-convolutional codes. We present a mathematical analysis of concatenated codes to show that even though their performance is worse than single codes at low signal-to-noise ratios (SNR), we expect them to perform better than single codes at high SNR. We present simulation results to show 10-bit and 11-bit RS-convolutional codes perform better than single codes at high SNR values. Our simulations also show that 11-bit RS-convolutional codes perform better than equivalent rate turbo codes at high SNR. The benefits of RS-convolutional codes cannot be overlooked for MB-OFDM as these codes have been used in other standards (eg [9]). We also present an understanding of why the 8-bit RS-convolutional codes under-perform and suggest ways to make them perform as well as 11-bit RS codes.

The rest of the paper is organised as follows. Section II presents the system model used in the simulations. Section III is a BER analysis of single and concatenated codes. Section IV presents the simulation results and Section V states the conclusions drawn from the investigation.

II. System Model

The block diagram of the system model used in this paper is shown in Fig. 1.

A. Channel Coding

Convolutional codes can be specified as CC(n,k,m), where n is the number of output bits, k is the number of input bits, and m is the constraint length of the encoder [8]. The codes used in this paper are CC($3,1,7$) and CC($2,1,3$). They have free minimum distances of 15 and 5 respectively. Decoding is performed using the Viterbi algorithm with hard decision [6].

Reed Solomon (RS) codes are block codes that operate on multiple bits rather than individual bits [10]. An RS(n,k,m) code is used to encode k m-bit symbols into blocks consisting of $n=2^m-1$ symbols. The encoder thus consists of m x k input bits and m x n output bits. The error

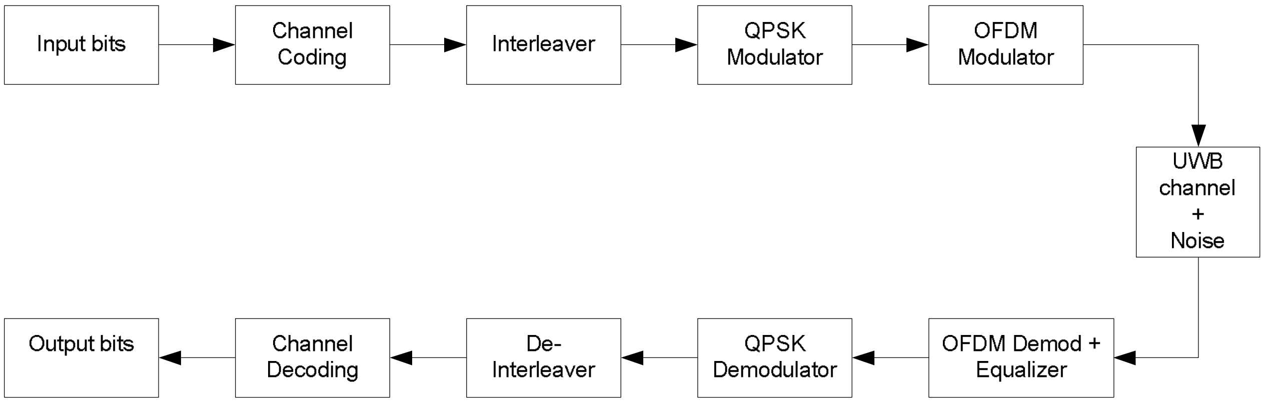

Fig. 1. MB-OFDM system block diagram

correcting capability of an RS code is measured by its minimum distance, $d_{min} = 2t + 1$, where $n\text{-}k = 2t$. The RS codes used in this paper are RS(*255,171,8*), RS(*1023,683,10*) and RS(*2047,1365,11*) whose minimum distances are 85, 341, and 683 respectively.

The turbo encoder utilized in this paper consists of two 1/2-rate recursive systematic convolutional (RSC) encoders with a random interleaver between the two encoders. The input bit stream plus two parity bits (one from each encoder) are transmitted resulting in 1/3-rate code [5]. Iterative turbo decoding using the maximum a posteriori (MAP) algorithm in 6 loops [11] is implemented. The input to this decoder are the bit log likelihood ratios (LRRs) of the received QPSK symbols.

The 53.3 Mbps data rate in the MB-OFDM scheme utilizes a 1/3-rate convolutional code. For the case of concatenated RS-convolutional codes, we concatenate 2/3-rate RS codes with 1/2-rate convolutional codes. This is so that the overall code rate is maintained at 1/3 in order to keep the throughput at 53.3 Mbps. We refer to the concatenated codes as CC(*n*,*k*,*m*)+RS(*n*,*k*,*m*) or RS-convolutional codes.

B. Interleaving

The interleaver is implemented according to the MB-OFDM standard in [1]. This includes a 100×3 symbol interleaver, a 10×10 bit interleaver and a cyclic shift.

C. Digital Baseband Modulation

Quadriphase-shift keying (QPSK) modulation with gray coding is used throughout this paper.

D. MB-OFDM Modulation

The MB-OFDM modulator is implemented according to the MB-OFDM scheme in [1]. The proposed UWB system employs OFDM with 122 modulated and pilot subcarriers out of a total of 128 subcarriers [1]. The 7.5 GHz band is divided into 4 band groups with three sub-bands and one band group with two sub-bands[12]. Each sub-band is 528 MHz wide. The MB-OFDM parameters are summarized in Table I. In the receiver, a zero forcing (ZF) equalizer is implemented. It utilizes a block type training sequence and the channel is assumed constant during one OFDM frame.

TABLE I
MB-OFDM PARAMETERS

Parameter	Value
Number of data Subcarriers	100
Number of defined pilot carriers	12
Number of guard carriers	10
Number of total subcarriers used	122
Subcarrier frequency spacing	528 MHz/128 = 4.125 MHz
FFT/IFFT period	242.42 nsec
Zero Pad duration	70.08 nsec

E. Channel Model

The statistical channel from the UWB channel modeling committee is utilized [13]. It is a block fading multipath channel model based on the Saleh-Valenzuela model [13]. There are four different types of channel models defined: CM1, CM2, CM3 and CM4. The estimated channel characteristics of the four different types of channels used in this paper can be found in [8]. The channel noise is additive white Gaussian.

III. BER ANALYSIS

For convolutional codes with hard-decision decoding, a union bound on the bit error probability at the output of the Viterbi decoder is given by [6]

$$P_b < \sum_{d=d_{free}}^{\infty} \beta_d P_2(d) \quad (1)$$

where $\{\beta_d\}$ are the coefficients in the expansion of the derivative of the convolutional code transfer function, T(D,N) evaluated at N=1, D indicates the distance of the sequence of the encoded bits for a path (in the code trellis) from the all zero path and N indicates the number of 1s in the information sequence of that path [6], and d_{free} represents the free distance of the convolutional code. The pairwise error probability, $P_2(d)$ can be estimated by the Chernoff upper bound: [6]

$$P_2(d) < \left[4p(1-p)\right]^{d/2} \quad (2)$$

with p as the bit error probability at the output of the QPSK demodulator, i.e. the uncoded bit error probability. For RS codes, the probability of bit error at the decoder output is bounded by [7]

$$P_{brs} < \sum_{i=t+1}^{n} \frac{i+t}{n} \binom{n}{t+1} p_s^i (1-p_s)^{n-i} \quad (3)$$

where p_s is the m-bit symbol error probability at the input of the RS decoder. At high E_b/N_o values, the bit error probably in (3) can be approximated by [7]

$$P_{brs} \approx \frac{2t+1}{n} \binom{n}{t+1} p_s^{t+1}. \quad (4)$$

In the case where a concatenated code consists of an outer RS code and an inner convolutional code, p_s is the m-bit symbol error probability at the output of the Viterbi decoder. A simple upper bound for p_s is given by [8]

$$p_s \le mP_b \quad (5)$$

with P_b given in (1). If we assume E_b/N_o is large enough such that $p << 1$, the first term in (1) will dominate. We can thus approximate (1) as

$$P_b \approx \beta_{d_{free}} \left[4p\right]^{d_{free}/2} . \quad (6)$$

Substituting (6) into (4), we have

$$P_{brs} = \frac{2t+1}{n} \binom{n}{t+1} \left(m\beta_{d_{free}}\right)^{t+1} \left[4p\right]^{d_{free}(t+1)/2} . \quad (7)$$

We see that (6) and (7) can be espressed in the form $Ap^{d_{free}/2}$ and $Bp^{d_{free}(t+1)/2}$ respectively, where A and B are given in the equations below:

$$A = \beta_{d_{free}} 4^{d_{free}/2} \quad (8)$$

and

$$B = \frac{2t+1}{n} \binom{n}{t+1} \left(m\beta_{d_{free}}\right)^{t+1} 4^{d_{free}(t+1)/2} . \quad (9)$$

Thus from (6) to (9), we see that the effect of the convolutional code is to offer a diversity order of $d_{free}/2$. Similarly, the diversity order provided by the concatenated code is $d_{free}(t+1)/2$. In this paper, we use a single CC(3,1,7) code whose free distance is 15. The concatenated code consists of CC(2,1,3) and 2/3-rate RS codes. The free distance of the convolutional code in the concatenated code is 5. Substituting these values in Equation (6) to (9) for 8-bit, 10-bit and 11-bit RS codes, we can derive approximate expressions for the bit error probability values for the various codes to be analyzed in this paper. The expressions are shown in Table II.

TABLE II
BIT ERROR PROBABILITY FOR VARIOUS CODES AS A FUNCTION OF UNCODED BIT ERROR RATE p

Code	BER
CC(3,1,7)	$10^6 p^{7.5}$
CC(2,1,3) + RS(255,171,8)	$10^{152} p^{107.5}$
CC(2,1,3) + RS(1023,683,10)	$10^{627} p^{425}$
CC(2,1,3) + RS(2047,1365,11)	$10^{1270} p^{852.5}$

From the various expressions in Table II, it can be seen that at high values of p (which in turn correspond to low values of E_b/N_o), convolutional codes will perform better than the concatenated codes since the coefficient term, which dominates the product in this case, is small. At low values of p, however, the concatenated codes will perform better than the convolutional code as the exponent term now dominates, and will thus make the corresponding products small. Among the concatenated codes, we expect the 8-bit RS codes to perform better than 10-bit RS codes, which in turn perform better than 11-bit RS codes at low E_b/N_o values. The inverse is true at high E_b/N_o values.

The presentation in this paper also includes turbo codes as a reference to indicate how better concatenated block and convolutional codes perform when compared to parallel concatenated convolutional codes. Parallel concatenated convolutional codes (PCCC) with interleaving, also known as turbo codes, mainly obtain their coding gain from the interleaving, which ensures a relatively sparse code, with very few nearest neighbors at the output of the encoder [6].

IV. SIMULATIONS AND RESULTS

A full system model was implemented in Matlab® according to the above described system for different channel codes. Only rate 53.3 Mbps was implemented. A set of 1000 channel realizations for all the four channel

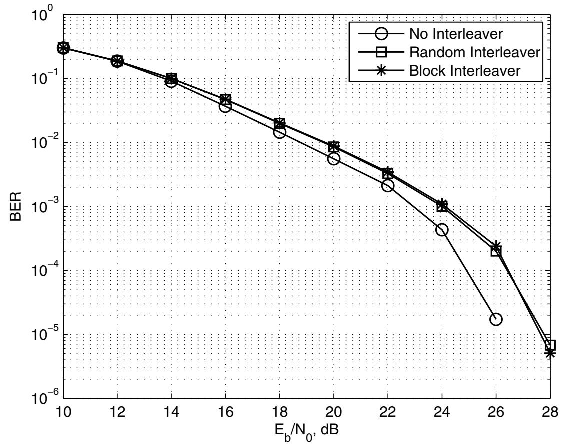

Fig. 2. Performance of concatenated RS(255,171,8) + CC(2,1,3) for different interleaver types in CM1

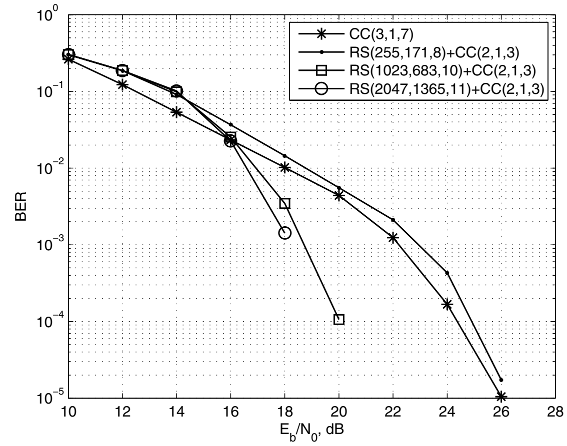

Fig. 3. Performance of convolutional codes and 8-bit, 10-bit and 11-bit RS-convolutional codes

types was used. In the following figures, E_b/N_o denotes the information bit energy to noise power density ratio.

We first investigate the use of a bit interleaver between concatenated RS and convolutional codes. An outer RS(*255,171,8*) code and an inner CC(*2,1,3*) code are used. The BER performance in CM1 is investigated. The results are shown in Fig. 2.

From Fig. 2, it can be seen that whether a block or random interleaver is used makes no difference. However, up to 1 dB less E_b/N_o is required when no interleaver is used. We therefore conclude that the use of a bit interleaver between the outer code and inner code degrades the performance of the system. The reason is that the bit interleaver, in breaking the burst errors at the output of the Viterbi decoder, only serves to spread the error to more RS symbols than before. This further confirms that the errors at the output of the Viterbi occur in bursts [8] and that a bit interleaver might actually increase the symbol error rate at the Viterbi output [14].

Next, we compare the BER performance of MB-OFDM when the channel coding is convolutional, 8-bit RS-convolutional, 10-bit RS-convolutional and 11-bit RS-convolutional codes in CM1. The results are shown in Fig. 3.

From Fig. 4, it can be seen that at low E_b/N_o values, just as expected from theory, the single code outperforms the concatenated code. At high E_b/N_o values the 10-bit and 11-bit RS-convolutional codes perform better than convolutional codes as expected. However, the 8-bit RS-convolutional code performs slightly worse than the single code at all E_b/N_o values. This is due to the number of RS symbols in error detected by the decoder, which is still more than the decoder can correct, i.e. it is larger than t+1. If the RS decoder detects more errors than it can correct, the decoded output is likely to be erroneous. We should then expect the BER performance to be worse than that of the 1/3-rate convolutional codes, since the only reliable decoding that has been done is by the inner 1/2-rate convolutional decoder. This is the reason why for low E_b/N_o values, the error performance of the concatenated code is worse than that of the single code. Consider the bits at the output of the 1/2-rate Viterbi decoder. The 8-bit RS decoder groups them into 2040 bits before decoding. The number of correctable symbol errors for a 2/3-rate 8-bit RS code is 43 symbols, giving 344 bits. Therefore, the decoder will successfully correct the detected errors if the number of erroneous bits at the output of the Viterbi decoder is at most 344 bits, situated in at most 43 symbols. Let us now consider an 11-bit RS decoder. In a group of 2047 symbols, equivalent to 22517 bits, at most 3762 bits should be in error for successful decoding. These errors should be located in at most 342 symbols. Obviously, the 11-bit decoder has a higher chance of having the number of symbol errors being within its correctable range than 8-bit codes. We can thus conclude that the coding gain from the 11-bit codes is from the large minimum distance of the code, which in turn is able to handle the errors at the output of the Viterbi decoder. This is the reason the error goes to zero at high E_b/N_o values since at these E_b/N_o values, the number of erroneous symbols in a codeword is less than t+1. This results in perfect recovery of the transmitted information sequence, and the bit error rate is zero. This analysis gives an insight into ways of making 8-bit RS-convolutional codes perform better than single codes. We could insert a codeword interleaver between the two concatenated codes. The effect of this interleaver would be to increase the minimum distance of the code. For example, we could utilize a block interleaver with a depth of 10 that interleaves among 10 8-bit RS codewords.

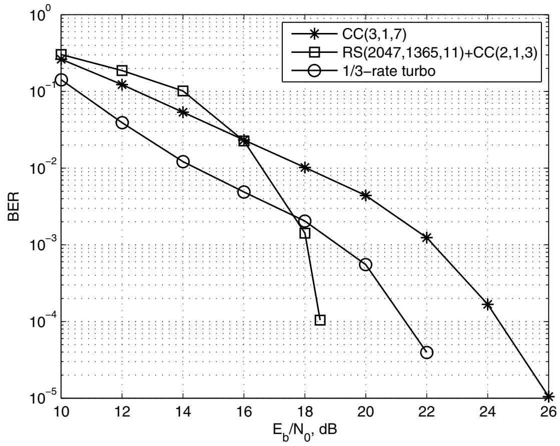

Fig. 4. Performance of convolutional codes, 11-bit RS-convolutional codes and turbo codes in CM1

This would increase the correction capability of the code to 430 symbols. The performance of the 8-bit RS codes paid in all this coding gain is, of course, the large delays introduced. In the case of 11-bit RS codes, the delay is about 25 OFDM frames or 328 µsec.

We further investigate the BER performance of CC(*3,1,7*), concatenated RS(*2047,1365,11*) and CC(*2,1,3*) and 1/3-rate turbo codes for CM1, CM2, CM3, and CM4 to show that the above analysis is valid for all UWB channel environments. Fig. 4 shows the results for CM1.

For low E_b/N_o, the convolutional codes have a 2 dB loss over turbo codes and up to a 2 dB gain over the concatenated code. At high E_b/N_o, however, the concatenated code outperforms both turbo and convolutional codes. For example, at E_b/N_o = 18.5 dB, the concatenated code has a gain of about 6 dB over the convolutional codes and about 3 dB over the turbo code. At an E_b/N_o of 20 dB, the RS decoder is able to detect and correct all the errors coming out of the Viterbi decoder. This is because the number of 11-bit symbols in error are now less than or equal to t+1. We thus have perfect recovery of the information bits. Compare this with turbo and convolutional codes whose BER values at 20 dB are 6×10^{-4} and 4×10^{-3} respectively. We observe a similar trend for CM2, CM3, and CM4 whose graphs are shown in Figure 5, Figure 6, and Figure 7 respectively. Therefore, the discussion above is also valid for all four channel types.

For CM4, we notice that the BER curves for convolutional and turbo codes floor. This is due to the large delay spreads associated with this NLOS fading multipath channel. This suggests that the zero pad length currently in use in the MB-OFDM scheme is not long enough to capture all the inter-symbol interference for the CM4 channel.

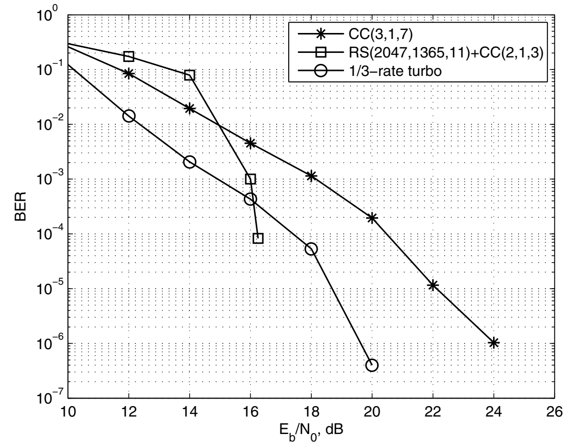

Fig. 5. Performance of convolutional codes, 11-bit RS-convolutional codes and turbo codes in CM2

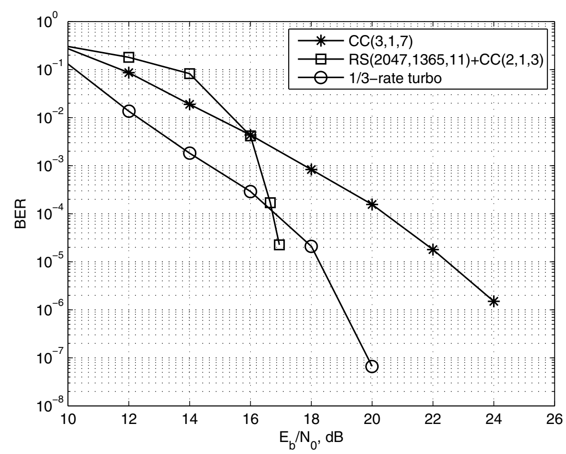

Fig. 6. Performance of convolutional codes, 11-bit RS-convolutional codes and turbo codes in CM3

V. Conclusion

The evaluation of the performance of a concatenation of a 2/3-rate RS code with 1/2-rate convolutional code was the focus of this paper. The analysis showed that concatenated codes perform better than single codes at high SNR. Simulation results also showed that the addition of either a block or a random interleaver between an outer RS code and inner convolutional code require up to 1 dB more E_b/N_o to achieve the same BER performance as for the case with no interleaver. This indicated that the interleaver spreads the burst errors coming out of the Viterbi decoder to even more RS symbols than before,

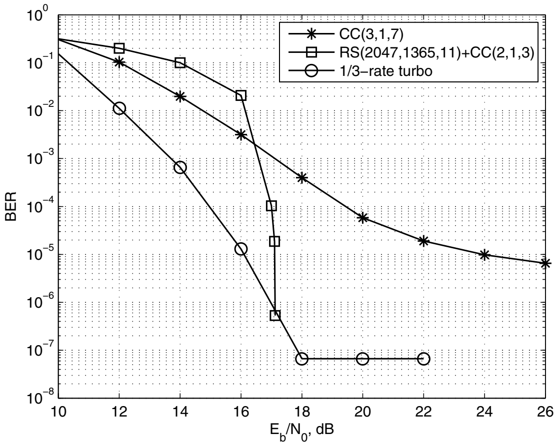

Fig. 7. Performance of convolutional codes, 11-bit RS-convolutional codes and turbo codes in CM4

hence the degradation in performance. The investigation further showed that there is no gain in using concatenated 8-bit RS-convolutional codes over convolutional codes at all the simulated E_b/N_o values. However, a gain of more than 4 dB is noticed when 10-bit and 11-bit RS-convolutional codes are employed at for high E_b/N_o values in CM1. Similar gains are observed for the other three channel types. The 11-bit RS-convolutional codes also out-perform turbo codes in this region. It can thus be concluded that a system operating in the 10 dB to 26 dB E_b/N_o range utilizing only convolutional codes will have a higher time averaged BER than a system utilizing concatenated RS-convolutional codes. This is because the concatenated code will result in a very low BER for E_b/N_o values greater than 18 dB. The analysis in this paper thus demonstrates that the use of a concatenated code will improve UWB system reliability and performance for an MB-OFDM system operating over a diverse SNR range. Future work will include analysis of the performance of the above codes for various code rate combinations of RS and convolutional codes as well as for other data rates.

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