MIMO capacity and multipath scaling in ultra-wideband channels

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Generalising previous results for narrowband multiple-antenna channels, a quadratic growth in multipath with the array size is reported for ultra-wideband indoor channels, based on experimental analysis. It is shown that the observed increase in multipath is sufficient to guarantee multiple-antenna capacity growth linearly with the array size, facilitating multi-gigabit future wireless systems.

Introduction: An ultra-wideband (UWB) system, with a bandwidth of over 500 MHz, has the ability to resolve most of the multipath signal incident at the receiver [1]. It is well known that multipath, or memory, boosts the channel capacity considerably [2]. UWB systems are therefore able to offer very high throughputs, with the current prototypes offering rates of several megabits per second at a short range. The information rate and coverage radius can be enhanced significantly with the use of multiple-antenna techniques. Spatial multiplexing, implemented by means of a multiple-input multiple-output (MIMO) antenna array configuration and appropriate signal coding, can increase the rate by orders of magnitude [3]. In a narrowband Rayleigh fading channel, the capacity gain of an $N_T \times N_R$ MIMO spatial multiplexing system is $\min\{N_T, N_R\}$. This rate enhancement is, however, conditional over dense multipath with uncorrelated fading. The number of resolvable multipath components (MPCs) is thus an important criterion for the prediction of MIMO performance. In a wideband multipath channel with L taps, the MIMO spatial multiplexing gain is upper-bounded by [3, 4]

$$\lim_{\rho\to\infty} \frac{C_{MIMO}(\rho)}{\log_{10}\rho} \le \min\{N_T, N_R, L\} \tag{1}$$

where C_{MIMO} is the MIMO capacity and ρ is the signal-to-noise ratio (SNR). According to this relation, the capacity of a multipath-rich MIMO channel, with $L \to \infty$, is limited only by the array configuration, $N_T \times N_R$. In this Letter, we will restrict our analysis to symmetric array dimensions, i.e. $N_T = N_R = N$, and to small arrays, i.e. $N \le 3$, as they are more practical for typical applications.

For C_{MIMO} to scale linearly with the array size in a Rayleigh channel, L must undergo a quadratic increase with N [5]. Intuitively, this requirement implies a dense scattering environment and sufficiently high multipath resolution. Our aim is to investigate the applicability of this result to indoor UWB channels, which do not generally follow Rayleigh amplitude statistics [1]. We are interested in the interplay between the UWB channel multipath, MIMO array size, fading correlation and achievable information rates.

Analysis method: We consider an $N \times N$ UWB system and represent the corresponding MIMO channel matrix by $\boldsymbol{H} \in \mathcal{C}^{N\times N\times N_F}$, with $\boldsymbol{H} = \{\boldsymbol{H}_f\}$ and $f = \{1, \ldots, N_F\}$, where $\mathbf{H}_f$ is the flat-fading MIMO channel matrix at frequency f, and N_F is the number of discrete frequency components of the UWB channel. Similar to [4], the UWB channel in our analysis spans the 3.1–10.6 GHz band allocated by the Federal Communications Commission (FCC), USA. A bandwidth of 500 MHz defines the lower limit on a UWB channel, under the current FCC regulations. We will consider the maximum bandwidth of 7.5 GHz in the allocated band, with $N_F = 1601$. As bandwidth is directly related to multipath resolution, this approach allows us to examine the maximum available channel memory. The frequency-domain spatial MIMO channel measurements, conducted in line-of-sight (LOS) and obstructed LOS (OLOS) environments, used for the analysis in this Letter, are described in [4]. The mean MIMO sub-channel fading correlation is then 0.5. With this procedure, a large ensemble of UWB MIMO channel realisations is obtained for each value of N at various indoor locations. The measured UWB channel matrix is then normalised such that each dimension has unit mean power, or $\|\boldsymbol{H}\|_F^2 = N^2 N_F$, where $\|.\|_F$ denotes the matrix Frobenius norm.

After converting the frequency-domain data to the time domain using the inverse discrete Fourier transform, a 25 dB noise threshold below the peak MPC amplitude is applied, followed by local maxima detection to identify the MPCs. The number of MPCs, $L_{N\times N}$, in the MIMO channel is calculated as the aggregate of the number of MPCs in the corresponding SISO sub-channels, i.e.

$$L_{N\times N} = \sum_{p,q=1}^{N} L_{p\times q}$$

Results: We now study the growth of $L_{N\times N}$ and C_{MIMO} with N, using the measured channel data. From [5], $L_{N\times N} = N^2 L_{1\times 1}$ is a necessary condition for $C_{N\times N} = NC_{1\times 1}$ in a narrowband MIMO channel. To verify this condition in the measured UWB channel, we first statistically analyse $\sqrt{L_{N\times N}}/N$, which we expect to be equivalent to $\sqrt{L_{1\times 1}}$. Fig. 1 shows the cumulative distribution function (CDF) of this quantity over the measurement ensemble for various array dimensions. A larger number of MPCs are received and resolved in the OLOS channel than in the LOS channel. This is due to greater scattering in the former and also its lower noise threshold in the absence of the direct component. The mean $\sqrt{L_{N\times N}}/N$ is approximately independent of N, and its numerical value is found to be equal to $\sqrt{L_{1\times 1}}$. The number of MPCs per spatial dimension thus remains approximately constant at 74 and 88 in LOS and OLOS scenarios, respectively, which is typical of a dense multipath channel with high temporal resolution. Therefore it can be concluded from our measured data that the condition of quadratic growth of channel memory with array size is satisfied.

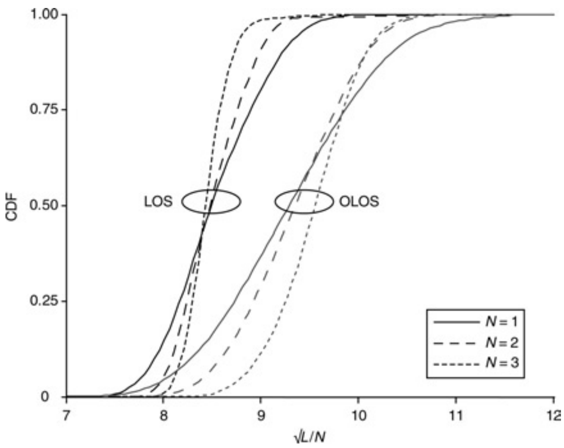

Fig. 1 *Cumulative distribution function of square root of multipath components per antenna, $\sqrt{L}/N$, in $N \times N$ MIMO UWB channels*

Next, we analyse the variation of the MIMO capacity with multipath. From the previous discussion, if the capacity-multipath relation is valid for the UWB channel, we should have for each MIMO channel realisation the relation

$$C_{N\times N} = C_{1\times 1}\sqrt{\frac{L_{N\times N}}{L_{1\times 1}}} \tag{2}$$

i.e. the capacity of the $N \times N$ MIMO channel should scale with the root of the number of MPCs. Assuming perfect channel state information (CSI) at the receiver but not at the transmitter, the capacity of the $N \times N$ UWB system can be evaluated as [4]

$$C_{N\times N} = \frac{1}{N_F}\sum_{f=1}^{N_F} \log_2 \det\left(I_N + \frac{\rho}{N}\boldsymbol{H}_f\boldsymbol{H}_f^{\dagger}\right) \tag{3}$$

We assume the average received SNR per dimension, ρ, to be 10 dB, and use (3) to evaluate the capacity for our MIMO channel measurement ensemble for each value of N. We then estimate the CDF of the capacity per root path, $C/\sqrt{L_{N\times N}}$. As shown in Fig. 2, the mean $C/\sqrt{L_{N\times N}}$ decreases only marginally as N increases, suggesting that the MIMO capacity scales approximately linearly with N. The small decrease can be attributed to the redisual sub-channel correlation that introduces a sublinearity into the MIMO capacity scaling [3]. The capacity per root path is lower in the OLOS channel than in the LOS channel because of the larger number of MPCs and lower average MPC energy in OLOS. To further investigate this effect, we analyse the capacity per spatial dimension, C/N, and find that its mean is not significantly affected by an increase in N, while the variance is reduced slightly owing to the increased channel hardening by antenna diversity.

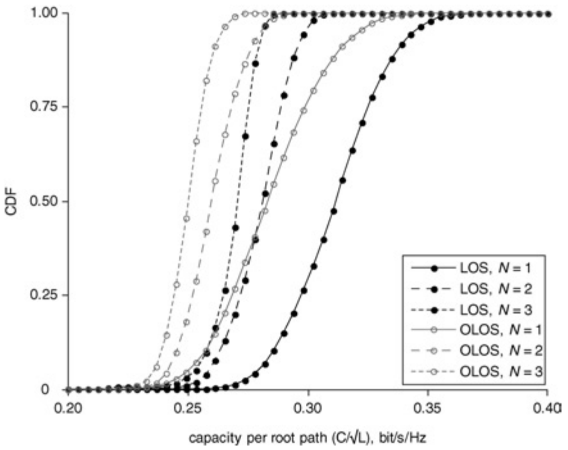

Fig. 2 *Cumulative distribution function of MIMO capacity per root path in $N \times N$ UWB channel*

We finally investigate the scaling of the mean MIMO capacity with N with the help of Fig. 3. For each measured MIMO channel, we estimate $L_{N\times N}$, and $C_{N\times N}$ using (3). The corresponding points are shown in the Figure using a scatter plot. In the OLOS channel, for a given N, the capacity can be seen to vary directly with $\sqrt{L_{N\times N}}$. This dependence is less obvious in LOS, where the $\sqrt{L_{N\times N}}$ variation for a given N across the measurement ensemble is smaller owing to the reduced impact of random scattering. The regression between the entire set of $\sqrt{L_{N\times N}}$ and $C_{N\times N}$ is also estimated, and is shown in the Figure. It is noticed that the capacity does indeed increase almost linearly with $\sqrt{L_{N\times N}}$, or with N, for the small arrays in this analysis. The slope of the LOS and OLOS regression lines are found to be 0.25 and 0.23 $\mathrm{bit/s/Hz/path}^{1/2}$, respectively. Additionally, the MIMO capacity upper bound, with an N-fold spatial multiplexing gain over the SISO channel, is also shown in terms of the $\overline{\sqrt{L_{N\times N}}}$ against the $C^{(ub)}_{N\times N} = N\overline{C_{1\times 1}}$ line, the slope of which is 0.31 and 0.28 $\mathrm{bit/s/Hz/path}^{1/2}$ in LOS and OLOS, respectively. From these results, we see that the UWB MIMO capacity nearly achieves the ideal capacity growth bound when the multipath grows quadratically.

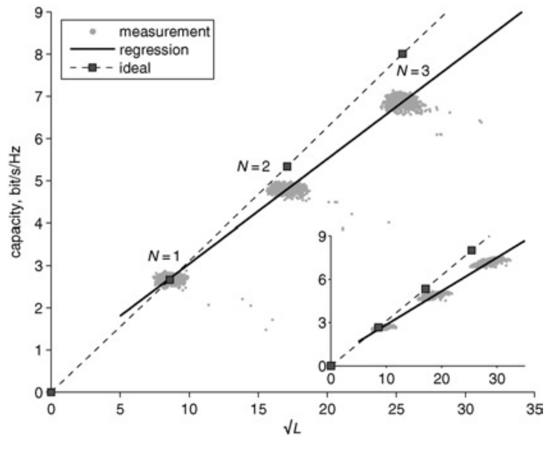

Fig. 3 *Scaling of MIMO capacity, C, with square root of paths, $\sqrt{L}$, in LOS UWB channel with $N \times N$* array configuration

Inset: results for OLOS channel

Conclusions: The above analysis demonstrates that the resolvable multipath obtained in the indoor UWB channel increases quadratically with the MIMO array size, N. Under this condition, the capacity is shown to increase almost linearly with N, and thus the relation between multipath and capacity scaling is established. From the perspective of system performance and design, these results establish that a small MIMO array can dramatically increase the UWB system capacity in a rich scattering environment. MIMO UWB techniques thus offer a means to realise practical multi-gigabit wireless networks.

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