

## Simultaneous mid-range power transfer to multiple devices

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Electromagnetic resonators strongly coupled through their near-fields [A. Karalis, J. D. Joannopoulos, and M. Soljačić, *Ann. Phys.* **323**, 34 (2008); A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, *Science* **317**, 83 (2007)] are able to achieve efficient wireless power transfer from a source to a device separated by distances multiple times larger than the characteristic sizes of the resonators. This midrange approach is therefore suitable for remotely powering several devices from a single source. We explore the effect of adding multiple devices on the tuning and overall efficiency of the power transfer, and demonstrate this scheme experimentally for the case of coupling objects of different sizes: a large source (1 m<sup>2</sup> in area) powering two smaller devices (each ≈0.07 m<sup>2</sup> in area). © 2010 American Institute of Physics.

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There has been substantial interest<sup>1-4</sup> recently in exploiting high  $Q$  electromagnetic resonances interacting via their near (i.e., nonradiative) electromagnetic fields as a means of transferring power wirelessly and efficiently over distances up to several times the characteristic sizes of the resonators. In terms of applications, this scheme could fill the gap between traditional inductive systems, which require the source and device to be separated by a gap smaller than their characteristic sizes, and radiative systems, which may be efficient at large distances if the electromagnetic radiation is properly collimated, but are sensitive to obstructions between source and device, require complicated tracking mechanisms if relative positions change, and pose more stringent safety concerns.<sup>5</sup>

In such a midrange strongly coupled approach,<sup>1</sup> a single source will couple strongly to devices placed within a volume several times larger than the volume of the source itself. One potentially important application enabled by this feature is using one source (or a small number of sources) to power a larger number of devices simultaneously. If the system is properly tuned, the interactions between resonators may be exploited so that the overall efficiency of the power transfer is significantly larger than the efficiency of the transfer from the source to each device in isolation.

Since our resonators have high quality factors ( $Q$ ), it is a good approximation to use the coupled-mode theory equations<sup>6</sup>

$$\dot{a}_m(t) = -[i\omega_m + (1 + x_m)\Gamma_m]a_m(t) + \sum_{n \neq m} i\kappa_{mn}a_n(t) + F_m(t), \quad (1)$$

where the indices label the different resonators. The complex-valued mode variables  $a_m(t)$  are normalized so that the energy contained in resonator  $m$  is  $|a_m(t)|^2$ ,  $\omega_m$  is the resonant angular frequency of the mode,  $\Gamma_m$  its intrinsic loss rate,  $x_m$  the normalized external loading that extracts the energy stored in mode  $m$  so it can be converted into useful work, and the  $\kappa_{mn}$  are real-valued coupling coefficients rep-

resenting the mutual interaction between the resonators.  $F_m(t)$  is the external driving term.

In the general case, finding a solution to Eq. (1) involves inverting an  $N \times N$  matrix ( $N$  being the total number of resonators involved), which can lead to a wide variety of phenomena.<sup>7</sup> For definiteness, we take a single resonator to be the source for the whole system, and label it 0 [so that in Eq. (1), only  $F_0(t) \neq 0$  and  $x_0=0$ ]. In the case where the source is much larger than the devices and the devices are not particularly clustered together, the mutual coupling between devices is much smaller than the direct coupling between the source and each device and we may neglect all off-diagonal terms in Eq. (1) except for  $\kappa_{0n}=\kappa_{n0}$  (where  $n=1, \dots, N-1$ ). This is precisely the regime of interest of this letter, and we show below that this is indeed satisfied for the system we study. The total power consumed at the resonator  $m$  is  $2(1+x_m)\Gamma_m|a_m|^2$ , of which a fraction  $x_m/(1+x_m)$  is sent into the resonator's load (which may in principle convert all of that power into useful work). We define the overall efficiency of the system as the ratio of the total power delivered to all loads over the power fed into the system by means of  $F_0(t)$ . We find that a condition for optimizing the overall efficiency is that all the devices be resonant at the same frequency  $\omega_d$ , and that the system be driven harmonically at  $\omega_d$ . The overall efficiency can then be shown to reduce to

$$\eta = \frac{\sum_{n=1}^{N-1} (x_n/(1+x_n)^2)(\kappa_{0n}^2/\Gamma_0\Gamma_n)}{1 + \sum_{n=1}^{N-1} (1/(1+x_n))(\kappa_{0n}^2/\Gamma_0\Gamma_n)}, \quad (2)$$

which is maximized for

$$x_1 = x_2 = \dots = x_{N-1} = \sqrt{1 + \sum_{n=1}^{N-1} \frac{\kappa_{0n}^2}{\Gamma_0\Gamma_n}}. \quad (3)$$

The dimensionless quantities  $\kappa_{mn}/\sqrt{\Gamma_m\Gamma_n}$  are also known as the “strong-coupling parameters,”<sup>1,2</sup> which represent, intuitively, the ratio of how fast energy is transferred between the resonators  $m$  and  $n$  to how fast it is dissipated due to intrinsic loss mechanisms at these resonators. It can be seen by inspection of Eqs. (2) and (3) that in this particular case of interest, the multiple devices look like one single effective device whose strong-coupling-squared is the sum of the strong-coupling-squared of the actual devices.<sup>8</sup>

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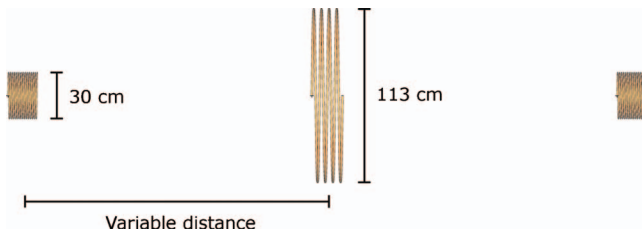


FIG. 1. (Color) Schematic representation of the geometrical arrangement of the experimental setup. For simplicity, the device coils are placed at identical distances on either side of the source (center coil). The image is to scale, with the center-to-center separation between the source and each device of 200 cm.

To demonstrate the principles outlined above experimentally, we built a large self-resonant coil<sup>2</sup> that would serve as a source for two smaller device coils, all designed to resonate at 6.5 MHz, which is in the range of optimal frequencies for this class of resonators.<sup>1</sup> The source coil is a helix made of four turns of 1/2" copper pipe, 113 cm in diameter, and 20 cm in height. The two devices are made of 15.25 turns of 1/4" copper pipe, 30 cm in diameter and 18 cm in height. In real-life applications, a source coil of this size (spanning an area of approximately 1 m<sup>2</sup>) could potentially be embedded in the walls or in the ceiling of a room, while the area of devices is comparable to those of some portable electronic devices and robots.<sup>9</sup> We placed the devices on opposite sides of the source (Fig. 1). To couple to each resonator we used smaller loop-and-capacitor “coupling” coils tuned to the same frequency as the larger self-resonant coils. The three coupling coils are then connected to appropriate driving and loading circuits.

By connecting each of the coupling coils to a separate port of a network analyzer and sweeping the driving frequency of the network analyzer around the resonant frequency of the coils, it is possible to extract all of the coupled-mode theory parameters in Eq. (1) from a fit of the measured input impedances. We found experimentally that the quality factor of the source coil was  $Q_0 = \omega / 2\Gamma_0 = 730 \pm 50$ . Due to imperfections in their (manual) fabrication, the two device coils had somewhat different quality factors, with device coil 1 having  $Q_1 = 1650 \pm 100$  and device coil 2 having  $Q_2 = 1850 \pm 100$ . The variation in the values of the  $Q$ 's is due in part to uncertainties in the data fit, and in part to the proximity of the coupling coils to the resonators.

The coupling between modes can be derived from standard coupled-mode theory arguments<sup>6</sup>

$$\kappa_{mn} = \frac{|\int d\mathbf{r} \mathbf{E}_n(\mathbf{r}) \cdot \mathbf{J}_m(\mathbf{r})|}{4\sqrt{\int d\mathbf{r} |\mathbf{B}_m(\mathbf{r})|^2 / (2\mu_0)} \int d\mathbf{r} |\mathbf{B}_n(\mathbf{r})|^2 / (2\mu_0)}. \quad (4)$$

In the approximation where the current distribution along the length of the self-resonant coils is a half-wave, each of the integrals in Eq. (4) can be quickly integrated numerically.<sup>10</sup> Since the integrals in the denominator are independent of the relative position between the resonators, they only need to be calculated once and one can very quickly estimate the coupling to any point in space. Although this analysis does not take into account the imperfections of the coils, the calculated values are within 10% of the experimental values of  $k = 2\kappa / \omega$  (Fig. 2), obtained in the same data fits as the experimental  $Q$  values.<sup>11</sup>

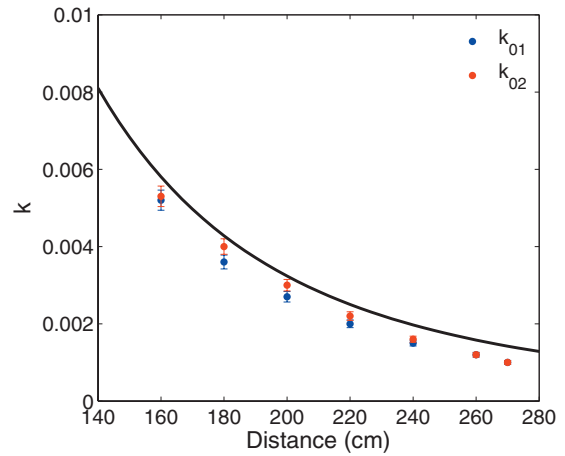


FIG. 2. (Color) Coupling  $k = 2\kappa / \omega$  between the source and each device as a function of center-to-center distance. Because of imperfections in the fabrication of the device coils, the measured coupling at the same distance for the two devices (blue and red) differs by  $\sim 10\%$ . Since  $\sqrt{Q_0 Q_{1,2}} \approx 1100$ , each device is strongly coupled to the source ( $k\sqrt{Q_0 Q_{1,2}} \approx 1$ ) over the entire range of distances. Also plotted is the coupling predicted (black) by a simple (and relatively easy to calculate) model that ignores the imperfections of the coils, but nevertheless gives predictions within 10% of the measured results.

We can now also check that the mutual coupling between the devices is much smaller than the coupling between each device and the source. We calculate from Eq. (4) that  $k$  values for the cross-coupling between the devices are smaller than the direct coupling the source and each device (Fig. 2) by at least a factor of 15 in each spatial configuration. Therefore, the approximation we made between Eqs. (1) and (2) is justified for the current system of interest.<sup>12</sup>

In order to effect power transfer at the resonant frequency, one can adjust the coupling between each coupling coil and its associated resonator by varying the distance between them or pitching the angle of the coupling coil, so that the system is properly matched to the 50  $\Omega$  ports of the network analyzer. After properly tuning the system, the efficiency can be directly measured by the network analyzer. We measured the efficiency between the source and each device separately, as well as the overall efficiency between the source and both devices simultaneously (Fig. 3). Note that, by virtue of Eq. (3), the matching in the case of simultaneous devices differs from that of a single device. The directly matched efficiency at the optimal frequency matches the efficiency predicted from the parameters derived from the fits to within 0.1%.

Once the matching is done with the help of a network analyzer, one can connect the coupling coils at the devices to a loading circuit with 50  $\Omega$  input impedance and drive the source with a 50  $\Omega$  rf amplifier, to provide significant amounts of power to the devices. In our setup, we were able to supply upwards of 25 W to each device (the power being dissipated in dummy loads) even when the devices were farther than 2 m from the source, the power level being constrained by the maximum output of the amplifier.

It can be readily seen from Fig. 3 that the relative improvement in overall efficiency due to having two devices coupled simultaneously to the source (green) compared to having each device couple separately to the source<sup>13</sup> (blue and red) is more significant when the devices are placed at longer distances from the source, i.e., when the coupling and strong-coupling parameters are lower. It can be seen from

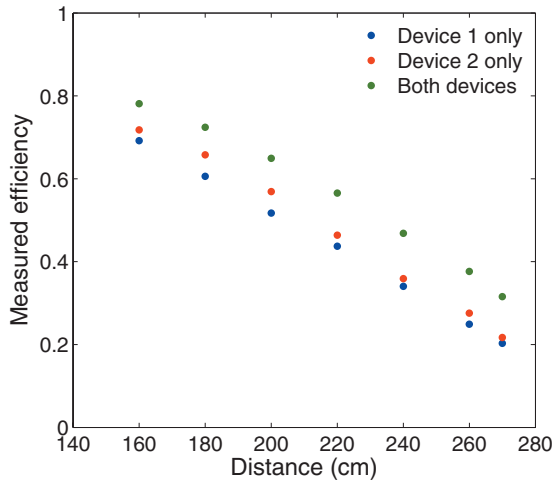


FIG. 3. (Color) Efficiency as a function of distance for the cases where each device alone is coupled to the source and where both devices are coupled simultaneously to the source, and properly matched according to Eq. (3). The losses due to the small resistance of the coupling coils have been subtracted out. Because of differences in the construction of the devices their  $Q$ 's and coupling to the source (see Fig. 2) are different, thereby leading to a somewhat higher efficiency for device 2.

Eqs. (2) and (3) that this conclusion holds for a larger number of devices, since the slope Eq. (2) [with  $x_n$  given by Eq. (3)], and therefore the incremental increase in efficiency when the strong-coupling parameters are increased (or equivalently, when a device is added to the system) is largest when  $\sum_{n=1}^{N-1} \kappa_{0n}^2 / (\Gamma_0 \Gamma_n) \sim 1$  (corresponding to an initial efficiency of  $\sim 20\%$ ). The approach of having one large source power multiple small devices distributed over a large volume may therefore lead to good overall efficiencies (e.g., greater than 50%) even in cases where the single device efficiencies are modest (e.g., worse than 20%).

In conclusion, we have derived analytically and shown experimentally for the case of two devices the effect on the overall efficiency and on the loading of the devices of adding multiple device resonators to a system of strongly coupled resonant modes. We find that the approach of powering multiple devices simultaneously can result in a good overall efficiency for the wireless power transfer even if the efficiency of the transfer to each individual device is relatively low.

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<sup>1</sup>A. Karalis, J. D. Joannopoulos, and M. Soljačić, *Ann. Phys.* **323**, 34 (2008).

<sup>2</sup>A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, *Science* **317**, 83 (2007).

<sup>3</sup>F. Segura-Quijano, J. García-Cantón, J. Sacristán, T. Osés, and A. Baldi, *Appl. Phys. Lett.* **92**, 074102 (2008).

<sup>4</sup>B. L. Cannon, J. F. Hoburg, D. D. Stancil, and S. C. Goldstein, *IEEE Trans. Power Electron.* **24**, 1819 (2009).

<sup>5</sup>*IEEE Std C95.1-2005, IEEE Standard for Safety Levels with Respect to Human Exposure to Radio Frequency Electromagnetic Fields, 3 kHz to 300 GHz* (IEEE, Piscataway, NJ, 2006).

<sup>6</sup>H. Haus, *Waves and Fields in Optoelectronics* (Prentice Hall, Englewood Cliffs, 1984).

<sup>7</sup>R. E. Hamam, A. Karalis, J. D. Joannopoulos, and M. Soljačić, *Ann. Phys.* **324**, 1783 (2009).

<sup>8</sup>Note that in case one needs to better control what fraction of the power is being drawn by a particular device  $m$ , it is possible to do so by varying  $x_m$ , and possibly  $\omega_m$  from their optimal values, although this would lower the overall efficiency.

<sup>9</sup>The fact that the resonators have a significant height is incidental to this particular coil design. The mutual coupling between resonators is predominantly determined by their areas and the distance between them (Ref. 1). It is in principle possible to design much thinner coils with performance characteristics similar to those demonstrated here (Ref. 2).

<sup>10</sup>Of the order of a second in MATLAB.

<sup>11</sup>Note that  $k$  and  $Q$  are dimensionless and that the strong-coupling parameter satisfies  $\kappa_{mn} / \sqrt{\Gamma_m \Gamma_n} = k_{mn} \sqrt{Q_m Q_n}$ .

<sup>12</sup>Note that in the  $\kappa_{mn}^2 / (\Gamma_m \Gamma_n) \ll 1$  regime, the energy exchanged between resonators  $m$  and  $n$  scales as  $\kappa_{mn}^2 / (\Gamma_m \Gamma_n)$ , so the direct energy exchanged between devices is estimated to be less than 1% of the energy exchanged between the source and each device.

<sup>13</sup>Separate coupling to each device could be achieved in real time by, for example, tuning each device to a separate resonant frequency and modulating the signal at the source so that it alternates between the different resonant frequencies of the devices, or alternatively by keeping the source's frequency fixed and modulating the resonant frequencies of the devices so that only one is resonant with the source at a given time.