The Metaphysics of Abstract Objects

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THE METAPHYSICS OF ABSTRACT OBJECTS*

What is—or should be—meant by the expression 'abstract object', and what sort of reasons could we have for supposing that such objects exist? These are the questions that I want to address in this paper. My strategy will be first to examine the general notion of an "object," then to consider several different conceptions of abstractness, and finally to discuss how existence claims in metaphysics are to be adjudicated, with special reference to the existence of such paradigmatically abstract objects as universals, numbers, and sets.

I. OBJECTS

What, in general, is an "object"? Here I shall examine two rival answers to this question, the *semantic answer* and the *metaphysical answer*, coming down eventually in favor of the latter.[1] According to the semantic answer, an object is to be conceived as a possible referent of a singular term (Gottlob Frege's view, as interpreted and defended by Michael Dummett) or as a possible value of a variable of quantification (W. V. Quine's view, encapsulated in his slogan "To be is to be the value of a variable").[2] The former version requires us to be able to define "singular terms" independently of the notion of an object, by appeal to their logical characteristics—that is, by appeal to the patterns of deductive inference characteristically sustained by sen-

* I am grateful for comments received when an earlier version of this paper was read to an audience at Queen's University/Belfast, and am particularly indebted to Alan Weir for his remarks on that draft.

[1] The present discussion continues one begun in my "Objects and Criteria of Identity," in Bob Hale and Crispin Wright, eds., *A Companion to the Philosophy of Language* (Cambridge: Blackwell, forthcoming).

[2] See, for example, Dummett, *Frege: Philosophy of Language* (Cambridge: Harvard, 1981, 2nd ed.), ch. 4; and Quine, "Speaking of Objects," in his *Ontological Relativity and Other Essays* (New York: Columbia, 1969).

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tences in virtue of their containment of singular terms. It must be emphasized that at this stage what is at issue is not the question of what objects *there are*, but rather the question of what it is *to be* an "object." According to the semantic answer, in its first version, we can only grasp the notion of an object via the notion of a singular term: singular terms are (potentially) *object denoting*, but this fact should be construed as explaining the notion of an object rather than as explaining the function of a singular term.

Now, of course, there can be empty singular terms, which fail to denote any existing object. But if a sentence containing a singular term is true, then, on the present account, that term does indeed denote an object. (Failure of a singular term to denote an object leads to a lack of truth value for the sentence containing that term.) But this threatens to produce a grossly overinflated ontology. Take, for instance, a sentence like 'The grin on John's face is broad', which could certainly be true. If it *is* true, however, advocates of the present account must apparently say that the singular term 'the grin on John's face' denotes an *object*, which actually exists. That seems extravagant, to say the least. The same applies, mutatis mutandis, for the second version of the semantic answer, as regards the sentence 'John is wearing a broad grin', which ostensibly involves quantification over grins. (That sentence would standardly be regimented in the form 'For some x, x is a grin and x is broad and John is wearing x'.)

In the face of these remarks, advocates of the semantic answer might appeal to the fact that the two sentences in question—'The grin on John's face is broad' and 'John is wearing a broad grin'—can plausibly be paraphrased by the sentence 'John is grinning broadly', in which no singular term or quantifier appears. But paraphrase is a symmetrical relation, and nothing within the theory of meaning is capable of telling us which of these sentences more accurately reflects the ontological commitments of those who utter them. The mere fact that a singular term can be "eliminated" by paraphrase cannot, by the semanticist's own lights, be taken to show that it is not "really" object denoting—for the semanticist has no independent account of objecthood to appeal to (and the same applies, mutatis mutandis, to the quantificational approach).

To the foregoing objection, the first sort of semanticist might perhaps be tempted to urge that *definite descriptions*, like 'the grin on John's face', are not "really" singular terms after all—taking, perhaps, a Russellian view of these as covertly quantificational. (Frege himself, of course, would not have taken such a view.) But this is not a plausible strategy, in the light of the fact that many definite de-

scriptions (though certainly not this one) plausibly *are* "object denoting"—for instance, 'the present Prime Minister of the U.K.'. In any case, the strategy seems likely only to lead to a replacement of the first version of the semantic answer by the second (that is, the quantificational version), which is really in no better shape.

Another way in which advocates of the semantic answer, in either version, might hope to defend it is by their insistence that all "genuine" singular terms or quantifier expressions need to be backed by the provision of *criteria of identity* for the objects denoted or quantified over (recall Quine's other famous slogan, "No entity without identity," and the Frege-Dummett thesis that singular terms can only be introduced in association with criteria of identity for their references). Then it might be urged that "grins" and the like can be provided with no such criterion, thus deflating what would otherwise be a grossly overblown ontology. In the first place, however, this move already appears to be a concession in the direction of admitting that metaphysical considerations independent of the theory of meaning are relevant to the questions of what an object is and what objects there are—for identity criteria are precisely *metaphysical* principles, telling us (as Locke would put it) what identity consists in for objects of given kinds. (It is true that they are also semantic principles, to the extent that the meaning of a kind term, like 'dog', cannot be adequately grasped without a grasp of the identity criterion governing objects of that kind, but that in no way undermines the point that has just been made.) Furthermore, it is in any case strongly arguable, on metaphysical grounds, that not *all* kinds of objects can be provided with criteria of identity, because in the case of objects of some kinds their identity *cannot* be taken to "consist in" anything else. For these "basic" objects—and, arguably, any ontology must include some—identity is primitive and irreducible.[3] (This may, for instance, be true of persons.)

I turn, then, to the metaphysical answer to the question 'What is an object?'. The answer I have in mind is simply that to be an *object* is to be an entity possessing *determinate identity conditions* (though not necessarily a *criterion* of identity, for the reason just given). If x and y are objects, there must be a "fact of the matter" as to whether or not x is identical with y. That is to say, the identity statement '$x = y$' must be of determinate truth value. (To put it this way is not to make any concession toward the semantic answer, because I do not believe that one

[3] See further my *Kinds of Being: A Study of Individuation, Identity, and the Logic of Sortal Terms* (Cambridge: Blackwell, 1989), ch. 2, and my "Primitive Substances," *Philosophy and Phenomenological Research*, LIV (1994): 531–52.

can assess whether an identity statement is of determinate truth value without recourse to independent metaphysical argument; thus, not too much should be read into any superficial similarity between the metaphysical answer as just stated and Dummett's semanticist characterization of "realism"[4] as a matter of a commitment to the principle of *bivalence*.) As an illustration of how the metaphysical answer may be applied, one reason why I am inclined to doubt whether so-called subatomic "particles" are properly to be thought of as objects is that it seems that in their case identity statements concerning them can genuinely be indeterminate.[5] (Note here, with regard to the issue of "wave-particle" duality, that waves—even those of the ordinary seaside variety!—are indeed not "objects" according to the metaphysical answer, because they lack determinate identity.)

Now, it is an implication of the metaphysical answer that there can be *entities* that are not objects. (Waves provide an example.) As we might put it, 'Not everything is a thing'—understanding 'thing' here to mean 'object'. Of course, according to the Quinean version of the semantic answer, this statement must necessarily be false: on that view, it is just trivially true that everything is a thing, since what the quantifier 'everything' ranges over is precisely *things*—which it does because "things" themselves, by this account, are precisely to be understood as what the quantifier ranges over. But the first version of the semantic answer seems to imply a different response—witness Frege's distinction between objects and *concepts*, the latter precisely not being things or objects. Be that as it may, I myself am certainly happy to countenance the existence of many entities that are not objects or things—much as P. F. Strawson[6] distinguishes between "particulars" and "nonparticulars." Some of these entities can be described as "ways things are," recalling to mind the scholastic distinction between *substance* and *mode*. For example, an object's individual shape and color can be thought of as "ways it is"—namely, as how it is colored and how it is shaped, respectively. But its color, say, is not "itself" an object, somehow related to the object of which it is the color. If it were an object, it would have determinate identity conditions, and yet it does not appear that it can have these. Supposing the colored object to be uniformly colored, it makes doubtful sense

[4] See, for example, his *The Logical Basis of Metaphysics* (Cambridge: Harvard, 1991), introduction.

[5] See my "Vague Objects and Quantum Indeterminacy," *Analysis*, LIV (1994): 110–14.

[6] *Individuals: An Essay in Descriptive Metaphysics* (New York: Methuen, 1959), pp. 226ff.

to ask whether "the color" of its top half is *numerically identical* with "the color" of its bottom half, or whether either or both of these is identical with "the color" of the whole object. Certainly, these questions cannot apparently be answered in a nonarbitrary and principled way. (Of course, the questions do make sense and trivially receive the answer 'Yes' if 'the color of *x*' is construed as referring to a universal: but here I am supposing it to refer to what used to be called an "individual accident.")

It will be recalled that, in objecting to the semantic answer, I remarked that by that account a singular term in a true sentence or statement must be object denoting. According to the metaphysical answer, no such conclusion can be drawn. On this view, for instance, I can allow the sentence 'The table at which I am writing is square' to be true, and allow, too, that the definite description here is as good an example of a "singular term" as any, and yet leave it open whether tables (including "this" one) really exist. And the reason for leaving this open need not have to do with any doubts one might harbor as to whether tables have determinate identity conditions. On metaphysical grounds, it might be plausible to hold, as Peter van Inwagen[7] does, that although table-shaped collections of particles exist, tables do not. As I shall explain more fully later, my view is that whether objects of a given kind should be thought actually to exist should, in general, turn on considerations of whether an inclusion of such objects in one's ontology has explanatory value.[8] On this score, tables do not fare all that well, since whatever facts they serve to explain can, it seems, be as easily explained by reference to table-shaped collections of particles, which must, at least in this world, be acknowledged to exist in any case. (In a world in which tables were *not* composed of such particles, matters might be otherwise.)

II. ABSTRACT ENTITIES

In contemporary discussions of abstract entities, we can find at least three different conceptions of abstractness at work. On the first conception, the term 'abstract' is used in opposition to the term 'concrete', with concrete entities being thought of as existing in space and time (or at least in time), while abstract entities are correspondingly thought of as being nonspatiotemporal in nature.[9] Let us call

[7] *Material Beings* (Ithaca: Cornell, 1990), ch. 13.

[8] See also my "Die Möglichkeit der Metaphysik," in J. Brandl, A. Hieke, and P. Simons, eds., *Metaphysik: Beitrage zum 3. Kongress der Österreichischen Gesellschaft für Philosophie* (Bonn: Academia, forthcoming).

[9] See, for example, Reinhardt Grossmann, *The Existence of the World* (New York: Routledge, 1992), p. 7.

abstract entities in this first sense *abstract*$_1$ entities. They would standardly be taken to include such items as numbers and universals.

On the second conception, an abstract entity is conceived as one logically incapable of enjoying a "separate" existence—separate, that is, from some other entity or entities—even though it may be separated "in thought" from that entity or those entities.[10] (Such separation "in thought"—a psychological process—seems to be what philosophers like John Locke understood by "abstraction"; but in calling the entities thus separated "abstract," we are now invoking a *metaphysical* distinction, defined in terms of the impossibility of their separate existence.) For example, *modes*—like the individual shape and color of a particular apple—come into this category. One can separate "in thought" the apple's color from its other features, but an apple's color cannot *exist* independently of the existence of other features of it, nor, indeed, independently of the existence of the apple as a whole.[11] I shall call abstract entities in this sense *abstract*$_2$ entities.

Finally, we have the third conception, according to which *abstract*$_3$ entities are, as I shall explain more fully in due course, entities that are conceived of as being introduced by way of *abstraction from concepts*, according to Fregean abstraction principles.[12] A paradigm example would be Fregean extensions (of concepts), purportedly introduced by Frege's fatal "basic law V" of the *Grundgesetze*. The three different conceptions of abstract entities cut across each other in various ways, and each has its own problems, as we shall see.

So far, I have deliberately spoken only of abstract entities rather than of abstract objects. On my view of what constitutes an "object," an abstract object—in any of the three senses of "abstract" just mentioned—will have to be an entity possessed of determinate identity conditions. Thus, by my account, modes are not abstract$_2$ objects, because they lack such conditions. This view contrasts with that of today's growing population of *trope* theorists, who speak of tropes as "abstract particulars" and regard individual substances (such as a particular apple) as bundles of compresent tropes—particular col-

[10] See, for example, Keith Campbell, *Abstract Particulars* (Cambridge: Blackwell, 1990), pp. 2–3.

[11] It is, incidentally, no simple matter to provide a definition of ontological dependency which properly represents its asymmetrical character: see further my "Ontological Dependency," *Philosophical Papers*, XXIII (1994): 31–48.

[12] For the background to this approach, which owes much to the work of Michael Dummett, see the discussion in Bob Hale, *Abstract Objects* (Cambridge: Blackwell, 1987), ch. 3.

ors, shapes, weights, and so forth.[13] My objection is not so much to the claim that such abstract entities exist, but to the supposition that they are objects, and indeed ones ontologically more basic than things like apples.

Let us, however, return to the first conception of abstractness, which contrasts it with concreteness. As I have indicated, this contrast is normally drawn in spatiotemporal terms, with abstract$_1$ entities being characterized as not existing "in" space and time.[14] But what does it mean to characterize them so? How could an object exist "outside" space and time? ('Outside' is a spatial preposition, so this way of talking can at best be metaphorical.) I do not think there is any very deep problem here, however. To exist *in* space and time is not to have a special kind of existence—for the notion of existence, like that of identity, is univocal. Rather, it is just to have certain sorts of properties and relations—spatiotemporal ones. Numbers do not have shapes (a "square" number is not square shaped!), nor do they undergo change, and it is facts like these, if any, which justify our description of them as not existing "in" space and time. Thus, one might be tempted to say that an object is abstract$_1$ if it necessarily lacks spatiotemporal properties and relations. As against this, it may be urged that a spatiotemporal relational property, like 'being thought of today by someone living in Vienna', can be possessed by a number, presumably without making it true that numbers exist "in" space and time. One might respond, however, either by urging that this is a "mere Cambridge" property (like Xanthippe's becoming a widow upon Socrates' death),[15] or alternatively by urging that for something to exist "outside" space and time it suffices that that thing should have no essential spatiotemporal properties and relations. I shall not pursue the issue further, not being convinced that it harbors any real difficulty for the conception of abstractness now under consideration. (I take a similar view of the objection that items like languages are abstract and yet not timeless, because they are said to undergo change; here I believe one must distinguish between a "language" conceived as a universal, which is timeless, and a "language" conceived as a social practice, which is not.[16])

[13] See Campbell, *Abstract Particulars*. He himself is sensitive to the identity problem (see pp. 135ff.) and as a result moves to a "field" theory conception of tropes.

[14] Difficulties for this way of drawing the contrast between abstract and concrete entities are raised in Gary S. Rosenkrantz, *Haecceity: An Ontological Essay* (Boston: Kluwer, 1993), pp. 56ff—but, as I imply below, I think these difficulties are not insuperable.

[15] On "mere Cambridge" properties, see H.W. Noonan, *Personal Identity* (New York: Routledge, 1989), pp. 162ff.

[16] See further my "Objects and Criteria of Identity."

One additional question which is worth raising, however, is whether abstract$_1$ objects, conceived of as existing "outside" space and time, can have causal powers, and whether it would matter if they could not. For some metaphysicians, possession of causal power is the very hallmark of real existence (and is one reason, for instance, why some have denied the existence of the void)—and some epistemologists, of course, espouse causal theories of knowledge, which would appear to rule out knowledge of the existence of causally impotent objects. My own view, as I shall make clearer later, is that some abstract$_1$ objects—notably certain universals—need to be invoked for explanatory purposes, even if it cannot be said that they themselves possess causal powers or enter into causal relations.

I turn, finally, to the third conception of abstractness. The idea here is that abstract$_3$ objects are objects "abstracted" from concepts in accordance with "Fregean" abstraction principles.[17] These principles are, in fact, a species of *identity criterion.*[18] Fregean identity criteria are exemplified by Frege's famous criterion for the identity of directions: the direction of line l is identical with the direction of line m if and only if lines l and m are parallel with one another. This particular criterion only invokes an equivalence relation on *objects* of a certain sort—lines—but other Fregean identity criteria invoke equivalence relations on *concepts* (in Frege's terms). A prime example is what is now known as "Hume's principle": the number of Fs is identical with the number of Gs if and only if the Fs and the Gs are one-one correlated with each other—where F and G are any concepts whatever (for example, they might be the concepts "fork laid on this table" and "knife laid on this table," or they might be the concepts "child in this classroom" and "book on this shelf"). The suggestion thus is that "Hume's principle" can be seen as "introducing" a kind of abstract$_3$ objects—the cardinal numbers—by way of a certain equivalence relation on concepts (the relation of one-one correspondence between objects falling under those concepts).

Another and more infamous example is provided by Frege's "basic law V," stating that the extension of F is identical with the extension of G if and only if all and only Fs are Gs—a principle which notoriously falls foul of "Russell's paradox," and which consequently can-

[17] See further Wright, *Frege's Conception of Numbers as Objects* (Aberdeen: University Press, 1983). My characterization of abstract$_3$ objects is indebted to an as yet unpublished lecture by Kit Fine, though nothing I say should be taken as representing his own views.

[18] See further my "Objects and Criteria of Identity," and my "What Is a Criterion of Identity?" *Philosophical Quarterly*, XXXIX (1989): 1–21.

not coherently be thought of as genuinely "introducing" a kind of abstract$_3$ objects, the "extensions" of concepts. Clearly, in the light of this failure, some restriction must be placed upon which Fregean abstraction principles can be thought of as legitimately introducing a kind of abstract$_3$ objects. (Note, though, that if such a principle is legitimate, any entities it introduces will certainly deserve to be called objects, because the principle itself will supply them with determinate identity conditions.)

It is not my concern here to consider how a principled distinction can be drawn between "legitimate" and "illegitimate" abstraction principles. But I do want to question the thought that what such principles, even if paradox-free, can do is really to specify a kind of abstract objects. My general point would be that criteria of identity—which is what these principles are—never serve unambiguously to determine the kind of objects to which they apply, for the very simple reason that many *different* kinds of objects are typically governed by the *same* criterion of identity. For instance, dogs and cats are objects of different kinds, but ones which share the same criterion of identity. (This is not to say that any cat can be identified with any dog—the point is merely that dogs and cats are both kinds of animal, and subkinds of the same kind must share the same criterion of identity—here, the criterion of identity for animals in general.)

"Hume's principle" certainly cannot tell us what sort of thing a cardinal number *is*—and this, indeed, is one lesson of Paul Benacerraf's[19] well-known problem. For all that "Hume's principle" tells us, either of the sets {0, {0}} and {{0}} *could* equally *be* the number 2, as indeed could infinitely many other distinct items. The idea that one can "introduce" a kind of objects simply by laying down an identity criterion for them really inverts the proper order of explanation. As Locke clearly understood, one must first have a clear conception of what kind of objects one is dealing with in order to extract a criterion of identity for them from that conception. (This is how he approaches the problem of personal identity.) So, rather than "*ab*stract" a kind of objects from a criterion of identity, one must in general "*ex*tract" a criterion of identity from a metaphysically defensible conception of a given kind of objects. The "Fregean" approach to abstract objects cannot, it seems, ultimately provide any insight into questions of ontology. These questions have to be addressed directly by metaphysical analysis and argument: one cannot conjure their answers into existence by laying down stipulative principles.

[19] "What Numbers Could Not Be," *Philosophical Review*, LXXIV (1965): 47–73.

Despite this criticism, I do not want to jettison altogether the notion of abstract$_3$ objects. That certain kinds of objects are governed by "Fregean" identity criteria may indeed set them apart in a special way. The kinds of objects in question are ones to which it is natural to make reference by means of *functional* expressions of appropriate sorts (expressions of the form 'the *f* of *a*').[20] Directions are a case in point (though their Fregean identity criterion does not invoke an equivalence relation on "concepts"). Directions apparently have to be thought of as directions *of* something—plausibly, indeed, of lines. This suggests a connection with the second notion of abstractness considered above, whereby an "abstract" object is one incapable of a "separate" existence. Arguably, although one can separate "in thought" a direction from any line of which it is the direction, one cannot conceive of a direction *existing* in the absence of any line possessing that direction. (This claim is more persuasive if one thinks of lines simply as unidimensional parts of space.) Perhaps, then, any residual value in the third, "Fregean" conception of abstractness lies in its association with the second conception.

III. UNIVERSALS AND PARTICULARS

An important distinction that we have not so far taken properly into account is the distinction between *universals* and *particulars*. (This is not the same as Strawson's distinction between particulars and nonparticulars, I should emphasize; the latter is more akin to my distinction between objects and those entities which are not objects.) Are all objects—including any abstract objects—particulars, as Locke[21] held? And how should we define the distinction between universals and particulars, in any case? I propose to define it in terms of *instantiation*.[22] A particular is something (not necessarily an object) that instantiates but is not itself instantiated. Universals, on the other hand, necessarily have instances (or at least are instanti*able*). Thus, I think of universals as *kinds*. But are universals thus conceived to be regarded as *objects*? (I ask this while still remaining neutral, as yet, regarding their actual existence.) By my account, universals will indeed qualify as objects if they have determinate identity conditions, and I think it plausible in many cases to hold that they do. For instance, the kind "horse" is surely determinately distinct from the kind "whale." Certainly, the kind "gold" is determinately distinct from the

[20] The idea that such objects are, in a strong sense, "ontologically dependent" entities is developed in my "Ontological Dependency."

[21] *An Essay concerning Human Understanding*, P.H. Nidditch, ed. (New York: Oxford, 1979), bk. III, ch. III, 1.

[22] Cf. my *Kinds of Being*, pp. 38–39.

kind "carbon." (Other kinds, such as the color kinds "yellow" and "orange," are obviously more problematic, in view of the vagueness of their boundaries.)

But are universals, if they exist, *abstract* objects, in any of the senses of "abstract" we have investigated? Plausibly, they are abstract$_1$, that is, nonconcrete, failing to exist "in" space and time. Of course, particular instances of them—particular horses and lumps of gold—exist in space and time, but it certainly is not obvious that the kinds of which they are instances themselves possess any essential spatiotemporal properties or relations. It is true that some metaphysicians think of each universal as being "wholly" present in each particular that instantiates it—thus, of "gold" as being "wholly" present in each piece of gold.[23] But I cannot see what saying this adds to saying that each piece of gold is indeed an instance of "gold," and I cannot see that saying the latter implies that the universal that they all instantiate itself has any essential spatiotemporal relations, that is, relations to parts of space and moments of time.

As for the second notion of abstractness, it would again seem plausible that universals are abstract$_2$ objects, for it is plausible to hold that their existence is "inseparable" from the existence of their particular instances—that, in short, there cannot be *uninstantiated* universals (a position which David Armstrong has also defended; *ibid.*, pp. 75ff.). Plausibly, not even God could have created the kind "horse" without creating some particular horses. (This claim need not be seen as conflicting with the previous claim that universals have no essential relations to space and time. Being necessarily instantiated by something that exists in space and time should not be thought of as a spatiotemporal relational property. For one thing, instantiation cannot properly be conceived of as a *relation* between particulars and universals, and thus as being a universal itself, on pain of generating a vicious regress. But even setting that consideration aside, the following analogy with sets—which are paradigmatically abstract objects—seems persuasive: a set of concrete objects necessarily has as its members things that exist in space and time, and yet that seems to be no good reason for saying that such a set itself participates in spatiotemporal relations.) I should add that it also seems clear that each universal is "inseparable" from certain others.

What about the question of whether universals could be thought of as abstract$_3$ objects, that is, as objects supposedly "introduced" by

[23] For discussion, see David M. Armstrong, *Universals: An Opinionated Introduction* (Boulder, CO: Westview, 1989), pp. 98–99.

abstraction from concepts? Here we run up against Frege's infamous paradox of the concept "horse."[24] Frege treated the expression '—is a horse' as expressing a concept and consequently as not denoting an object. But when we ask which concept it expresses, the obvious answer is: the concept "horse"—or, if one prefers, the concept of being a horse. Either way, what we now ostensibly have is a singular term denoting a concept, whereas the official doctrine is that singular terms are object denoting. But even glossing over this problem, a more serious difficulty arises if we try to formulate a "Fregean" abstraction principle with a view to abstracting concepts as abstract$_3$ objects from concepts themselves, for the obvious candidate for such a principle would be something of the form: the concept *F* is identical with the concept *G* if and only if it is *necessarily* the case that all and only *F*s are *G*s. (The modality invoked here might be interpreted in various alternative ways—for instance, in such a way that the right-hand side of the biconditional means something like, 'Any thinker who grasped both concepts would think it true but uninformative that something is *F* if and only if it is *G*'.[25]) But such a principle is apparently in no better shape than Frege's "basic law V."

Abandoning, now, the notion of abstract$_3$ objects as altogether too problematical, we can ask whether, in the first and second senses of abstractness, there might actually *be* any abstract objects, either universals or particulars. My view is that there indeed are. Universals, as we have seen, appear to be abstract objects in both of these senses, and I believe that we need to invoke the existence of certain universals because they figure essentially in natural laws governing the behavior and composition of all particulars that instantiate those universals.[26] This consideration applies most obviously to natural kind universals, such as the kinds "horse" and "gold," as opposed to artifactual kinds like "table." There are laws governing the behavior and composition of all particulars instantiating the kind "gold" which they obey only in virtue of being instances of that kind—such as, that they are soluble in aqua regia and are composed of atoms containing seventy-nine protons in their nucleus.

[24] See "On Concept and Object," in *Translations from the Philosophical Writings of Gottlob Frege*, P.T. Geach and M. Black, eds. (Cambridge: Blackwell, 1960, 2nd ed.).

[25] Such a principle is perhaps superficially reminiscent of the sort of thing invoked by Christopher Peacocke for the purpose of individuating what *he* calls "concepts," though Peacocke explicitly distances himself from the "Fregean" view of concepts in this context: see his *A Study of Concepts* (Cambridge: MIT, 1992), p. 2.

[26] See my *Kinds of Being*, ch. 8, and cf. Armstrong, *What Is a Law of Nature?* (New York: Cambridge, 1983).

But what about abstract particulars—do any objects in this category actually exist? As regards abstract$_2$ objects, I have already challenged the claim of some metaphysicians that tropes provide an example of such particulars—because I do not believe that "tropes" can be seen as having determinate identity conditions and so do not think that they can qualify as objects at all. I allow that modes exist, but not as "objects." I am happy to allow that events qualify as abstract$_2$ objects, however: an event, such as the death of a particular horse, qualifies as an "object" because it has, I believe, determinate identity conditions, but at the same time it qualifies as abstract$_2$ because it cannot exist "separately" from the horse whose death it is. As we might again put it, not even God could have created the death of this horse without creating this very horse. (Notice, by the way, how such an event is naturally referred to, as here, by means of a functional expression, of the form 'the *f* of *a*'.) Furthermore, I believe that we must allow that events do actually exist—that is, we must include them in our ontology—because they figure indispensably in singular causal explanations. An event like the death of a particular horse has its own distinctive causes and effects. One cannot be an "eliminativist" about events in general, in the way one might hope to be about artifacts (like tables) in general.

Events are clearly *concrete* objects, however, in that they essentially possess at least temporal properties and relations. We are still left, thus, with the question of whether there are any abstract$_1$ objects that are particulars—particular objects that, like universals, do not exist "in" space and time. Two obvious candidates are sets and numbers, which I shall discuss in the next and final section. Another traditional candidate would be God, but I do not feel equipped to discuss this possibility here.

IV. NUMBERS AND SETS

In what follows, I shall confine myself, for simplicity, to the consideration of the natural numbers, 0, 1, 2, 3,.... These are often taken to be abstract$_1$ particulars, and indeed are often identified, for theoretical purposes, with certain sets—2 with the set {0, {0}}, for instance. Sets, likewise, are commonly regarded as abstract$_1$ particulars—even when their members are concrete objects, as in the case of the set of planets of the sun. My own view, however, is that the natural numbers are *universals*—kinds—rather than particulars, and indeed that they are kinds of sets, that is, that they are kinds whose instances are sets.[27] Hence, on this view, numbers are certainly not themselves sets, any more than the kind "dog" is itself a dog. Specifically, the number

[27] See my "Are the Natural Numbers Individuals or Sorts?" *Analysis*, LIII (1993): 142–46.

2 is, by this account, the kind of two-membered sets (there is no circularity here). That is, each two-membered set is "a" 2, quite in the way in which each particular dog is a dog.

Rather than hold, as is common, that the notion of number is to be explained in terms of the notion of set, I hold the reverse of this. I hold that the notion of a set is precisely the notion of a number of things. A set just *is* "a number of things."[28] This is much preferable to saying that a set is a "collection" of things, which is at best a metaphor. Nothing literally "collects" the members of a set, such as the set of planets of the sun—unless it be a Fregean "concept" under which they all fall. But—not to speak of the fact that there are not "enough" concepts for every set to be "collected" by one—in the case of many sets such a concept is at best only made available once the membership of the set has already been specified, as in the case of the set whose members are the front door of my house, North America, and the positive square root of 2. Yet even this set is just "a number of things"—a threesome—and by my account just an instance of the numerical kind 3. It is, quite literally, "a" 3.

Let us reflect further on the nature of the "set" of planets of the sun. We say 'The planets are 9', which is equivalent to saying 'The number of the planets is 9'. I would compare these statements respectively with 'Fido is a dog' and 'The genus of Fido is dog'. Moreover, I see only a grammatical distinction between the plural noun phrase 'the planets' and the singular term 'the set of the planets'.[29] The words 'set of the' in the latter term are only there to transform a plural noun phrase into a singular one, a transformation which exists merely to satisfy certain idiosyncrasies of idiom. Thus, I treat a plural noun phrase like 'the planets' as denoting a set, construed in my way as being, quite simply, a number of things. Who can deny that the planets are indeed "a number of things"? The statements 'The set of the planets is 9 in number', 'The number of the planets is 9', and 'The planets are 9' are all ways of saying the same thing—that a certain number of things is "a" 9, a ninesome. (Here it may be inquired what, on my view, it is to *be a member of* a set. I answer that to be a member of a set is just to be one of a certain number of things—for instance, to be one of the planets, or to be one of the following things: my front door, North America, and the positive square root of 2 (*ibid.*, p. 213). I accept that this notion of

[28] Interestingly enough, this is precisely how the *Concise Oxford Dictionary* defines a set.

[29] Cf. Peter Simons, "Plural Reference and Set Theory," in Barry Smith, ed., *Parts and Moments: Studies in Logic and Formal Ontology* (München: Philosophia, 1982).

"being one of a certain number of things" cannot be further reduced, but do not consider that it is in any way mysterious. Formally, if armed with plural quantification, we may define '$a \in b$' as '$(EX)(X = b \,\&\, a{:}X)$'—reading the latter as 'There are some things which are b and a is one of them'.[30])

Suppose this view of sets as simply instances of numerical kinds is accepted. Should we then think of sets as *abstract objects*, in any of the senses of "abstract" discussed earlier? That sets so conceived qualify as *objects* is supported by the fact that the principle of extensionality provides them with determinate identity conditions, at least to the extent that their members have these—this being the principle that if x and y are sets, then x is identical with y if and only if x and y have the same members. That they are abstract$_2$ surely follows from the fact that, with the exception of the empty set, a set cannot exist "separately" from its members: not even God could create a set without creating its members. As for whether sets are abstract$_1$—nonspatiotemporal—this might appear to depend on whether or not their members are.[31] But even with a set like the set of the planets, it is—as we saw earlier—far from obvious that *it* has any essential spatiotemporal properties and relations, even though its members obviously do.

Finally, do sets *exist*? That is—to put it in my terms—*are* there really any instances of the numbers? If there are not, then, on the assumption that uninstantiated universals cannot exist, neither will the numbers themselves exist. Once again, we should bear in mind here considerations of explanatory need. And, certainly, it appears that numbers play an important role in natural laws—for instance, in the Newtonian inverse square law of gravitation and in specifying the values of various constants of nature, such as Planck's constant.[32] Likewise, they play an important role in measurement.

Of course, it might also be argued, with some plausibility, that the numbers exist *of necessity* if they exist at all, and even that they must exist if anything at all exists.[33] And sets appear to come so cheaply that it is hard to see why anyone should deny their existence who accepts the existence of their members—deny, for instance, that the

[30] I say more about plural quantification in my "Noun Phrases, Quantifiers, and Generic Names," *Philosophical Quarterly*, XLI (1991): 287–300.

[31] This appears to be Penelope Maddy's view; see her *Realism in Mathematics* (New York: Oxford, 1990), pp. 50ff.

[32] Here, of course, one needs to bear in mind the arguments of Hartry Field in his *Science without Numbers* (Cambridge: Blackwell, 1980).

[33] I make the latter claim myself in my "Are the Natural Numbers Individuals or Sorts?"

set of the planets exists while accepting that each of the planets exists. If each of them exists, then, surely, *the planets* exist, and this, I have urged, is all it means to say that the "set" of the planets exists. And if they exist, "a" 9 exists, and so 9 exists. But 9 surely could not exist if none of the other natural numbers existed.

Even so, one cannot help doubting whether concerns like this really have much metaphysical importance, in the way that other ontological concerns do. Does it, in the end, really matter whether the numbers actually exist—in anything like the way in which it matters whether space and time or persons actually exist? I find it hard to suppose so. Perhaps it is enough, for mathematical purposes, that numbers *could* exist, if indeed that is a possibility distinct from that of their *actually* existing.

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