Permuted Solutions

This is a jigsaw puzzle yielding a Young tableaux. After solving it you get the Young tableaux

```
YRSQHTPW
OEAVZI
UJKFM
NDXC
GLB
```

Reading the rows as cycles you get the permutation (YRSQHTPW)(OEAVZI)(UJKFM)(NDXC)(GLB)

These are analogues of Pascals triangles with the outer diagonals of 1s replaced with other famous sequences (Recaman's sequence, Fibonaccis, triangular numbers and primes). Then numbers between 0 and 25 are replaced by letters and numbers greater than 25 by *s. Here's the solution:

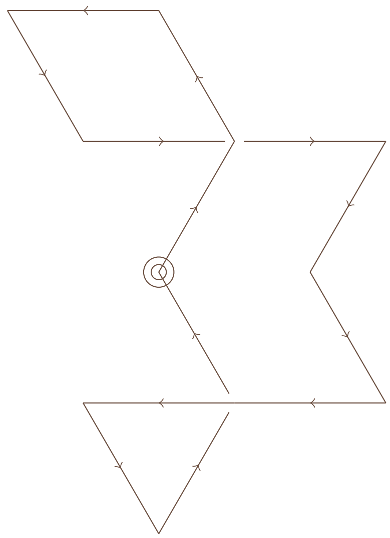
```
                  1
               3     1
            6     4     2
         2    10     6     3
      7    12    16     9     5
  13   19     *    25    14     8
20    *     *     *     *    22    13
                     0
                  1     2
               3     3     3
            6     6     6     5
        10    12    12    11     7
     15    22    24    23    18    11
  21     *     *     *     *     *    13
*     *     *     *     *     *     *    17
```

Writing down the letters in this order yields the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
CXSHOWEFBYVTDMIKULNGARZJQP
```

The three equations give an encoding of paths as positive integers. A ray at angle 0 is encoded as a "0", at angle $\pi/3$ as "1", ... and at angle $5\pi/3$ as "5." These digits are then concatenated into a number expressed in base 6.

Using this encoding in reverse on the number given below the grid produces a path, starting from the center point:



Reading the letters along this path yields the cycle (BURDIMLCNHSZV).

We encode letters as nonzero elements of the finite field $\mathbb{F}_{27} = \mathbb{F}_3[\alpha]/(\alpha^3 + 2\alpha^2 + 1)$. The specified function then defines an ordering of the elements of the field, from which we extract the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
UOVFZMCQLXEGKSYNBAHDWPRIJT
```

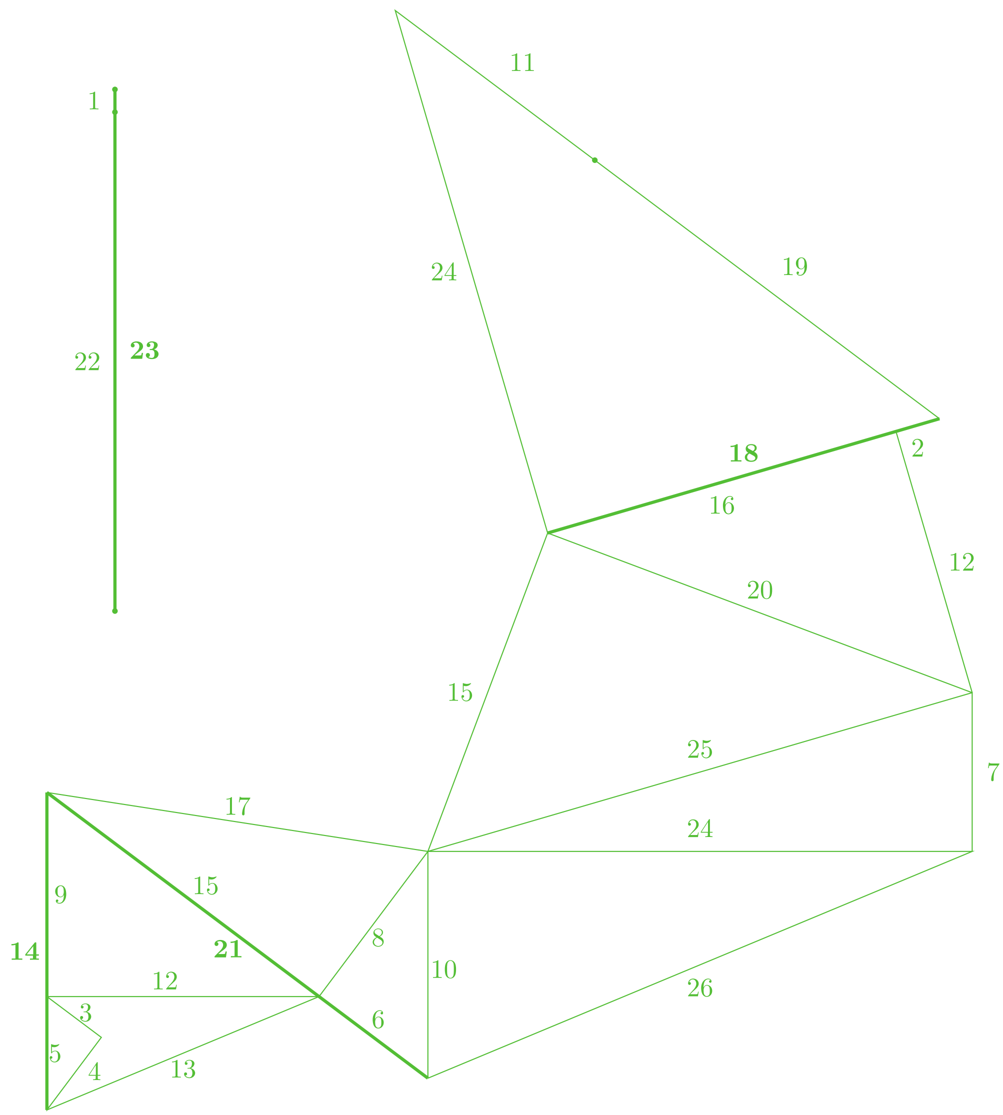
The given polynomial factors as

$$(z + (149i + 914))(z + (1912i + 1219))(z - (2335i + 3523))(z - (22158i + 15822)).$$

Roots with negative coefficients correspond to transpositions and with positive coefficients correspond to 3-cycles (these are the sign of the cycle). The product of the cycles gives the desired permutation $(9, 14)(12, 19)(3, 5, 23)(8, 22, 15)$ or (IN)(LS)(CEW)(HVO)

The diagram consists of right triangles with integer side lengths, each length between 1 and 26 appearing at least once. Determining all of the lengths yields the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
RWJTHZMNVYDABOCXFEUPIGSKLQ
```



This is just a product of transpositions, expressed as ligatures.
(AN)(DR)(EW)(ZO)(MB)(IE)(BU)(YE)(RS) = (AN)(BUM)(DRS)(EYIW)(OZ)

Encode letters into vectors in $\mathbb{F}_3^3$ by taking the base three expansion of the corresponding number:

$$\mathrm{K} \to 11 \to (102)_3 \to \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}.$$

The given matrix then defines an automorphism of $\mathbb{F}_3^3$, yielding the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
EGHACDFBLQJKMOPINXTVWYRSUZ
```

The words appearing around the 26-gon are satellites and exploration missions associated to various solar system bodies, some of them written backward.

A	Rhea	Saturn	N	Oberon	Uranus
B	Beagle	Mars	O	Stereo	Sun
C	Curiosity	Mars	P	Phoebe	Saturn
D	Deimos	Mars	Q	Paaliaq	Saturn
E	Ganymede	Jupiter	R	Suttungr	Saturn
F	Ferdinand	Uranus	S	Proteus	Neptune
G	Galatea	Neptune	T	Margaret	Uranus
H	Hydra	Pluto	U	Umbriel	Uranus
I	Cassini	Saturn	V	Viking	Mars
J	Janus	Saturn	W	Wind	Earth
K	Kiviuq	Saturn	X	Nix	Pluto
L	Luna	Earth	Y	Ymir	Saturn
M	Mimas	Saturn	Z	Zarya	Earth

Starting from any point you like, aim “across the solar system” at the orbit of the corresponding body; aiming counterclockwise (the direction of planetary orbits) for words written forward and clockwise for words written backward. For example, “Galatea” is a moon of Neptune, so the next point should be $(13 - 8)$ points counterclockwise around the perimeter.

The resulting permutation has two cycles: (QWFSYTKULIRZDNVOEACGH)(JMXBP).

Each letter corresponds to one of the first 26 primes, and the given equations show the product of the corresponding letters.

2	J	19	M	47	Z	73	E
3	P	23	A	53	D	79	W
5	B	29	S	59	K	83	X
7	T	31	I	61	V	89	U
11	R	37	L	67	Q	97	C
13	F	41	G	71	O	101	Y
17	H	43	N				

This yields the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
JPBTRFHMASILGNZDKVQOEWXUCY
```

Each variable is replaced by a letter so that the resulting paragraph of text uses sensible choices of variable names for the objects being discussed (though the mathematics is non-sensical). The assignment is as follows:

Let T be an equilateral triangle of area A and set r as the radius of its incircle C. Suppose that the origin O is contained within C, and choose a point (x, y, z) with prime distance p from O. Let G be the group of isometries fixing C and set ℓ, w and h as the length, width and height of the smallest bounding box B containing all translates of (x, y, z) under G.

Now fix an irreducible polynomial f defining a field K of characteristic p. Let V be a vector space over K of even dimension n and choose a matrix M in $SU(V)$. Then obviously $M_{i,j} < H$. QED

This yields the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
ANDBUERHLSITJGKMZXVPOQYWCF
```

The given description is of a Turing machine that operates on a tape with Morse code. Valid input is a tape filled with spaces, with a Morse code letter somewhere to the right of the head. The Turing machine starts in state 0 and runs, overwriting the tape. When it halts, the result is the Morse code for another letter. This yields the permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
TXYRAFUVNWGLMIOJZESPDHKBCQ
```

The given relations describe the symmetries of the shapes of unknown upper and lower case letters (without serifs, etc). τ represents reflection across the y-axis, σ rotation by 180 degrees around the center of a lower case x and ρ rotation by 180 degrees around the center of an upper case X.

$\sigma(\mathrm{n}) = \mathrm{u}$	$\mathrm{j} \subset \mathrm{i} \cup \mathrm{g}$	$\mathrm{x, s, o, z} \in \mathrm{Stabilizer}(\sigma)$
$\sigma(\mathrm{p}) = \mathrm{d}$	$\mathrm{F} \subset \mathrm{E}$	$\mathrm{x, c, o} \in \mathrm{Stabilizer}(\tau\sigma)$
$\tau(\mathrm{b}) = \mathrm{d}$	$\mathrm{Y} \subset \mathrm{X} \cup \mathrm{I}$	$\mathrm{t, w, i, l, v, A, T, I, H, V,} \in \mathrm{Stabilizer}(\tau)$
$\tau(\mathrm{q}) = \mathrm{p}$	$\mathrm{P} \subset \mathrm{R}$	$\mathrm{t, l, K, C, I, E, H, D} \in \mathrm{Stabilizer}(\tau\rho)$
$\rho(\mathrm{M}) = \mathrm{W}$	$\mathrm{Z} \subset \mathrm{X} \cup \mathrm{E}$	$\mathrm{N, O, Z} \in \mathrm{Stabilizer}(\rho)$

This yields the permutation

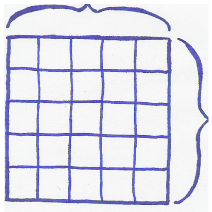
```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
AMRBTWNQUXKCFYISPEHOGJZLDV
```

The fact that the numerator and denominator are written backwards suggests looking at the Taylor series expansion at 0. Doing so yields

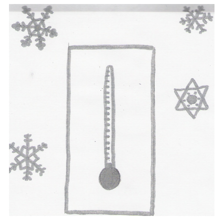
$$x + 2x^2 + 7x^3 + 4x^4 + 9x^5 + 6x^6 + 3x^7 + 8x^8 + 5x^9 + 10x^{10} + 15x^{11} + 12x^{12} + 17x^{13} + 14x^{14} + 11x^{15} + 16x^{16} + 13x^{17} + 18x^{18} + 23x^{19} + 20x^{20} + 25x^{21} + 22x^{22} + 19x^{23} + 24x^{24} + 21x^{25} + 26x^{26} + 31x^{27}$$

This gives the permutation

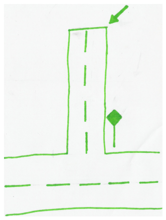
```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABGDIFCHEJOLQNKPMRWTYVSXUZ
```


area


blind

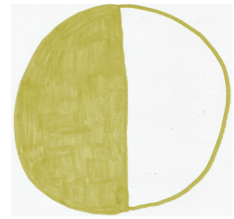

cold

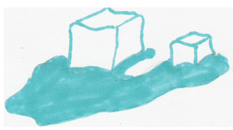

drink


end


foot


gothic

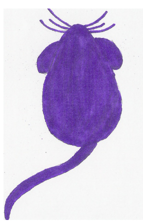

half

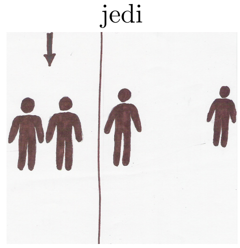

ice


jedi


key

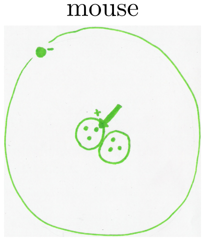

love

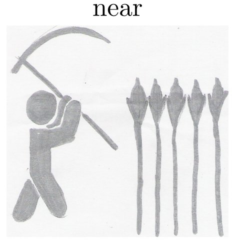

mouse

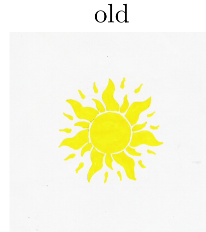

near

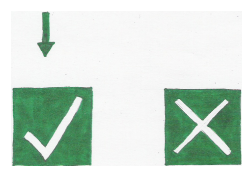

old

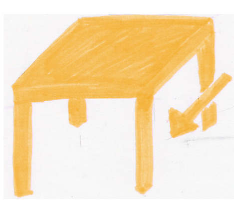

peace

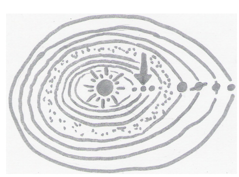

quark


reap


sun

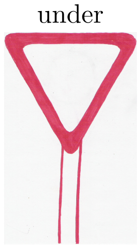

true


under


venus


well


xray


yield


zombie

Note that the colors of the pictures match the colors of the 13 permutation mini-puzzles in the first half. When you correctly identify a picture and apply the corresponding permutation you should get a new word:

AREA→TEAT	HALF→HATE	OLD→CAT	UNDER→MARYS
BLIND→PLANT	ICE→NEW	PEACE→START	VENUS→ROMAN
COLD→SITH	JEDI→WARN	QUARK→FIRED	WELL→SILL
DRINK→FALSE	KEY→FAR	REAP→LOCK	XRAY→LEAD
END→HOT	LOVE→SHOW	SUN→TWO	YIELD→TRAIN
FOOT→SEEK	MOUSE→FIGHT	TRUE→OVER	ZOMBIE→VOLUME
GOTHIC→LEPTON	NEAR→HEAD		

Most of these new words are the "opposite" of one of the other words. There are seven words, however, from which you need to remove a letter in order to obtain the opposite.

Picture	Opposite	Extra Letter
AREA	VOLUME	
BLIND	SEE	K
COLD	HOT	
DRINK	EAT	T
END	START	
FOOT	HEAD	
GOTHIC	ROMAN	
HALF	TWO	
ICE	FIRE	D
JEDI	SITH	
KEY	LOCK	
LOVE	HATE	
MOUSE	CAT	
NEAR	FAR	
OLD	NEW	
PEACE	WAR	N
QUARK	LEPTON	
REAP	SOW	H
SUN	RAIN	T
TRUE	FALSE	
UNDER	OVER	
VENUS	MARS	Y
WELL	ILL	S
XRAY	LEAD	
YIELD	FIGHT	
ZOMBIE	PLANT	

There is one picture for each letter (in alphabetical order, which should help with the identification). Tranforming via the permutations from the first part and then taking the opposite yields a new permutation of A through Z:

Picture	Transformed	Opposite
AREA	EAT	DRINK
BLIND	PLANT	ZOMBIE
COLD	SITH	JEDI
DRINK	FALSE	TRUE
END	HOT	COLD
FOOT	SEE	BLIND
GOTHIC	LEPTON	QUARK
HALF	HATE	LOVE
ICE	NEW	OLD
JEDI	WAR	PEACE
KEY	FAR	NEAR
LOVE	SOW	REAP
MOUSE	FIGHT	YIELD
NEAR	HEAD	FOOT
OLD	CAT	MOUSE
PEACE	START	END
QUARK	FIRE	ICE
REAP	LOCK	KEY
SUN	TWO	HALF
TRUE	OVER	UNDER
UNDER	MARS	VENUS
VENUS	ROMAN	GOTHIC
WELL	ILL	WELL
XRAY	LEAD	XRAY
YIELD	RAIN	SUN
ZOMBIE	VOLUME	AREA

Applying the resulting permutation

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
DZJTCBQLOPNRYFMEIKHUVGWXSA
```

to the extra letters yields the answer, "NUTFLUSH."