Recovery of atmospheric flow statistics in a general circulation model without nonlinear eddy-eddy interactions

Paul A. O'Gorman[1] and Tapio Schneider[1]

Received 22 August 2007; revised 3 October 2007; accepted 12 October 2007; published 16 November 2007.

[1] The closure problem of turbulence arises because nonlinear interactions among turbulent fluctuations (eddies) lead to an infinite hierarchy of moment equations for flow statistics. Here we demonstrate with an idealized general circulation model (GCM) that many atmospheric flow statistics can already be recovered if the hierarchy of moment equations is truncated at second order, corresponding to the elimination of nonlinear eddy-eddy interactions. Some, but not all, features of the general circulation remain the same. The atmospheric eddy kinetic energy spectrum retains a −3 power-law range even though this is usually explained in terms of an enstrophy cascade mediated by nonlinear eddy-eddy interactions. Our results suggest that it may be possible to construct fast general circulation models that solve for atmospheric flow statistics directly rather than via simulation of individual eddies and their interactions. **Citation:** O'Gorman, P. A., and T. Schneider (2007), Recovery of atmospheric flow statistics in a general circulation model without nonlinear eddy-eddy interactions, *Geophys. Res. Lett.*, *34*, L22801, doi:10.1029/2007GL031779.

1. Introduction

[2] The study of turbulent flows has focused on idealized isotropic and homogeneous geometries in which mean flows vanish and nonlinear interactions among eddies are of central importance in determining the higher-order flow statistics [*McComb*, 1992]. Large-scale (>500 km) turbulence in the atmosphere is an example of turbulent flow in which other effects such as interactions of eddies with the mean flow are important, and for which, as the results presented here suggest, a focus on nonlinear eddy-eddy interactions may be inappropriate. The most energetic transient eddies in the atmosphere—the familiar cyclones and anticyclones—have a length scale of ~3000 km [*Shepherd*, 1987; *Straus and Ditlevsen*, 1999]. Baroclinic instability is the main energy source for these eddies, and the length scale of the linearly most unstable wave (given by the Rossby deformation radius) and the energy-containing eddy length scale are of similar magnitude. The inverse eddy energy cascade to length scales much larger than the Rossby deformation radius that can occur in two-dimensional or quasigeostrophic models is not seen in the atmosphere, although energy transfer from eddies to the zonal-mean flow at scales larger than the Rossby deformation radius does occur [*Shepherd*, 1987]. Simulations and theory suggest that the absence of this kind of inverse energy cascade is not an accident but comes about through the effect of the eddies on the mean thermal structure of the atmosphere [*Schneider and Walker*, 2006]. Thus, nonlinear eddy-eddy interactions (e.g., the advection of eddy fluctuations in temperature by eddy fluctuations in the wind), which could lead to an inverse energy cascade, may be relatively unimportant in determining the energy-containing length scale of atmospheric eddies.

[3] Eddy-eddy interactions are also commonly invoked to explain the shape of the atmospheric eddy kinetic energy spectrum, which has an approximate n^{-3} power-law range in spherical wavenumber n at length scales smaller than the energy-containing eddy length scale [*Boer and Shepherd*, 1983; *Nastrom and Gage*, 1985; *Straus and Ditlevsen*, 1999; *Schneider and Walker*, 2006]. The n^{-3} range is commonly explained by analogy with two-dimensional or quasigeostrophic models of atmospheric flow, which can exhibit an enstrophy cascade with an n^{-3} energy spectrum [*Charney*, 1971; *Salmon*, 1998]. Enstrophy is proportional to the mean-square vorticity (in two dimensional models) or potential vorticity (in quasigeostrophic models), and inertial-range cascades involving eddy-eddy interactions are postulated for it and the kinetic energy because they are both quadratic invariants of the inviscid flow [*Kraichnan*, 1967]. An n^{-3} spectrum might also be expected over a wavenumber range in which vorticity or potential vorticity is effectively advected as a passive scalar. Such an energy spectrum is sufficiently steep that spectrally nonlocal eddy-eddy interactions may be important (such as strain of small-scale vorticity by large-scale eddies), leading to a logarithmic correction to the power-law spectrum [*Kraichnan*, 1971; *Salmon*, 1998]. In the atmosphere, however, an inertial range in which only eddy-eddy interactions are important does not occur. Other terms such as conversion from potential to kinetic energy, transfer of energy to the mean flow, and eddy dissipation (primarily in the planetary boundary layer) are known to be important in the spectral energy budget over a range of length scales [*Lambert*, 1987; *Straus and Ditlevsen*, 1999]. Thus, it is not clear why the energy spectrum of the atmosphere should follow an n^{-3} power law, and the enstrophy-cascade explanation for the atmospheric energy spectrum has been questioned [*Vallis*, 1992]. Furthermore, the baroclinic lifecycle experiments of *Gall* [1976] suggest (albeit indirectly) that the shape of the energy spectrum may not depend on eddy-eddy interactions.

[4] Rather than studying the processes that determine the mean state and energy spectrum in two-dimensional or quasigeostrophic systems, we use an atmospheric general circulation model (GCM), which more faithfully resolves the large-scale turbulence of the atmosphere by representing all important terms in the spectral energy budget and by allowing for dynamical changes in the thermal stratification. In removing the eddy-eddy interactions in a GCM, we

[1]California Institute of Technology, Pasadena, California, USA.

0094-8276/07/2007GL031779$05.00

directly test their role in determining the eddy length scale, the eddy energy spectrum of atmospheric turbulence, and the general circulation of the atmosphere. The removal of eddy-eddy interactions represents a turbulence closure equivalent to a truncation of the moment equations at second order by setting third-order eddy moments (third-order cumulants) to zero [*Herring*, 1963; *Marston et al.*, 2007]. While we eliminate the nonlinear interactions that involve only eddies, we do retain the effects of eddies on the mean flow because of the important role of eddy heat and momentum fluxes in equilibrating the mean flow.

2. Model Description

[5] We use the idealized GCM of *Schneider and Walker* [2006]. The model has a thermally insulating and uniform lower boundary, and the forcing is time-independent and axisymmetric. Consequently, there are no stationary waves, but the transient eddies that are the focus of this study are well resolved.

[6] The idealized GCM integrates the primitive equations on the sphere using the pseudospectral method [*Bourke*, 1974], with a horizontal resolution of T127 and with 30 unequally spaced σ-levels in the vertical. The planetary boundary layer has a fixed height of 2.5 km, and surface fluxes of momentum are calculated using a constant roughness length of 5 cm. Radiative processes are represented by linear relaxation of temperatures toward a radiative-equilibrium state that is axisymmetric, statically unstable in the lower troposphere, and which has a pole-to-equator surface temperature contrast of 90 K. The GCM does not account for moist processes, but a quasi-equilibrium convection scheme mimics some effects of moist convection by relaxing a temperature profile to a convective profile with lapse rate 6.8K km^{-1} when the temperature profile is less stable than the convective profile; see *Schneider and Walker* [2006] for details. An exponential cutoff filter [*Smith et al.*, 2002] is applied to all fields for numerical stability, with a damping timescale of 2 hours at the smallest resolved length scale and with zero damping for wavenumbers smaller than 50.

[7] Simulations with and without eddy-eddy interactions were spun up over 600 days to reach statistically steady states, and averages were taken over a subsequent 400 days.

3. Removal of Eddy-Eddy Interactions

[8] We decompose flow fields into a mass-weighted zonal mean and eddies. Defining eddies as fluctuations about a zonal mean is not the only possible choice, but it is computationally more efficient than, for example, using a temporal mean.

[9] Eddy-eddy interactions are removed by modifying the evolution equations for temperature and horizontal momentum so that eddy fluctuations are not advected by the eddy wind. Modifications are made only to nonlinear terms related to horizontal and vertical advection and to the metric terms in the horizontal momentum equations. These modifications do not alter the evolution equations for mean temperature or mean momentum, but they do alter the evolution equations for higher moments.

[10] For example, we do not allow for the advection of temperature fluctuations by wind fluctuations except insofar as it affects the evolution of the mean temperature. The meridional advection of temperature, T, contributes to the evolution of temperature in the full GCM as

$$\begin{aligned}\frac{\partial T}{\partial t} &= -v\,\frac{\partial T}{\partial y} + ...,\\ &= -\overline{v}\,\frac{\partial \overline{T}}{\partial y} - \overline{v}\,\frac{\partial T'}{\partial y} - v'\,\frac{\partial \overline{T}}{\partial y} - v'\,\frac{\partial T'}{\partial y} + \dots, \end{aligned} \qquad (1)$$

where v is the meridional wind and y is meridional distance. We denote by $\overline{T}$ and T' the zonal-mean and eddy temperatures, respectively, with $T = \overline{T} + T'$, and with a similar decomposition for the three-dimensional wind field. The surface pressure acts as a density in σ-coordinates, and so zonal means are defined on σ-levels with a surface pressure-weighting. Although we use Cartesian coordinates and derivatives here for simplicity, the equations of motion in the GCM are solved in spherical coordinates. The advection of temperature in the simulation without eddy-eddy interactions is written as

$$\frac{\partial T}{\partial t} = -\overline{v}\,\frac{\partial \overline{T}}{\partial y} - \overline{v}\,\frac{\partial T'}{\partial y} - v'\,\frac{\partial \overline{T}}{\partial y} - \overline{v'\,\frac{\partial T'}{\partial y}} + \dots, \qquad (2)$$

which includes only the mean part of the nonlinear eddy-eddy term.

[11] Taking the zonal mean of surface pressure times the modified evolution equations gives the result that the evolution equations for mean temperature and mean momentum are unchanged. Although we use surface pressure-weighted means, the global energy (the sum of enthalpy and horizontal kinetic energy in the hydrostatic approximation) is not quite conserved because of a term involving the horizontal eddy convergence of mass. This term arises because we define eddy and mean quantities on σ-levels and would not arise if the vertical coordinate was pressure (in which case, however, boundary terms would arise, which present similar difficulties).

4. Comparison of Simulations

4.1. Instantaneous Vorticity Fields

[12] Figure 1 shows typical instantaneous vorticity fields in the mid-troposphere in the full simulation and in the simulation without eddy-eddy interactions. In both simulations, as on Earth, most of the eddies are generated by baroclinic instability in extratropical storm tracks. The vorticity field of the full simulation is being wrapped and stretched into filaments by the cyclones and anticyclones, but the eddies in the simulation without eddy-eddy interactions do not advect themselves. The vorticity structures are being sheared apart by the mean zonal wind, which is an example of an important eddy-mean interaction that is present in both simulations [*Shepherd*, 1987; *Huang and Robinson*, 1998]. The synoptic variability (e.g., frontal occlusions) and higher-order statistics of the flow are clearly affected by eddy-eddy interactions, but below we show that several important low-order statistics are similar in both simulations.

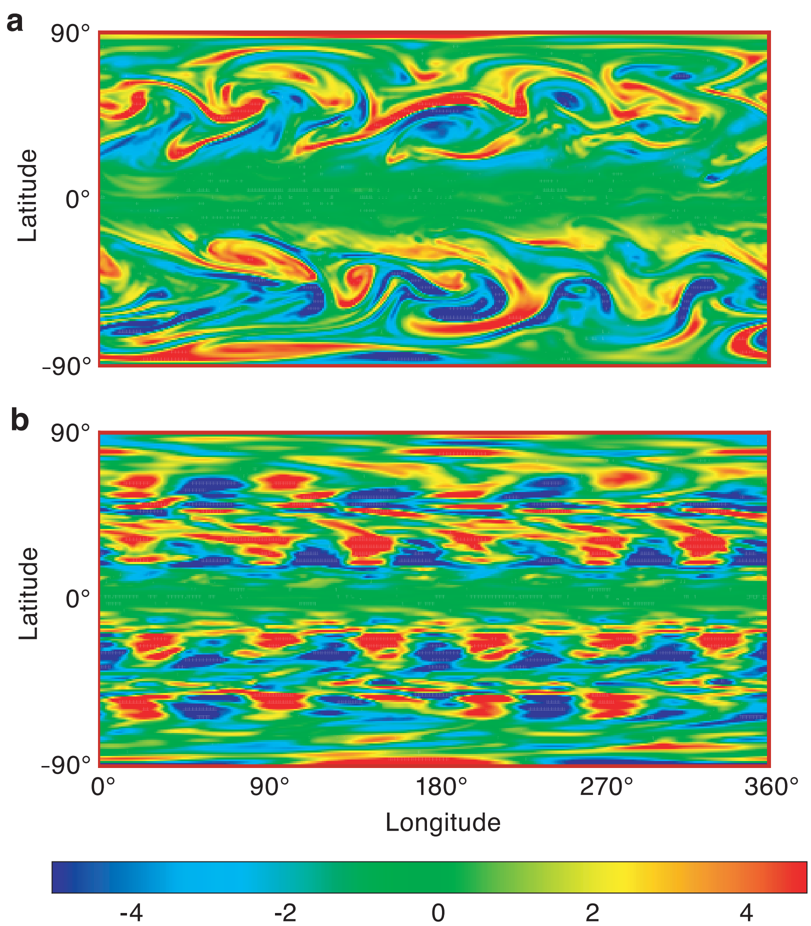

Figure 1. Typical instantaneous vorticity fields (10^{-5} s^{-1}) in (a) the full simulation and (b) the simulation without eddy-eddy interactions. The horizontal surface shown is in the mid-troposphere at $\sigma = 0.5$. The fields are shown at times after the simulations have reached statistically steady states.

4.2. Eddy Statistics

[13] The vertically averaged eddy kinetic energy spectrum gives the contribution at each horizontal wavenumber to the eddy kinetic energy and is shown in Figure 2 for both simulations. The spectra shown are defined using spherical wavenumbers [*Boer and Shepherd*, 1983] and are calculated as the mass-weighted vertical average of the eddy kinetic energy spectrum on σ-levels. Contributions from zonal wavenumber zero are omitted because they are associated with the zonal-mean flow. The total eddy kinetic energy is greater by a factor of 2.1 in the simulation without eddy-eddy interactions, representing a significant difference between the two simulations. We rescale the spectrum from the simulation without eddy-eddy interactions by this factor in Figure 2 to facilitate comparison of the shapes of the spectra. A possible explanation for the discrepancy in total eddy kinetic energy is that the scrambling of structures by the eddy-eddy interactions reduces the efficiency of conversion from potential to kinetic energy, leading to smaller eddy kinetic energy in the full simulation.

[14] The length scales of the dominant eddies appear comparable in the two vorticity fields (Figure 1), and this is quantitatively confirmed by the eddy kinetic energy spectra (Figure 2). The eddy kinetic energy spectrum peaks at spherical wavenumber 8 in both simulations, so that the energy-containing eddy length scale is almost exactly the same. The similarity of the eddy length scale in both simulations implies that an inverse energy cascade involving eddy-eddy interactions is not important in setting the eddy length scale. Consistent with this picture, the estimated Rossby deformation radius corresponds to wavenumbers 7 in the full simulation and 10 in the simulation without eddy-eddy interactions. This suggests that conversion from potential to kinetic energy occurs at comparable length scales in both simulations. The Rossby deformation radius is estimated following the method of *Schneider and Walker* [2006] using near-surface static stabilities and temperature gradients, except that we determine the tropopause using the conventional criterion as a 2 K km^{-1}-isoline of temperature lapse rate.

[15] It is striking that the shape of the energy spectrum in the full simulation is replicated over a wide range of wavenumbers in the simulation without eddy-eddy interactions. The approximate n^{-3} power-law range in the full simulation extends from the wavenumber at the maximum of the energy spectrum to a roll-off at spherical wavenumbers greater than ~60 caused by the damping filter that is used to stabilize the simulations at small scales. The energy spectrum of the simulation without eddy-eddy interactions also exhibits an approximate n^{-3} power-law range, but at spherical wavenumbers greater than ~40, there is evidence that the spectral decay becomes shallower. The

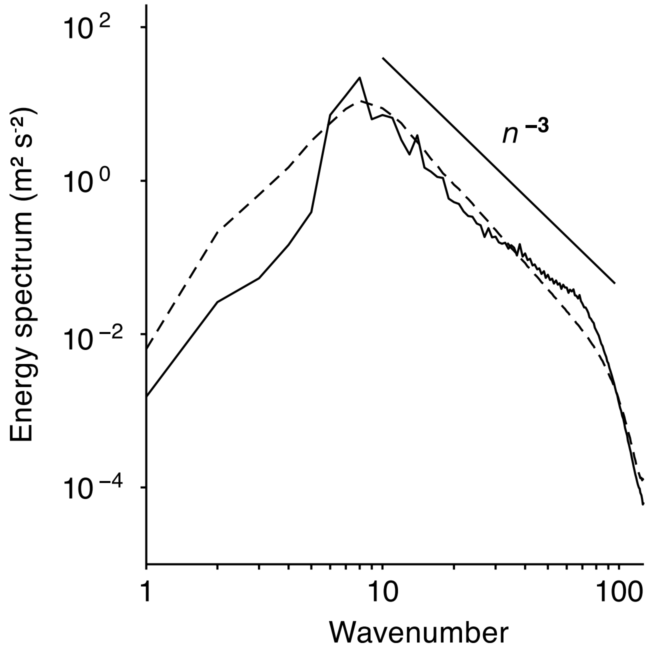

Figure 2. Vertically averaged eddy kinetic energy spectrum vs. spherical wavenumber for the simulation without eddy-eddy interactions (solid line) and the full simulation (dashed line). The total eddy kinetic energy of the simulation without eddy-eddy interactions is 2.1 times greater, and its energy spectrum has been divided by this factor for ease of comparison of the spectral shapes. Contributions from zonal wavenumber zero (the zonal mean) are omitted in calculating the eddy energy spectra. The solid straight line shows a reference power law of n^{-3}.

shallowing of the spectrum is reminiscent of the $n^{-5/3}$ mesoscale range that is seen in observational data near the tropopause [*Nastrom and Gage*, 1985], but here it is more likely an indication that eddy-eddy interactions prevent a build-up of energy at these relatively small length scales.

[16] That the shape of the eddy energy spectrum is recovered without eddy-eddy interactions over a wide wavenumber range helps resolve the paradox that the atmospheric energy spectrum has the power-law decay that would result from a enstrophy cascade in an inertial range even though there are significant terms in the atmospheric spectral energy budget other than those related to eddy-eddy interactions. Our results show that these other terms would by themselves lead to a similar energy spectrum. We also found similar results for other parameter settings in the GCM and using an idealized GCM [*Held and Suarez*, 1994] with different radiation and boundary layer schemes and without a convection scheme. Therefore, our results for the energy spectrum do not appear to be artifacts of specific parameter settings or parameterizations. The approximate n^{-3} spectrum in the simulation without eddy-eddy interactions may be explained by an unconventional enstrophy cascade in which the zonal-mean component plays a central role [cf. *Bartello and Warn*, 1988], but this is difficult to assess given the number of important terms in the spectral energy budget. Although we have only considered the atmosphere here, similar considerations may apply to the energy spectrum in the ocean at length scales smaller than the oceanic Rossby deformation radius.

[17] The eddy energy spectrum of the simulation without eddy-eddy interactions (Figure 2) is more jagged than the spectrum of the full simulation. One effect of eddy-eddy interactions, then, is to smooth the spectrum by transferring energy between wavenumbers. Eddy-eddy interactions also tend to isotropize the eddies in the horizontal. The two-dimensional spectral energy distribution reveals isotropization of the eddy energy in the full simulation for length scales smaller than the eddy length scale but anisotropy at all length scales in the simulation without eddy-eddy interactions.

4.3. Mean Circulations

[18] The general circulation of the simulation without eddy-eddy interactions shares many features with that of the full simulation, but there are also significant differences. The mean zonal wind in both simulations exhibits upper-level westerly jets, which illustrates that extratropical jets can form as a result of eddy-mean flow interactions alone, without nonlinear eddy-eddy interactions (Figure 3). However, the simulation without eddy-eddy interactions has a second jet in each hemisphere, and the Eulerian-mean circulation has corresponding extra eddy-driven cells. The mean circulation of the simulation without eddy-eddy interactions is compressed in the meridional direction relative to that of the full simulation. A possible explanation is that although most eddy energy resides at the energy-containing eddy length scale in the full simulation, and there is no general cascade of energy to larger scales, upscale energy transfer can still be expected to occur in the region of spectral space close to zonal wavenumber zero [*Rhines*, 1975; *Vallis and Maltrud*, 1993], leading to eddy

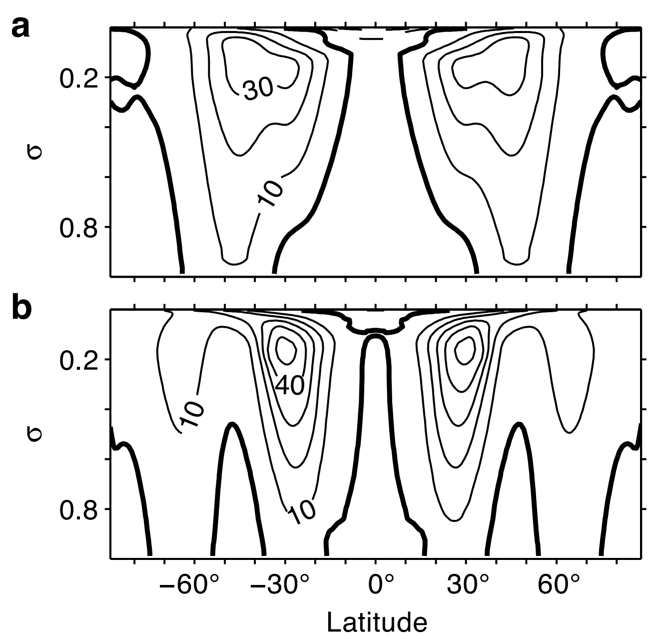

Figure 3. Mean eastward wind (m s^{-1}) in the meridional plane in (a) the full simulation and (b) the simulation without eddy-eddy interactions. The mean is a zonal, time, and interhemispheric average with mass weighting. The thick solid lines are the zero-wind lines.

energy at larger meridional length scales, which can then interact with the zonal mean flow. Consistent with this explanation, the energy spectrum of the full simulation at small wavenumbers (<8), while relatively small in magnitude, is larger than the rescaled energy spectrum of the simulation without eddy-eddy interactions (Figure 2).

5. Conclusions

[19] We have shown that eddy-eddy interactions are not essential for several features of large-scale flow in the atmosphere, although they are important for some features, such as the magnitude of the eddy kinetic energy. Our results indicate that the eddy length scale is not directly determined by eddy-eddy interactions, and so it may be possible to make simple modifications to the simulation without eddy-eddy interactions to better match the full simulation. For example, the addition of stochastic noise and damping to the linearized equations of motion has been shown to capture some of the effects of eddy-eddy interactions in a GCM [*Zhang and Held*, 1999] and in quasigeostrophic models [*Delsole and Farrell*, 1996]. This also opens up the possibility of constructing a GCM in which climate statistics are obtained by solving a closed set of moment equations directly rather than by explicitly resolving eddies [*Marston et al.*, 2007], although further approximations might be needed. Such a GCM would have an advantage in terms of conceptual simplicity. It could also be computationally more efficient than a full GCM, particularly for simulations with statistical zonal symmetry [*Marston et al.*, 2007].

[20] **Acknowledgments.** This work was supported by the National Science Foundation (Grant ATM-0450059), the Davidow Discovery Fund, and a David and Lucile Packard Fellowship. The numerical simulations were performed on Caltech's Division of Geological and Planetary Sciences Dell cluster.

References

Bartello, P., and T. Warn (1988), A one-dimensional turbulence model; enstrophy cascades and small-scale intermittency in decaying turbulence, *Geophys. Astrophys. Fluid Dyn.*, *40*, 239–259.
Boer, G. J., and T. G. Shepherd (1983), Large-scale two-dimensional turbulence in the atmosphere, *J. Atmos. Sci.*, *40*, 164–184.
Bourke, W. (1974), A multi-level spectral model. I. Formulation and hemispheric integrations, *Mon. Weather Rev.*, *102*, 687–701.
Charney, J. G. (1971), Geostrophic turbulence, *J. Atmos. Sci.*, *28*, 1087–1095.
Delsole, T., and B. F. Farrell (1996), The quasi-linear equilibration of a thermally maintained, stochastically excited jet in a quasigeostrophic model, *J. Atmos. Sci.*, *53*, 1781–1797.
Gall, R. (1976), Structural changes of growing baroclinic waves, *J. Atmos. Sci.*, *33*, 374–390.
Held, I. M., and M. J. Suarez (1994), A proposal for the intercomparison of the dynamical cores of atmospheric general circulation models, *Bull. Am. Meteorol. Soc.*, *75*, 1825–1830.
Herring, J. R. (1963), Investigation of problems in thermal convection, *J. Atmos. Sci.*, *20*, 325–338.
Huang, H. P., and W. A. Robinson (1998), Two-dimensional turbulence and persistent zonal jets in a global barotropic model, *J. Atmos. Sci.*, *55*, 611–632.
Kraichnan, R. H. (1967), Inertial ranges in two-dimensional turbulence, *Phys. Fluids*, *10*, 1417–1423.
Kraichnan, R. H. (1971), Inertial-range transfer in two- and three-dimensional turbulence, *J. Fluid Mech.*, *47*, 525–535.
Lambert, S. J. (1987), Spectral energetics of the Canadian Climate Centre general circulation model, *Mon. Weather Rev.*, *115*, 1295–1304.
Marston, J. B., E. Conover, and T. Schneider (2007), Statistics of an unstable barotropic jet from a cumulant expansion, *J. Atmos. Sci.*, in press.
McComb, W. D. (1992), *The Physics of Fluid Turbulence*, Oxford Univ. Press, New York.
Nastrom, G. D., and K. S. Gage (1985), A climatology of atmospheric wavenumber spectra of wind and temperature observed by commercial aircraft, *J. Atmos. Sci.*, *42*, 950–960.
Rhines, P. B. (1975), Waves and turbulence on a beta-plane, *J. Fluid. Mech.*, *69*, 417–443.
Salmon, R. (1998), *Lectures on Geophysical Fluid Dynamics*, Oxford Univ. Press, New York.
Schneider, T., and C. C. Walker (2006), Self-organization of atmospheric macroturbulence into critical states of weak nonlinear eddy-eddy interactions, *J. Atmos. Sci.*, *63*, 1569–1586.
Shepherd, T. G. (1987), A spectral view of nonlinear fluxes and stationary-transient interaction in the atmosphere, *J. Atmos. Sci.*, *44*, 1166–1178.
Smith, K. S., G. Boccaletti, C. C. Henning, I. Marinov, C. Y. Tam, I. M. Held, and G. K. Vallis (2002), Turbulent diffusion in the geostrophic inverse cascade, *J. Fluid Mech.*, *469*, 13–48.
Straus, D. M., and P. Ditlevsen (1999), Two-dimensional turbulence properties of the ECMWF reanalyses, *Tellus, Ser. A*, *51*, 749–772.
Vallis, G. K. (1992), Problems and phenomenology in two-dimensional turbulence, in *Non-linear Phenomena in Atmospheric and Oceanic Sciences*, edited by G. F. Carnevale and R. T. Pierrehumbert, pp. 1–25, Springer, New York.
Vallis, G. K., and M. E. Maltrud (1993), Generation of mean flows and jets on a beta plane and over topography, *J. Phys. Oceanogr.*, *23*, 1346–1362.
Zhang, Y., and I. M. Held (1999), A linear stochastic model of a GCM's midlatitude storm tracks, *J. Atmos. Sci.*, *56*, 3416–3435.

P. A. O'Gorman and T. Schneider, California Institute of Technology, MC 100-23, Pasadena, CA 91125, USA. (pog@caltech.edu)