Reliability and Route Diversity in Wireless Networks

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***Abstract*— We study the problem of communication reliability and diversity in multi-hop wireless networks. Our aim is to develop a new network model that better takes into account the fading nature of the wireless physical layer. To that end, we use the outage probability model for a fading channel to develop a probabilistic model for a wireless link. This model establishes a relationship between the link reliability, the distance between the communicating nodes and the transmission power. Applying this probabilistic model to a multi-hop network setting, we define and analyze the end-to-end route reliability and develop algorithms for finding the optimal route between a pair of nodes. The relationship between the reliability of the optimal route and the consumed power is studied. The idea of route diversity is introduced as a way to improve the end-to-end route reliability by taking advantage of the wireless broadcast property, the independence of the fade state between different pairs of nodes, and space diversity created by multiple intermediate relay nodes along the route. We give analytical results for the improvements due to route diversity in some simple network topologies and present simulations for more general networks.**

I. Introduction

The area of ad-hoc and sensor networks has received a lot of attention in the research community over the past several years. In this paper, we look at the problem of routing and reliability in these networks.

Motivated by results from propagation of electromagnetic signals in space, the amount of energy required to establish a link between two nodes is usually assumed to be proportional to the distance between the communicating nodes raised to a constant power. This fixed exponent, referred to as the path-loss exponent, is usually assumed to be between 2 to 4. In this model, it is assumed that the information is received by the intended destination with certainty if the source transmits the information at a minimum power level dictated by its distance to the intended destination. We will refer to this model as the *deterministic link model* in this paper since the set of nodes that receive the transmitted information is known with certainty based on the transmission power level chosen by the transmitter.

Due to this relationship between the distance between nodes and the required power, it is usually beneficial, in terms of energy savings, to relay the information through a multi-hop route in an ad-hoc network. Figure 1 shows an example of a multi-hop route between two nodes.

The deterministic model for a wireless link, however, may not be very realistic for describing one of the most important effects in wireless communication, the multi-path fading. The received signal in a wireless link is the sum of signals reflected by different scatterers in the propagation environment. A link is said to be in a deep fade state when the reflected signals add destructively at the receiver. Naturally, a higher transmission power is required to establish a link between two nodes when the channel between them is in deep fade. Since the fade state of a link changes over time, the amount of energy required to transfer a unit of information between any two nodes changes over time as well. The simple deterministic model for a wireless link does not take into account this time varying nature of the wireless propagation medium.

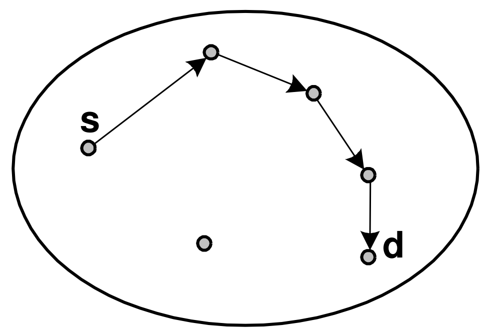

Fig. 1. Multi-hop Routing

Depending on how fast channel changes occur, the time varying nature of the wireless channel can be addressed in two different ways. If the channel changes relatively fast, coding can be done to average the effect of fading. This type of averaging effect is the motivation behind the *ergodic capacity* model for fading channels (see [9]). To achieve this type of average behavior, however, very long delays might be imposed on the data. In situations where the ergodic capacity is achievable, the deterministic model for the wireless link may still be applicable with some minor changes. We will not get into the details here as our goal in this paper is to propose an alternative model for a wireless link which is more suitable for scenarios in which this type of average behavior is not appropriate. The case of delay sensitive data and slowly changing channel is one example.

An alternative model for the wireless link in this scenario is the *outage probability* model, see [9], [5], and [6]. In this model, the instantaneous capacity of a wireless link is treated as a random variable. A link is said to be in outage when the instantaneous capacity supported by the link is less than the transmission rate. The reliability of a link, i.e. the probability of correct reception at the receiver, is modeled as a function of the transmission rate, the transmitted power, the distance between the communicating nodes, and the channel fade state. In this paper, we assume that the fade state is not known to the transmitter. Under this setting, the transmitter can control the probability of successful reception at its intended receiver by adjusting the transmission rate or power. We refer to this model as the *probabilistic link* model.

There are several ways to avoid losing data when the channel is in outage, such as coding over a long period of time, employing ARQ protocols, or obtaining transmitter side channel information. However, in this study, we focus on the reliability of a link without using any of such techniques. This approach allows us to isolate the issue of obtaining diversity through routing, and the results developed here can be readily applied in combination with other forms of diversity techniques.

Our analysis starts by looking at the reliability of a point-to-point communication link. In section II, we develop a model of how the channel fade state and the distance between the communicating nodes affects the probability of correct reception of the transmitted information at the receiver This would give us the mathematical formulation for the *probabilistic link* model.

In section III, we extend the probabilistic link model to a network setting. In a network setting, we first define and analyze the reliability for a fixed route and then develop algorithms for finding the optimal route between a source-destination pair of nodes. The trade-off between the route reliability and the consumed power is studied. To our knowledge, this is the first attempt to introduce the concept of route reliability and the end-to-end reliability versus power trade-off in a network setting. More precisely, this is the first time that network layer routing algorithms and route properties, such as reliability and power, are studied based on the outage probability model at the physical layer. This model has the potential to open the door for a wide-range of research on wireless network reliability.

In section IV, we introduce the idea of *route diversity* as a way to improve route reliability by taking advantage of the wireless broadcast property and the independence of fade state between different pairs of nodes. We give analytical results on improvements due to route diversity in some simple network topologies and show how route diversity can fundamentally change the trade-off between the route reliability and the consumed power.

The idea of route diversity is motivated by the work done in [1], [2], [3], and [4]. Most pervious results have been focused on two-hop networks, and the analysis has been based on the information theory results for relay channels. References [1], [2], and [3] look at the effect of cooperation among nodes in increasing the capacity or reducing the outage probability in a fading network. In [1] and [3], the authors described several protocols for benefiting from the space diversity created by the relays in an ad-hoc network. They look at the trade-off between the capacity and the outage improvement in a two-hop ad-hoc network. This analysis ignores the deterministic part of link attenuation due to the distance between nodes and assume all link fading factors are independent and identically distributed Rayleigh random variables. While [4] looks at the asymptotic benefit of relay nodes in improving the capacity in an ad-hoc network. Their analysis only takes into account the deterministic part of link attenuation due to the distance between nodes. Their results mainly deal with how the capacity scales as a function of the number of nodes in the network.

II. Probabilistic Link Model

In this section, we develop the analytical framework for the probabilistic link model. This framework determines the relationship between the probability of successful reception, the distance between the communicating nodes, and the transmission power in a point-to-point single-user flat Rayleigh fading link. We model the received signal as:

$$\mathsf{y} = \mathsf{a}\ \mathsf{x} + \eta,$$

where x is the transmitted signal, η is the additive received noise, a is the signal attenuation due to propagation in the wireless point-to-point link, and y is the received signal. We assume the received noise, η, is zero mean additive white Gaussian noise with average power of σ_η^2. In general, attenuation, a, depends on the distance between the communicating nodes and the fade state of the channel. We use d to represent the distance between the communicating nodes and f to represent the fading state of the channel. To emphasize this dependence, we can express a as an explicit function of these two parameters:

$$\mathsf{y} = \mathsf{a}(\mathsf{f}, \mathsf{d})\mathsf{x} + \eta. \tag{1}$$

In a system with mobile nodes and a constantly changing propagation environment, both f and d change over time. However, we assume a system where f and d remain constant for a long period of time compared to the typical transmission block length. Furthermore, we assume that the transmission blocks are long enough that coding can be done to average over the Gaussian noise. Given these assumptions, the link between two nodes is a simple AWGN channel and the capacity, i.e., the amount of information that can be reliably transmitted through this channel (see[11]), is given by:

$$\mathsf{C}(\mathsf{f}, \mathsf{d}, |\mathsf{x}|^2, \sigma_\eta^2) = \log(1 + \frac{|\mathsf{a}(\mathsf{f}, \mathsf{d})|^2|\mathsf{x}|^2}{\sigma_\eta^2})$$

To simplify this notation, we decompose $\mathsf{a}(\mathsf{f}, \mathsf{d})$ into two independent components corresponding to the small scale fading and the large scale path loss (see [10]). More specifically, we assume:

$$|\mathsf{a}(\mathsf{f}, \mathsf{d})|^2 = \frac{|\mathsf{f}|^2}{\mathsf{d}^\mathsf{k}},$$

where k is the propagation power loss exponent, usually assumed to be between 2 to 4. Simplifying the capacity formula using this form for a, and simplifying the notation by using $\frac{|x|^2}{\sigma_\eta^2} = \mathsf{snr}$, we get:

$$C(f, d, \mathsf{snr}) = \log(1 + \frac{|f|^2}{d^k}\ \mathsf{snr}). \tag{2}$$

A. Outage Formulation

Eq. (2) gives the instantaneous capacity of the point-to-point link defined by (1). An outage event is said to have occurred (see [5]) when the transmission rate, R bits/channel-use, is above the instantaneous capacity of the link, i.e.

$$\{\text{Outage}\} \stackrel{\text{def}}{=} \{C(f, d, \mathsf{snr}) < R\}. \tag{3}$$

One parameter of interest in communication systems is the probability of error at the receiver. An error occurs if the channel is in outage or if the channel is not in outage but there is a decoding error. In our analysis, we assume that the probability of decoding error is almost zero when channel is not in outage. Under this assumption, outage is the dominating error event. Hence:

$$P_{\text{Error}} \approx P(\text{Outage})$$

We focus our attention on calculating the outage probability. Based on the definition given in (3), the outage probability is given by:

$$\begin{aligned} P_{\text{Outage}} &= P\{C(f, d, \mathsf{snr}) < R\} \\ &= P\{\log(1 + \frac{|f|^2}{d^k}\mathsf{snr}) < R\} \\ &= P\{\frac{|f|^2}{d^k} < \frac{2^R - 1}{\mathsf{snr}}\}. \end{aligned} \tag{4}$$

Similar to the approach taken in [1] and other works in this area, we normalize the transmission rate by absorbing its effect into the snr term. So we define:

$$\mathsf{snr}_{\text{norm1}} = \frac{\mathsf{snr}}{2^R - 1}. \tag{5}$$

Equation (4) simplifies to:

$$P_{\text{Outage}} = P\{\frac{|f|^2}{d^k} < \frac{1}{\mathsf{snr}_{\text{norm1}}}\}. \tag{6}$$

For the case that fading, f, is random and distance, d, is known to the transmitter, the outage probability simplifies to:

$$\begin{aligned} P_{\text{Outage}} &= P\left(\frac{|f|^2}{d^k} < \frac{1}{\mathsf{snr}_{\text{norm1}}}\right) \\ &= F_{|f|^2}(\frac{d^k}{\mathsf{snr}_{\text{norm1}}}). \end{aligned}$$

where $F_{|f|^2}$ is the CDF of $|f|^2$. In our analysis, we model fading as a Rayleigh random variable. For Rayleigh fading with $E\left[|f|^2\right] = \mu$, the CDF is given by:

$$F_{|f|^2}(x) = 1 - \exp(\frac{x}{\mu}).$$

Hence:

$$P_{\text{Outage}} = 1 - \exp(\frac{-d^k}{\mu\ \mathsf{snr}_{\text{norm1}}}). \tag{7}$$

To simplify the notation, we can absorb the effect of μ into the value of $\mathsf{snr}_{\text{norm1}}$ by defining $\mathsf{snr}_{\text{norm2}}$ as:

$$\mathsf{snr}_{\text{norm2}} = \mu\ \mathsf{snr}_{\text{norm1}}.$$

For notational convenience, we drop the subscript in the subsequent analysis. The probability of successful reception, or equivalently the reliability, for a Rayleigh fading link with fixed distance is given by the following simple expression:

$$P_{\text{Succ}}(d, \mathsf{snr}) = \exp(-\frac{d^k}{\mathsf{snr}}). \tag{8}$$

B. Link Outage-Power Trade-Off

Eq. (8) gives the probability that a link is established between two nodes located a distance d apart when the transmitter transmits the information at the normalized power level of snr. There is a trade-off between the success probability and the transmitter power level. This trade-off can be visualized by plotting the success probability versus the transmitted snr. However, it turns out that plotting the outage probability vs. the snr is more insightful. Figure 2 shows the outage probability as a function of the transmitted snr for a point-to-point link. A Log-log scale is used to increase the range of values covered by the plot.

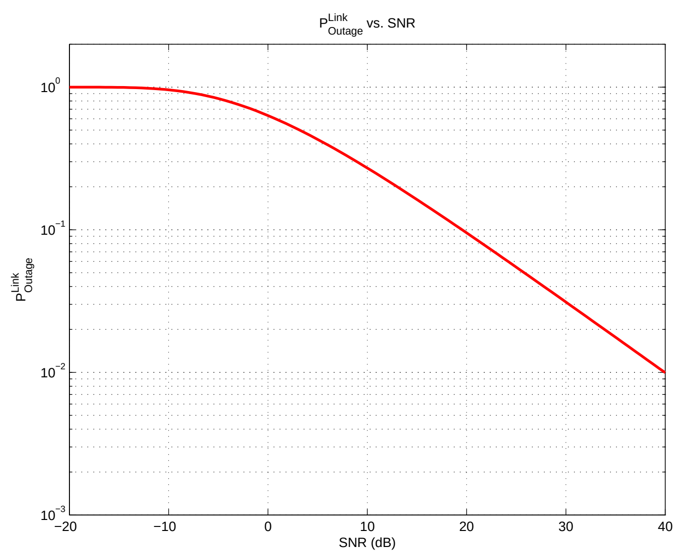

Fig. 2. Link Outage Probability vs. the Transmitted Power

It is clear that the outage probability decays linearly in the log-log plot in the high-snr regime. The *diversity gain*, d, defined as

$$\lim_{\mathsf{snr}\to\infty} \frac{P_{\text{Outage}}(\mathsf{snr})}{\log(\mathsf{snr})} = -d \tag{9}$$

characterizes the relationship between the outage probability and the snr in the high-snr regime. For a point to point link with a single transmitting and receiving antenna, it is known that the outage probability decays as snr^{-1} in the high-snr regime, i.e. the diversity gain is 1. See [8] for advanced coverage of this topic and the effect of multiple antenna on this relationship. In the next section, we look at a similar plot for a multi-hop route and introduce the idea of diversity gain in a network setting.

III. Reliability at the Network Layer

A route is a sequence of nodes through which the information is relayed from a source to a destination, i.e.

$$\mathsf{Route} \quad = \quad (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$$

where, $\mathsf{r}_0 = \mathsf{s}$, $\mathsf{r}_\mathsf{h} = \mathsf{d}$, and h is the number of hops. We assume the network operates based on a time division protocol under which successive transmissions along a route happen in consecutive transmission slots. Route $(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d})$ is identical to a sequence of h point-to-point links, where for the i^{th} link, relay $\mathsf{i}-1$ is the transmitter and relay i is the receiver, $\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$ is the transmitted signal-to-noise power, and $\mathsf{d}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$ is the distance between the nodes. We define the event of successful end-to-end transmission as the event that all h transmissions are successful and the *End-to-End Reliability* is defined as the probability of this event. We assume that the fading factors for different links are independent and identically distributed Rayleigh random variables. Based on this assumption and using results from (8), the end-to-end reliability can be written as:

$$\begin{aligned}\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} &= \prod_{\mathsf{i}=1}^{\mathsf{h}} \exp\left(-\frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right) \\ &= \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right). \quad (10)\end{aligned}$$

The corresponding total amount of power spent for successful end-to-end transmission is

$$\mathsf{SNR}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})}_{\mathsf{Total}} = \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}. \quad (11)$$

In the subsequent analysis, we use the route reliability defined by (10) and the *route outage probability*, ρ, given below interchangeably.

$$\rho^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} = 1 - \mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})}. \quad (12)$$

Our goal is to find optimal route selection and power allocation algorithms that maximize the end-to-end reliability for a fixed power. There are three different questions in connection with the end-to-end reliability and power that we consider:

1) What is the end-to-end reliability if the maximum transmitted power per link is fixed?
2) What is the minimum total power required to achieve a guaranteed level of end-to-end reliability?
3) What is the maximum end-to-end reliability for a fixed total power?

The first problem is motivated by the fact that in some cases the transmitted power by each node might be limited due to hardware constraints or to limit the interference level to other nodes. The second problem is a power allocation problem, where the objective is to minimize the total consumed power subject to a guaranteed level of end-to-end reliability. The last problem is also a power allocation problem, where the objective is to maximize the end-to-end reliability of a route subject to a total power constraint.

1) **Maximum End-to-End Reliability for a Fixed Maximum Transmission Power Per Link**

Assuming the transmitted signal-to-noise ratio at each link is limited to $\mathsf{SNR}_{\mathsf{Link-Max}}$, the corresponding route reliability can be readily calculated using (10). For a fixed route, $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$, the end-to-end reliability is given by:

$$\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} = \exp\left(-\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{SNR}_{\mathsf{Link-Max}}}\right). \quad (13)$$

According to this expression, the end-to-end reliability is a monotonically decreasing function of $\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$. This quantity can be treated as the cost metric for route selection. The most reliable route between two nodes is the route that minimizes this cost metric. We refer to route selection algorithm based on this cost metric as the **M***inimum* **O***utage* **R***oute*, *MOR*, algorithm.

Lemma 1: The most reliable route between nodes s and d in a fixed multi-hop wireless network where the fading parameters of different links are independent Rayleigh random variables and the maximum transmitted snr at each node is limited to $\mathsf{SNR}_{\mathsf{Max-Link}}$ is the route

$$(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d}) = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$$

that minimizes

$$\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}},$$

and the reliability of this route is given by (13).

2) **Minimum End-to-End Power for a Guaranteed End-to-End Reliability**

The problem of minimizing the end-to-end power for a fixed route, $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$, and fixed end-to-end reliability, $\mathsf{Reliability}_{\mathsf{Min}}$, is formulated by the following constrained optimization problem:

$$\begin{aligned}\min \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} \\ \text{s.t.} \quad & \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right) \geq \mathsf{Reliability}_{\mathsf{Min}}. \quad (14)\end{aligned}$$

Since exponential is a monotonically increasing function, the constraint must be satisfied with equality at the optimal solution. So, the optimization problem is equivalent to:

$$\begin{aligned}\min \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} \\ \text{s.t.} \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}} = -\ln(\mathsf{Reliability}_{\mathsf{Min}}). \quad (15)\end{aligned}$$

The Lagrangian for this problem is given by:

$$\mathsf{L}(\mathsf{snr}_{\mathsf{r_0r_1}},\cdots,\mathsf{snr}_{\mathsf{r_{h-1}r_h}},\lambda)$$
$$=\sum_{i=1}^{h}\mathsf{snr}_{\mathsf{r_{i-1}r_i}}+\lambda\left(\sum_{i=1}^{h}\frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{snr}_{\mathsf{r_{i-1}r_i}}}+\ln(\mathsf{Reliability_{Min}})\right).$$

The partial derivatives with respect to the transmitted snr at each intermediate relay is:

$$\frac{\partial \mathsf{L}}{\partial \mathsf{snr}_{\mathsf{r_{i-1}r_i}}} = 1-\lambda\,\frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{snr}^2_{\mathsf{r_{i-1}r_i}}}.$$

Setting these first order conditions to 0 and solving for the optimal transmitted snr, we get:

$$\widehat{\mathsf{snr}}_{\mathsf{r_0r_1}} = \sqrt{\lambda\ \mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}} \tag{16}$$

Substituting these into the constraint and solving for the optimal λ, we get:

$$\sqrt{\widehat{\lambda}} = \frac{\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}}{-\ln(\mathsf{Reliability_{Min}})}. \tag{17}$$

Substituting this back into (16), the optimal transmitted signal-to-noise ratio for each node is given by:

$$\widehat{\mathsf{snr}}_{\mathsf{r_{i-1}r_i}} = \frac{\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}}{-\ln(\mathsf{Reliability_{Min}})}\ \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}. \tag{18}$$

The resulting optimal end-to-end power is given by:

$$\begin{aligned}\widehat{\mathsf{SNR}}_{\mathsf{Total}} &= \sum_{i=1}^{h}\widehat{\mathsf{snr}}_{\mathsf{r_{i-1}r_i}}\\ &= \sum_{i=1}^{h}\left(\frac{\sum_{j=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{j-1}r_j}}}}{-\ln(\mathsf{Reliability_{Min}})}\right)\ \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}\\ &= \frac{\left(\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}\right)^2}{-\ln(\mathsf{Reliability_{Min}})}.\end{aligned} \tag{19}$$

For easier future reference, we state this result in lemma 2.

Lemma 2: For a fixed route $(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})$, the minimum required total power to guarantee the end-to-end reliability of $\mathsf{Reliability_{min}}$ is

$$\widehat{\mathsf{SNR}}_{\mathsf{Total}} = \frac{\left(\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}\right)^2}{-\ln(\mathsf{Reliability_{Min}})},$$

and the power allocation scheme that achieves this total consumed power is

$$\widehat{\mathsf{snr}}_{\mathsf{r_{i-1}r_i}} = \frac{\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}}{-\ln(\mathsf{Reliability_{Min}})}\ \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}.$$

From lemma 2, we know that for any route, $(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})$, and under optimal power allocation scheme, the total power required to achieve a desired level of end-to-end reliability is a monotonically increasing function of $\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}$. Hence, the minimum power route is the route among all possible routes between two nodes that minimizes this sum. We refer to this route selection scheme as **M***minimum* **E***nergy* **R***oute*, *MER*, algorithm.

Theorem 1: The minimum power route between nodes s and d in a multi-hop wireless network where the fading parameters for different links are independent Rayleigh random variables to achieve guaranteed end-to-end reliability of $\mathsf{Reliability_{Min}}$ is the route

$$(\mathsf{s},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{d}) = (\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})$$

that minimizes

$$\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}},$$

and the corresponding end-to-end power is given by (19).

3) **Maximum End-to-End Reliability for a Fixed End-to-End Power**

The problem of achieving maximum end-to-end reliability for a fixed route, $(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})$, and fixed end-to-end power, $\mathsf{SNR_{Total-Max}}$, can also be formulated by the following constrained optimization problem:

$$\begin{aligned}\max \quad & \exp\left(-\sum_{i=1}^{h}\frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{snr}_{\mathsf{r_{i-1}r_i}}}\right)\\ \text{s.t} \quad & \sum_{i=1}^{h}\mathsf{snr}_{\mathsf{r_{i-1}r_i}} \le \mathsf{SNR_{Total-Max}}.\end{aligned} \tag{20}$$

This problem can be solved using a technique very similar to the approach used to solve (14). In fact, as it is elaborated in the next section, these two problems are dual of each other. Skipping the details of the optimization solution, we simply present the solution to (20) in lemma 3.

Lemma 3: For a fixed route $(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})$ and for a fixed end-to-end power of $\mathsf{SNR_{Total-Max}}$, the maximum end-to-end reliability is

$$\mathsf{Reliability_{Optimal}} = \exp\left(-\frac{\left(\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}\right)^2}{\mathsf{SNR_{Total-Max}}}\right), \tag{21}$$

and the optimal power allocation that achieves this reliability is

$$\widehat{\mathsf{snr}}_{\mathsf{r_{i-1}r_i}} = \mathsf{SNR_{Total-Max}}\frac{\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}}{\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}}.$$

From lemma 3, we know for any route, $(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})$, and under the optimal power allocation scheme the end-to-end reliability is a monotonically decreasing function of $\sum_{i=1}^{h}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}$.

Hence, the maximum reliability route is the route that minimizes this sum. We state this result in the following theorem.

Theorem 2: The most reliable route between nodes s and d in a fixed multi-hop wireless network where the fading parameters of different links are independent Rayleigh random variables and the maximum end-to-end power is limited to $\mathsf{SNR}_{\mathsf{Total-Max}}$ is the route

$$(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d}) = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$$

that minimizes

$$\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}},$$

and the corresponding end-to-end reliability is given by (21).

A. Optimal Reliability-Power Curve

A careful reader might notice that the two optimization problems that we looked at in the last section, formulated in (14) and (20), are in fact dual problems. Hence, it is not surprising that the cost metric in both cases turned out to be $\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}$. To clarify this point, we present a graphical illustration of the relationship between the end-to-end reliability and power under optimal power allocation among the nodes along the route.

For any fixed route, different power allocation schemes result in different end-to-end reliability and consumed power. If we were to characterize each power allocation scheme only by the total consumed power and the resulting end-to-end reliability, each allocation scheme could be represented by a point in the two dimensional plot of the end-to-end reliability vs. the total power. Certain allocation schemes are optimal, i.e., either minimize the total power consumed to achieve a guaranteed end-to-end reliability or maximize the end-to-end reliability for a fixed consumed power.

In problem 2, we found the optimal power allocation to minimizes the total power subject to a guaranteed end-to-end reliability. Graphically, this optimization corresponds to moving along the horizontal line in figure 3 and finding the allocation scheme that minimized the total consumed power for the end-to-end reliability of $\mathsf{Reliability}_{\mathsf{min}}$. We found that the reliability and power corresponding to the optimal allocation are related by the following relationship:

$$\widehat{\mathsf{SNR}}_{\mathsf{Total}} = \frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{-\ln(\mathsf{Reliability}_{\mathsf{Min}})}. \tag{22}$$

In problem 3, we found the optimal power allocation to maximize the end-to-end reliability for a given end-to-end power. This corresponds to moving along the vertical line in figure 3 and finding the allocation scheme that maximizes the reliability for $\mathsf{SNR}_{\mathsf{Total-Max}}$ We found that the resulting end-to-end reliability for this optimal allocation is:

$$\mathsf{Reliability}_{\mathsf{Optimal}} = \exp\left(-\frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right) \tag{23}$$

Clearly, the curve specified by (22) and (23) are identical. This set of optimal power allocation can be represented by a single curve in the two dimensional plot of the end-to-end reliability vs. total power as shown in figure 3. We refer to this curve as the *Optimal Reliability-Power Trade-off* curve. The relationship between the end-to-end reliability and consumed power for power allocation schemes on this curve is given by (23).

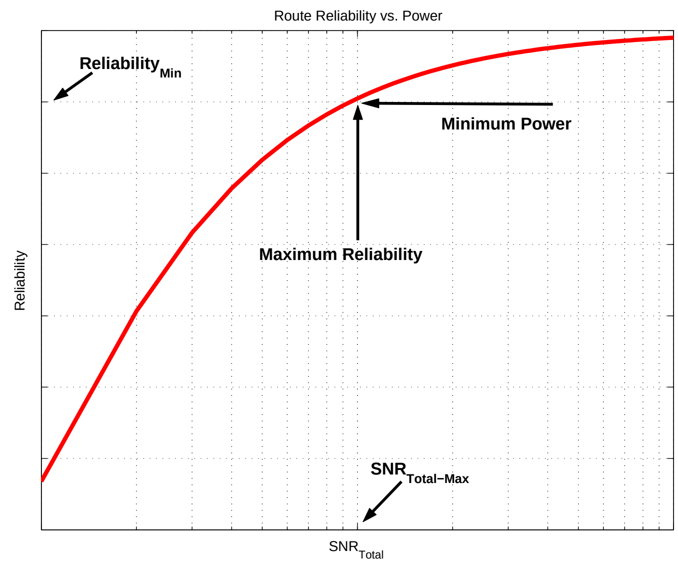

Fig. 3. Route Reliability vs. Power

B. Route Outage-Power Trade-off

Similar to the case of a point-to-point link, we consider the trade-off between the route outage and power. This type of analysis gives insight on how much power is required to achieve a desired outage probability or how the outage probability decreases as more power is spent on communication.

For the case that the maximum transmitted power at each link was limited to $\mathsf{SNR}_{\mathsf{Max-Link}}$, (13) gives the end-to-end reliability. We get more insight into the relationship between power and reliability by looking at the route outage probability define in (12). Writing (13) in terms of the outage probability, ρ, we have:

$$\begin{aligned} 1-\rho &= \exp\left(-\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{SNR}_{\mathsf{Max-Link}}}\right), \\ \ln(1-\rho) &= -\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{SNR}_{\mathsf{Max-Link}}}. \end{aligned}$$

For small values of ρ, we can use the approximation of $\ln(1-\rho) \approx \rho$ to simplify this relation to:

$$\rho \quad \approx \quad \frac{\sum_{i=1}^{h} d_{r_{i-1}r_i}^{k}}{\mathsf{SNR}_{\mathsf{Max-Link}}}. \tag{24}$$

The relationship between reliability and power with optimal power allocation, i.e. the optimal reliability-power curve discussed in the last section, is given by (23). Writing (23) in terms of the route outage probability and following a similar approach, we find:

$$\rho \quad \approx \quad \frac{\left(\sum_{i=1}^{h} \sqrt{d_{r_{i-1}r_i}^{k}}\right)^2}{\mathsf{SNR}_{\mathsf{Total}}}. \tag{25}$$

From (24) and (25), we observe that route outage decays as $\mathsf{SNR}_{\mathsf{Max-Link}}^{-1}$ and $\mathsf{SNR}_{\mathsf{Total}}^{-1}$, respectively, in the high-snr regimes. It is not surprising that we observe this type of relation as these relationships are very similar to what we observed in the first section for a point-to-point link. In the last section, it is shown how diversity at the route level can fundamentally change the form of this trade-off.

IV. Route Diversity

The motivation behind the *Route Diversity* idea that we introduce in this section is to improve the end-to-end route reliability by taking advantage of the wireless broadcast property and the independence between different Rayleigh fading links. To clarify this idea, let's look at a simple example. Assume that in the network shown in figure 4, the most reliable route is selected as shown. Without diversity, a successful end-to-end relaying would require 3 successful point-to-point transmissions. We refer to this strategy as the *Non-Diversified Routing Scheme*. Due to the broadcast and the fading nature of the wireless propagation environment, the information transmitted by s may be received correctly by, for example, r_1 while r_0 fails to receive that information. Hence, it is not accurate to only define the event of successful end-to-end relaying as previously mentioned. For instance, as shown in figure 5, d can receive the information directly from s in the first transmission slot, from r_0 in the second transmission slot or from r_1 in the third transmission slot. We refer to this routing scheme as the *Diversified Routing Scheme*.

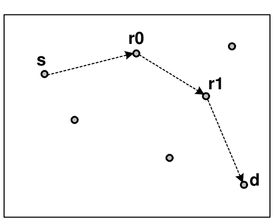

Fig. 4. Simple Route

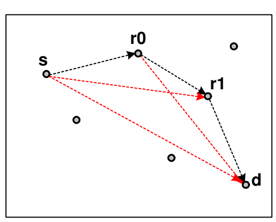

Fig. 5. Diversified Route

In the subsequent analysis, we assume that the path-loss exponent, k, is 2. We also assume that the maximum transmitted snr by each node is fixed. This is the quantity we represented by $\mathsf{SNR}_{\mathsf{Max-Link}}$ in the previous section. However, for simpler notation, we drop the subscript and denote this quantity as snr in the analysis that follow. The reader should keep this in mind in interpreting our results. Our aim is to find how the end-to-end outage probability varies with the maximum transmitted power level under the diversified routing scheme and compare the result with relation given in (24).

Before looking at the end-to-end outage in a diversified route, we need to give a more precise description of the relaying process under the diversified routing scheme. Assume $\mathsf{Route}_{\mathsf{optimal}} = (r_0, r_1, \cdots, r_h)$ is selected as the most reliable route to the destination. In the diversified routing scheme that we analyze in this section, nodes operate according to the following rules: node i, $\mathsf{i} \geq 1$, transmits in slot $\mathsf{i}+1$ if it has received the information in time slots $\{1, \cdots, \mathsf{i}\}$. Otherwise, no transmission takes place in time slot $\mathsf{i}+1$. Given this protocol, successful relaying of the information from r_0 to r_h takes no more than h time slots and no more than h units of transmitted power. This route is defined to be in outage if the information is not received by r_h by the end of time slot h. The end-to-end route outage probability is defined as the probability of this event. Defining the diversified routing scheme according to these rules is the simplest way that would allow us to compare the end-to-end reliability in the non-diversified and the diversified route on a fair basis, We can compare the end-to-end reliability since both routing schemes use the same number of transmission slots and the same amount of total power.

As the first step in analyzing the benefits of route diversity, we start by looking at a small, i.e., a 2-hop network. We then present some asymptotic analysis that applies to very large line networks. Finally, we extend some of these results to a line network with a finite number of nodes.

A. Example 1: Two Hop-Networks

This example focuses on a 2-hop network constructed by uniformly placing a relay node inside the circle centered at the mid-point between s and d, see figure 6.

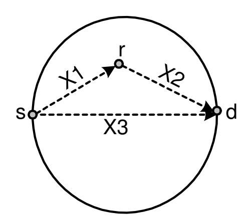

Fig. 6. 2-Hop Disk Network

The Minimum Outage Route (MOR) in this network is $(\mathsf{s}, \mathsf{r}, \mathsf{d})$. The non-diversified outage probability for this route is:

$$\begin{aligned} \rho_{\mathsf{Non-Diversified}}^{(\mathsf{s,r,d})} &= 1 - P_{\mathsf{Succ}}(x_1, \mathsf{snr}) P_{\mathsf{Succ}}(x_2, \mathsf{snr}) \\ &\approx \frac{x_1^2 + x_2^2}{\mathsf{snr}} \end{aligned} \tag{26}$$

where the approximation is valid in the high-snr regime. In the diversified scheme, successful relaying requires either a successful direct transmission, $\{\mathsf{s} \rightarrow \mathsf{d}\}$, or successful multi-hoping, $\{\mathsf{s} \rightarrow \mathsf{r}\}$ followed by $\{\mathsf{r} \rightarrow \mathsf{d}\}$. The reliability of this route is given by:

$$\begin{aligned}
\mathsf{Reliability}^{(\mathsf{s},\mathsf{r},\mathsf{d})}_{\mathsf{Diversified}} &= \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_3, \mathsf{snr}) + (1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_3, \mathsf{snr})) \\
&\quad \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_1, \mathsf{snr})\mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_2, \mathsf{snr}), \\
&= \exp(-\frac{\mathsf{x}_3^2}{\mathsf{snr}}) + \\
&\quad (1 - \exp(-\frac{\mathsf{x}_3^2}{\mathsf{snr}}))\exp(-\frac{\mathsf{x}_1^2 + \mathsf{x}_2^2}{\mathsf{snr}}) \\
&\approx 1 - \frac{\mathsf{x}_3^2(\mathsf{x}_1^2 + \mathsf{x}_2^2)}{\mathsf{snr}^2}. \qquad (27)
\end{aligned}$$

where the approximation is valid in the high-snr regime. Hence, the route outage probability is simply:

$$\rho^{(\mathsf{s},\mathsf{r},\mathsf{d})}_{\mathsf{Diversified}} \approx \frac{\mathsf{x}_3^2(\mathsf{x}_1^2 + \mathsf{x}_2^2)}{\mathsf{snr}^2}. \qquad (28)$$

Based on this expression, we observer that the end-to-end outage probability decays as snr^{-2} for the diversified routing scheme. This is a significant improvement over the snr^{-1} decay observed in (26) in the absence of diversity.

It should be noted that both (26) and (28) are valid for any two hop network. Hence, a similar type of improvement in the relationship between the end-to-end route outage and power can be seen in any two hop network. Furthermore, this gain is achieved through route diversity and does not require any coding, ARQ, or transmitter side information.

The actual end-to-end outage is highly dependent on the network topology. To eliminate this dependence and to get a sense of the average improvement due to route diversity, one can take the expectation of (26) and (28) over the location of the relay node. For the network shown in figure 6, where the node is located uniformly in circle with radius of 1 unit, these expectations can be calculated as given:

$$\begin{aligned}
\rho^{(\mathsf{s},\mathsf{r},\mathsf{d})}_{\mathsf{Non-Diversified}} &\approx \frac{1.2}{\mathsf{snr}}, \\
\rho^{(\mathsf{s},\mathsf{r},\mathsf{d})}_{\mathsf{Diversified}} &\approx \frac{4.7}{\mathsf{snr}^2}. \qquad (29)
\end{aligned}$$

Figure 7 show the exact values and the approximation for the average route outage probabilities. It should be emphasized that the average route outage does not necessarily correspond to any particular network, but it shows that route diversity can significantly improve the end-to-end outage on an average basis.

B. Example 2: Disconnect Probability

Consider a network in which nodes are distributed on a line and the distance between neighboring nodes are independent exponential random variables with parameter λ. Assume that the destination is located a large number of hops away to the right of the source node. Although it may be possible to calculate the exact end-to-end outage probability as a function of the maximum transmitted power level, the location of the relay nodes, and the number of hope, we will take a different approach in analyzing the benefit of route diversity. We define the disconnect event for a node as the event that the node is not connected to any node located to it's right. Without diversity, this event is equivalent to the event that the link between the node and its immediate right neighbor is in outage, see figure 8. With diversity, however, this event is equivalent to the event that all the links between the node and all the nodes to its right are in outage, see figure 9. Clearly the second event has a lower probability as it is a subset of the first event. We are interested in analytically calculating these probabilities and observing how these quantities depends on the snr.

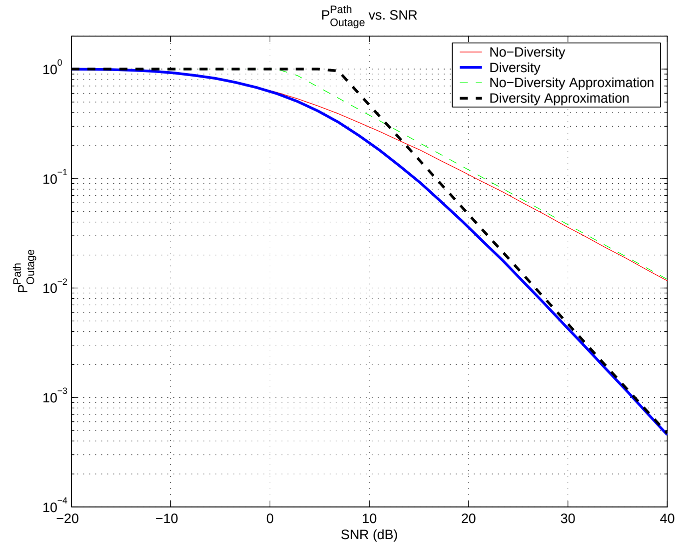

Fig. 7. The Average Reliability is a Disk Network

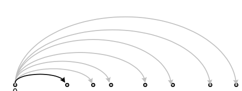

Fig. 8. Disconnect, No-Diversity

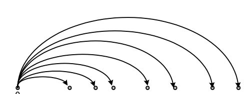

Fig. 9. Disconnect, with Diversity

For a given network realization, i.e. given the distance to the neighboring node is d_r, the probability of the disconnect event without diversity is given by:

$$1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_\mathsf{r}, \mathsf{snr}) \approx \frac{\mathsf{d}_\mathsf{r}^2}{\mathsf{snr}} \qquad (30)$$

The disconnect probability depends on a particular realization of the network, i.e., on the value of d_r. To eliminate this dependency, we take the expectation over the distribution of d_r to find the average disconnect probability for a link. This probability is only a function of the transmitted snr as given below:

$$\begin{aligned}
\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{snr}) &\approx \mathsf{E}_{\mathsf{d}_\mathsf{r}}\left[\frac{\mathsf{d}_\mathsf{r}^2}{\mathsf{snr}}\right] \\
&\approx \frac{2}{\lambda^2 \mathsf{snr}} \qquad (31)
\end{aligned}$$

Calculating the probability of the disconnect event with diversity requires a different approach. We start by dividing the line into segments of length δ. For small values of δ, the number of nodes in a line segment of length δ is approximately a Bernoulli random variable. i.e. there is a node in the segment with probability $\lambda\delta$ or there is no node with probability $1-\lambda\delta$. Furthermore, the number of nodes in non-overlapping line segments are independent random variables, see [7] for details. This approximation is prefect in our case as we will take the limit of $\delta \to 0$ to get the desired result. For small values of δ, let's define the disconnect event for segment located at distance $m\delta$ away from the transmitter as the event that the information is not received by any node in the line segment $(m\delta, (m+1)\delta]$. This event includes both the case that there is no node in this line segment or there is a node and transmission fails due to bad fading. The probability of this event can be calculated as:

$$P_{Disconnect}(m\delta, snr) = 1 - P_{Succ}(m\delta, snr)\lambda\delta$$

where $P_{succ}(d, snr)$ is given in (8). Let $P_{Disconnect}(x, y, \delta, snr)$ be the probability that the information transmitted by a node located at location 0 is not received by any node between $(x, y]$ where this segment is broken down into segments of length δ. This probability can written in terms of $P_{Disconnect}(m\delta, snr)$ calculated above as:

$$P_{Disconnect}(x, y, \delta, snr) = \prod_{i=\frac{x}{\delta}}^{\frac{y}{\delta}} P_{Disconnect}(i\delta, snr)$$

Taking the natural log of both sides, we get:

$$\begin{aligned} \ln(P_{Disconnect}(x, y, \delta, snr)) &= \sum_{i=\frac{x}{\delta}}^{\frac{y}{\delta}} \ln(P_{Disconnect}(i\delta, snr)) \\ &= \sum_{i=\frac{x}{\delta}}^{\frac{y}{\delta}} \ln(1 - P_{Succ}(m\delta)\lambda\delta). \end{aligned}$$

Taking the limit $\delta \to 0$:

$$\ln(P_{Disconnect}(x, y, snr)) = -\int_x^y \lambda \; P_{Succ}(l, snr)dl \quad (32)$$

where we used the approximation of $\ln(1-x) \approx -x$ for small values of x in the last step. For the case when the path-loss exponent $k = 2$, we have

$$P_{Succ}(d, snr) = \exp(-\frac{d^2}{snr}),$$

and the above integral can be calculated easily based on the complementary error function:

$$\begin{aligned} &\ln(P_{Disconnect}(x, y, snr)) \\ &= -\lambda\sqrt{snr}\int_{\frac{x}{\sqrt{snr}}}^{\frac{y}{\sqrt{snr}}} e^{-t^2}dt. \\ &= -\frac{\lambda\sqrt{\pi snr}}{2}\left(\operatorname{erfc}\left(\frac{y}{\sqrt{snr}}\right) - \operatorname{erfc}\left(\frac{x}{\sqrt{snr}}\right)\right) \quad (33) \end{aligned}$$

Let $P_{Disconnect}(snr)$ be the disconnect probability, i.e. the probability that a node is not connected to any node to its right. Assuming an infinitely long line, this probability is obtained by evaluating (33) for $x = 0$ an $y = \infty$. We have:

$$\begin{aligned} \ln(P_{Disconnect}(snr)) &= \ln(P_{Disconnect}(0, \infty, snr)) \\ &= -\frac{\sqrt{\pi}\ \lambda^2 snr}{2} \end{aligned}$$

Hence:

$$P_{Disconnect}(snr) = \exp\left(-\frac{\sqrt{\pi}\ \lambda^2 snr}{2}\right) \quad (34)$$

Comparing (31) and (34), it is clear that without diversity, the disconnect probability decays linearly with snr^{-1} while with diversity, the disconnect probability decays exponentially with snr.

C. Multi-Hop Line Network

Similar to the last example, consider a network in which nodes are distributed on a line and the distance between neighboring nodes are independent exponential random variables with parameter λ. We focus our attention on calculating the outage probability for the route between two nodes separated by $N-1$ intermediate relays. The minimum outage route in this network is the successive hoping route in which each nodes transmits the information to its immediate next neighbor in the direction of the destination. With out diversity, a route is a sequence of N independent links. The outage probability of each link is calculated in (31). It can be shown, see [12], that the end-to-end outage probability in the high-snr regime is:

$$\rho^{(0,1,\cdots,N)}_{Non-Diversified} \approx N\frac{2}{\lambda^2 snr}. \quad (35)$$

We now look at how the route outage probability changes for the diversified routing scheme. We look at two different schemes.

1) Full Relay Diversity: The full relay diversity scheme operates exactly as described in the first part of section IV. We start by looking at 2-hop line network as shown in figure 10. In this case, we can readily use the result from (28). Taking the expectation to find the average route outage probability, we have:

$$\begin{aligned} \rho^{(0,1,2)}_{Diversified} &\approx E_{d_1,d_2}\left[\frac{(d_2+d_1)^2(d_1^2+d_2^2)}{snr^2}\right] \\ &\approx \frac{80}{\lambda^4\ snr^2} \end{aligned} \quad (36)$$

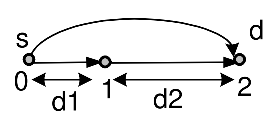

Fig. 10. 2-Hop Line Network

Figure 11 shows the exact values and the approximation for the average route outage probabilities in a 2-hop line network.

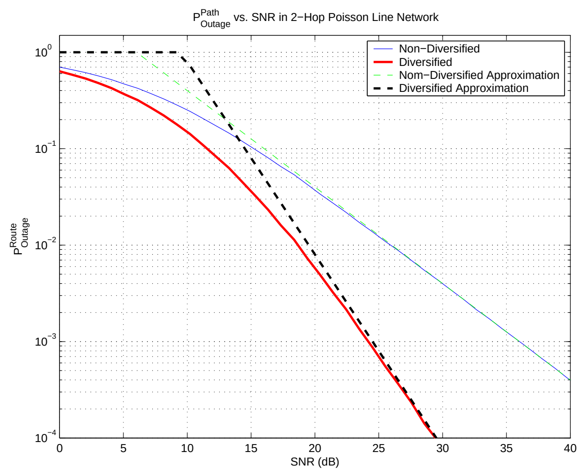

Fig. 11. 2-Hop Line Network

It is clear that our approximation are quite good for high values of snr. A similar analysis can be done for a 3-hop network. It can be shown that the end-to-end outage probability decays as $(\lambda^2 \mathsf{snr})^{-3}$ in that case. Based on this analysis, we conjecture that for a route with N hops, the outage probability decays as $(\lambda^2 \mathsf{snr})^{-\mathsf{N}}$ in the high-snr regime. In order to achieve this type of behavior, the high-snr approximation for individual link outage probability must be valid for the worst link, i.e. for the link between the source, node 0, and the destination, node N. This requires a very high level of transmitted snr. In fact, a higher value of snr is needed as N increases. Therefore, this behavior is not applicable to a large network. Hence, next we consider a limited diversity scheme where the high-snr approximation is appropriate

2) Limited Relay Diversity: The only difference between this scheme and the full diversity scheme is that in the limited diversity scheme with degree L, node i can only receive the information from nodes $\{\mathsf{i}-\mathsf{L}+1, \cdots, \mathsf{i}-1\}$. For example, figure 12 shows all the paths in a 6-Hop line network with a diversity limit of 2. The motivation behind this strategy is two-fold: first, this type of diversity requires coordination only among nodes located close to each other, which might be more reasonable than coordination among all nodes along the selected route as needed under the full diversity scheme. Furthermore, this approach allows us to show some interesting analytical results for the benefits of route diversity in a multi-hop line network.

The exact analysis of the end-to-end reliability is complicated due to the strong correlation between the different events that contribute to a successful end-to-end relaying. For example in figure 12, we are interested in calculating the probability that a route exists between node 0 and node 6 using any of the point-to-point links shown in the figure. The probability of success for these links are strongly correlated. This correlation arises from the dependence of the point-to-point link success probability on the node locations. For example, in figure 12, distance between nodes 1 and 2, affects the probability of success for links $\{1 \to 2\}$, $\{0 \to 2\}$, and $\{1 \to 3\}$.

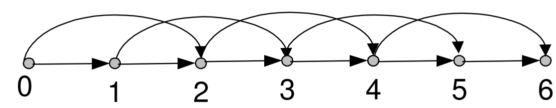

Fig. 12. Limited Cooperation in a Line Network

To eliminate this correlation, we consider the diversity scheme shown in figure 13, where we have eliminated the links between $\{1 \to 3\}$ and $\{3 \to 5\}$. Hence, the total reliability of all the routes shown in figure 13 is less than the reliability of the routes shown in figure 12. Finding the probability of outage for figure 13 give an upper-bound for the probability of outage for figure 12.

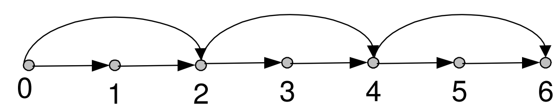

Fig. 13. Upper Bounding for the Outage Probability

So we have:

$$\begin{aligned}&\mathsf{Reliability}^{(0,1,\cdots,\mathsf{N})}\\ &\geq \mathsf{Reliability}^{(0,\cdots,\mathsf{L})} \times \cdots \times \mathsf{Reliability}^{(\mathsf{N}-\mathsf{L},\cdots,\mathsf{N})}\end{aligned}$$

Of course, for this to make sense N must to divisible by L. Essentially, we have divided the line into segments of size L and used the result from unlimited diversity in each segment. For the case of a Poisson line, the internode distances are independent from each other. Hence:

$$\mathsf{Reliability}^{(0,1,\cdots,\mathsf{N})} \quad \geq \quad \left(\mathsf{Reliability}^{(0,1,\cdots,\mathsf{L})}\right)^{\frac{\mathsf{N}}{\mathsf{L}}}. \tag{37}$$

Using the result from the last section and the above inequality, we can find the upper bound for the end-to-end outage probability in a line network. We skip all the intermediate steps, see [12], and only give the final result. For $\mathsf{L}=2$, it can be shown that:

$$\rho^{(0,1,\cdots,\mathsf{N})} \quad \leq \quad \frac{\mathsf{N}}{2}\frac{80}{\lambda^2\ \mathsf{snr}^2}. \tag{38}$$

In figure 14, the exact end-to-end outage probability and bound for a 6-Hop line network under diversity limit of $\mathsf{L}=2$ are shown. The bound is clearly not very tight. However, finding this bound allows us to get an idea of how the relation between reliability and power changes due to route diversity. Comparing (35), and (38) it is clear that without diversity the end-to-end outage probability decays as $(\lambda^2\mathsf{snr})^{-1}$. When diversity is limited to $\mathsf{L}=2$ nodes, the end-to-end outage probability decays as $(\lambda^2\mathsf{snr})^{-2}$. We conjecture that for diversity limit of L, the end-to-end outage decays as $(\lambda^2\mathsf{snr})^{-\mathsf{L}}$.

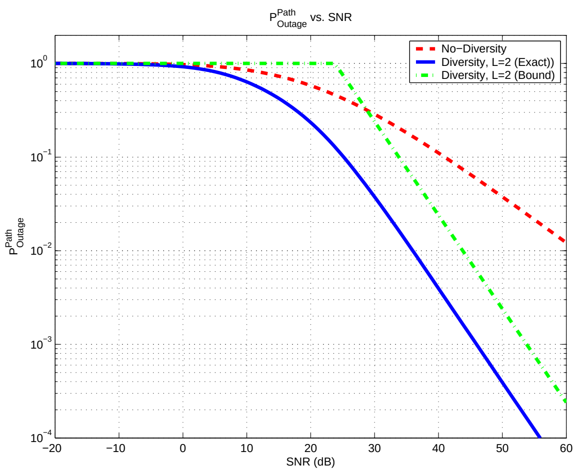

Fig. 14. Outage for 6-Hop Poisson Line Network

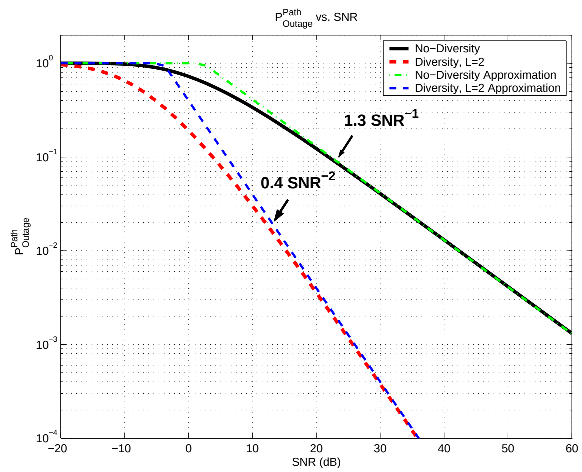

Fig. 16. Average Outage Probability

D. Simulations

Extending the analysis of a line or a two-hop network to a random planar network appears to be very difficult. In this part, we give some simulation results confirming that similar behavior is achievable in more general networks..

The network that we will look in this section is constructed by uniformly placing 10 nodes inside a circle with radius of 1. The source and the destination nodes are placed at the two opposite ends of a diameter of the circle. Figure 15 shows one particular realization of such a network. For each realization, the most reliable route is selected based on the algorithm discussed in lemma 1. For each realization, we calculate the end-to-end outage probability for the non-diversified routing scheme, and compared that to the outage probability for the limited diversity scheme with the limit of $\mathsf{L} = 2$. We averaged the outage probabilities for 1000 difference realization of the network. Figure 16 shows the resulting average outage probability vs. snr curves. It can be seen that even limited diversity has significantly increased the slope of these curves, confirming the conjecture that relay diversity can fundamentally change the trade-off between the end-to-end outage probability and the transmitter power.

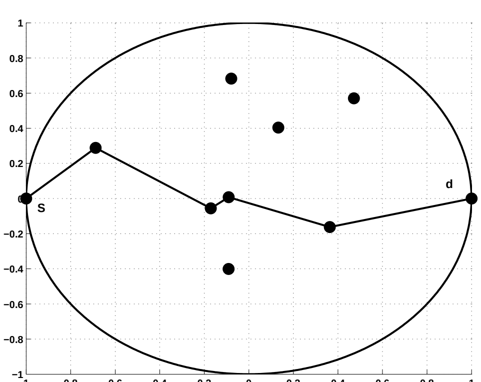

Fig. 15. Simulation Network

V. Summary and Conclusions

We studied the problem of route reliability in a multi-hop wireless network. Our analysis started by looking at the reliability of a point-to-point communication link. Based on this analysis, we proposed a new probabilistic way of looking at a wireless link. We used this probabilistic model to look at reliability in wireless network setting. In a network setting, we first defined and analyzed the reliability for a fixed route and then developed algorithms for finding the optimal route between a source-destination pairs of nodes. We looked at three different formulation for the routing problem: finding the most reliable route for a fixed maximum transmitted snr per link, finding the most reliable route for a fixed end-to-end power, and finding the minimum power route for a guaranteed end-to-end reliability. We showed that the last two problem are dual of each other. Based on this duality, we found the optimal trade-off curve between the end-to-end reliability and the end-to-end power consumption. It was shown that the trade-off between the end-to-end reliability and consumed power in a route is very similar to the trade-off between the transmission power and reliability in a link.

The idea of route diversity was introduced as a way to improve the end-to-end reliability by taking advantage of wireless broadcast property and the independence of the fade parameters between different pairs of nodes. We gave analytical results for improvements due to route diversity in some simple network topologies. The results observed in this section closely resembled the reliability improvements due to space diversity in multiple-antenna system (see [8]).

The model proposed in this paper can open the door to a new area of research on communication reliability at the network layer. The trade-off among different route properties, such as the end-to-end reliability, the delay, or the total consumed power should be studied to help draw a better picture of the actual limits of communication in a multi-hop wireless network. In this context, route diversity appears to have the potential to fundamentally change these trade-offs.

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