Joint Node Placement and Assignment for Throughput Optimization in Mobile Backbone Networks

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***Abstract*—We study the novel hierarchical architecture of Mobile Backbone Networks. In such networks, a set of Mobile Backbone Nodes (MBNs), which are envisioned to be airborne, are deployed to provide an end-to-end communications capability for the terrestrial Regular Nodes (RNs). We address the joint problem of placing a fixed number K MBNs, and assigning each RN to exactly one MBN, using two optimization objectives. The first is the Maximum Fair Placement and Assignment (MFPA) problem in which the objective is to maximize the minimum throughput obtained by any RN. The second is the Maximum Throughput Placement and Assignment (MTPA) problem, in which the objective is to maximize the aggregate throughput of the RNs. We develop an optimal polynomial time algorithm for the MFPA problem for any K, and an optimal polynomial time algorithm for the MTPA problem for $K \leq 2$. We also develop lower complexity approximation algorithms and present simulation results comparing the performance of the various algorithms.**

I. Introduction

Wireless Sensor Networks (WSNs) and Mobile Ad Hoc Networks (MANETs) can operate without any physical infrastructure (e.g. base stations). However, it has been shown that it is sometimes desirable to construct a *backbone* over which reliable end-to-end communication can take place [6],[8]. In particular, if some of the nodes are more capable than others, these nodes can be dedicated to providing the backbone. Such networks, termed *Mobile Backbone Networks*, have been recently been studied in [18],[25],[22], [19], [20], [21].

Based on [18] and [25], a Mobile Backbone Network was defined in [22] as composed of two types of nodes. The first type includes static or mobile nodes (e.g. sensors or MANET nodes) with limited capabilities. These nodes are referred to as *Regular Nodes* (RNs). The second type includes mobile nodes with superior communication, mobility, and computation capabilities as well as greater energy resources. These nodes are termed *Mobile Backbone Nodes* (MBNs). The main purpose of the MBNs is to provide a mobile infrastructure facilitating network-wide communication.

The mobile backbone network architecture is particularly suitable for battlefield communications where terrestrial nodes such as foot-soldiers or ground vehicles have limited communications capability, and their mobility is constrained by their mission. In such a setting, Unmanned Air Vehicles (UAVs) can serve as Mobile Backbone Nodes for facilitating the communication connectivity between the ground nodes. The UAVs have rapid and nearly unconstrained mobility (relative to the ground nodes), and superior communication capability due to both their elevated position and their relatively high-power. Thus, our novel architecture can be used to design a self-organizing and robust network architecture for heterogeneous mobile networks consisting of both terrestrial and airborne nodes.

An implicit assumption in previous formulations of the Mobile Backbone Network construction problem is that an arbitrary number of MBNs are available for deployment, and the goal is to minimize the number actually deployed. Indeed, this problem formulation was given in [22] as the *Connected Disk Cover* (CDC) problem. Specifically, the CDC problem aims to place the *minimum* number of MBNs such that (i) All RNs are covered by at least one MBN, and (ii) The MBNs form a connected network. In many scenarios however, a more appropriate (and perhaps realistic) assumption would be that the number of available MBNs is *fixed* a-priori, and the objective is to do the "best we can" with these fixed resources.

Note, however, that the CDC-type formulation for MBN placement arises very naturally given the assumption of a discrete communications model, such as the "disk" connectivity model. In such a model, two nodes can communicate if they are within some fixed range, and cannot otherwise. However, while the disk model is a good first-order communications model, a more realistic model would account for the fact that the data rate at which two nodes can reliably communicate is actually a continuous function of the received Signal-to-Interference-and-Noise Ratio (SINR). The SINR in turn, depends on the wireless channel conditions and underlying PHY/MAC protocols (i.e. the System model). In this paper and for the specific context of Mobile Backbone Networks, we distill these issues into the following general model: The "throughput" achieved by an RN transmitting to its assigned MBN is a *decreasing* function of (i) The distance between the RN and MBN, and (ii) The total number of RNs assigned to

Manuscript received 1 July 2011; revised 25 April 2012. This work was supported by NSF grants CCR-0325401 and CNS-0915988, by ONR grant N000140610064, and by a grant from Draper Laboratory. A preliminary version of this work appeared in IEEE INFOCOM 2008, Phoenix, Apr. 2008 [19].

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Digital Object Identifier 10.1109/JSAC.2012.120612.

that MBN. The idea is that the first factor models the loss due to wireless propagation, and the second models loss due to interference caused by multiple RNs trying to access a single MBN. We elaborate further on the mathematical specifics of the model, as well as provide examples in section III.

With the above communications model, we formulate the backbone construction problem in a manner significantly different from previous works, and thereby requiring significantly different solution methodologies. In particular, we consider the joint problem of *placing* a fixed number of MBNs, and *assigning* each RN to exactly one MBN, such that a throughput objective is maximized. We consider two objective functions, yielding two separate problems. The objective of the Maximum Fair Placement and Assignment (MFPA) Problem is to maximize the throughput of the minimum throughput RN. While the objective of the Maximum Throughput Placement and Assignment (MTPA) problem is to maximize the aggregate system throughput (i.e. sum of the throughputs achieved by each RN).

It should be noted that in contrast to previous backbone construction problem formulations that use a simple disk communication model, the MFPA/MTPA involve a non-trivial assignment component. Specifically, a solution needs to balance assigning RNs to their closest MBNs and not assigning too many RNs to any particular MBN. Thus for the overall problems, not only do K MBNs need to be placed at their optimal locations, but once placed there are K^N different RN to MBN assignments, among which the optimal one must be chosen, where N is the number of RNs.

By exploiting the geometric structure of the problem, we are able to develop an optimal polynomial time algorithm for the MFPA problem for fixed K and an optimal solution for a restricted version of the MTPA problem for $K \leq 2$. As will be described later, the key lies in exploiting certain geometric properties of the placement portion of the problem, and certain combinatoric structure for the associated assignment subproblem. To the best of our knowledge, this is the first work to consider the MBN placement and assignment problem with the objective of optimizing throughput objectives (as opposed to just connectivity objectives as is typically done with the disk communications models). As such, the main contribution of this paper is the development of a new framework for the backbone construction problem under throughput objectives (i.e., the MFPA and MPTA objectives), and the development of optimal algorithms[1] for the MFPA and MTPA problems.

This paper is organized as follows. In Section II we review related work and in Sections III and IV we formulate the problem and give illustrative examples. Section V presents an optimal solution for the MFPA problem. In section VI, we discuss solutions for a restricted version of the MTPA problem. In section VII, we present approximation and heuristic algorithms for both problems. Finally, in section VIII we evaluate the performance of the algorithms via simulation.

II. Related Work

The Mobile Backbone Architecture was originally presented in [18],[25] (and references therein), where the authors assume that the RNs and MBNs are already placed, and a-priori form a connected network. Thus the focus of [18],[25] is on developing system-level protocols for routing, scheduling, and MBN election. In contrast, the focus of this paper is on the fundamental problem of how to place the MBNs and assign RNs to MBNs, such that a network performance objective is optimized.

Somewhat along these lines is the work of [22] [21], in which the specific network performance objective is end-to-end connectivity. In particular, [22] [21] formulate the *Connected Disk Cover* (CDC) problem, which aims to place the minimum number of MBNs such that (i) All RNs are covered by at least one MBN, and (ii) The placed MBNs form a connected network. These papers develop approximation algorithms for solving the CDC problem, under the assumption that an arbitrary number of MBNs are available and a disk connectivity model.

The MBN placement problem under the more general communications model considered in this paper, is somewhat related to the base station selection/placement problem considered for cellular and indoor wireless systems, e.g. [4],[23],[24],[15],[11]. However, there are several aspects that differentiate the work of this paper from the base-station placement problem. First, the optimization in our work includes both the MBN placement *and* the RN to MBN assignments. In contrast, much of the cellular work use trivial solutions to the assignment problem (e.g., assign users to the nearest base-station) and optimize throughput via the base station placement and power control. Another key difference is that practical considerations for cellular base station placement usually a-priori restricts the set of possible locations to a discrete set of candidates. This restriction typically results in solution methodologies along the lines of simple heuristics, or large scale optimization tools (e.g. Mixed-Integer-Linear-Programming or Genetic Algorithms). In contrast, in this paper we do not limit the placement of MBN to a restricted set of locations, and exploit the geometry of the problem to obtain optimal MBN placements.

We note that the basic idea of clustering nodes to form a hierarchical architecture has been extensively studied in the context of wireless networks (e.g. [5],[8]). Yet, the idea of deliberately controlling the motion of specific nodes in order to maintain some desirable network property (e.g. lifetime or connectivity) has been introduced only recently (e.g. [22],[14],[17], [12]), [20].

To the best of our knowledge, our work in [19], is the first to consider the placement and assignment problem with the objective of throughput optimization. Bulding upon [19] Craparo et. al. [26] [27] developed a polynomial-time constant-factor approximation algorithm for the MFPA problem as well algorithms for a variant of the MFPA problem in which the number of regular nodes is maximized for a given throughput value.

III. Problem Formulation

We consider a set of N Regular Nodes (RNs), distributed in the plane and assume that a set of $K < N$ Mobile Backbone

[1] We use the term "optimal algorithm" to denote an algorithm that finds an optimal solution, i.e. an "exact algorithm".

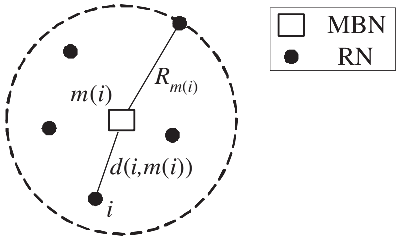

Fig. 1. Example of a Cluster.

Nodes (MBNs) are to be deployed[2]. We denote the set of RNs by $P = \{1, 2, \ldots, N\}$ and the set of MBNs by $M = \{m_1, m_2, \ldots, m_K\}$. For every RN i, let $m(i)$ denote the MBN to which i has been assigned, (e.g $m(i) = k$ if i is assigned to m_k), and let $d(i, m(i))$ represent the distance between them. In general, let $d(i, j)$ represent the distance between nodes i and j. Next, for every MBN m_k, let P_k denote the set of RNs assigned to it. We refer to the tuple of an MBN and its assigned RNs as a *cluster*. For cluster k corresponding to (m_k, P_k), we define the *cluster radius* R_k as, $R_k = max_{j \in P_k} d(j, m_k)$. The number of RNs assigned to MBN m_k, $|P_k|$, is refered to as the *cluster size*. An example of a cluster is shown in Fig. 1. For the communications model, we assume that the throughput of an RN i transmitting to its assigned MBN $m(i)$ is some *function* $H\big(d(i, m(i)),\ |P_{m(i)}|\big)$, that is *decreasing* in both its arguments. As mentioned earlier, the dependence of $H()$ on $d(i, m(i))$ models wireless propagation loss, and the dependence on $|P_{m(i)}|$ reflects loss due to interference at MBN $m(i)$. Note that in this communications model we assume that RNs from different MBNs do not interfere with each other, e.g. different clusters operate on different frequencies channels. This assumption is consistent with deployment of cellular systems where clusters (cells) are kept from interfering from one another through the use of different frequencies, or CDMA codes. While suffiicent frequency spectrum may not always be available to allow the use of different frequency bands for the clusters, the non-interfering cluster assumption can serve as a useful approximation. For example, in CDMA cellular systems, interference from different cells is relatively small and such an assumption yields a rasonable approximation [28]. Moreover, frequency reuse can be used to improve the frequency spectrum utilization, while keeping interference between the clusters to a minimum.

In order to gain some intuition about the form $H()$ could take, consider the following two system examples: (i) Slotted Aloha-based, and (ii) CDMA-based. In the Slotted Aloha based model, we assume that all RNs assigned to an MBN m_k transmit within a slot with equal probability, $1/|P_k|$. Additionally, we associate a throughput loss due to attenuation that is proportional to $d^{-\alpha}$ for an RN located a distance d away from m_k, where α represents the path loss exponent. This could, for example, reflect extra coding that needs to be used in order to deal with the propagation loss. The resulting throughput of a node i in this system is therefore simply the probability that exactly one RN transmits in a slot, multiplied by the attenuation loss, i.e.,

$$
\begin{aligned}
TP_{SA}(i) &= \tfrac{1}{|P_{m(i)}|}\left(1 - \tfrac{1}{|P_{m(i)}|}\right)^{|P_{m(i)}|-1}\left(\tfrac{1}{d(i,m(i))^{\alpha}}\right) \\
&\approx \tfrac{1}{e \cdot |P_{m(i)}| \cdot d(i,m(i))^{\alpha}} \\
&\triangleq H_{SA}\Big(d(i, m(i)), |P_{m(i)}|\Big) \qquad (1)
\end{aligned}
$$

where we have left out most of the constants for simplicity, and we use the approximation that $(1 - 1/x)^{x-1} \to 1/e$ even for small values of $x \geq 1$. Note that (1) is of the desired form for $H()$, i.e. decreasing in both $d(i, m(i))$ and $|P_{m(i)}|$. Next, consider a CDMA-based system in which power control is employed. Specifically, in order to combat the near-far problem, all RNs assigned to an MBN $m(i)$ equalize their received power (equal to 1, for simplicity) at $m(i)$ to that of the farthest away RN. Thus the throughput achieved by every RN within a cluster is the same, and is proportional to its Signal-to-Interference-and-Noise Ratio (SINR) at $m(i)$, i.e.,

$$
\begin{aligned}
TP_{cdma}(i) &= \tfrac{1/R^{\alpha}_{m(i)}}{(1/R^{\alpha}_{m(i)})(|P_{m(i)}|-1)+\eta} \\
&= \tfrac{1}{|P_{m(i)}|+\eta \cdot R^{\alpha}_{m(i)}-1} \\
&\triangleq H_{cdma}\Big(R_{m(i)}, |P_{m(i)}|\Big) \qquad (2)
\end{aligned}
$$

where η represents the noise at MBN $m(i)$, and $R_{m(i)}$ the radius of cluster $m(i)$. Again, note the form of the throughput function is as desired, since it is decreasing in both distance and cluster size. For the purpose of intuition, we will carry these two examples throughout the paper, whenever possible directly applying to them the general results that we derive.

We now give a precise formulation for the two problems that will be addressed in this paper: (i) The Maximum Fair Placement and Assignment (MFPA) Problem and (ii) Maximum Throughput Placement and Assignment (MTPA) problem.

Problem MFPA: Given a set of RNs (P) distributed in the plane, place K MBNs (M) and assign each RN i to exactly one MBN $m(i)$ such that the following is maximized:

$$\min_{i \in P} TP(i) = \min_{i \in P}\left\{H\Big(d(i, m(i)), |P_{m(i)}|\Big)\right\} \qquad (3)$$

Problem MTPA: Given a set of RNs (P) distributed in the plane, place K MBNs (M) and assign each RN i to exactly one MBN $m(i)$ such that the following is maximized:

$$\sum_{i \in P} TP(i) = \sum_{i \in P} H\Big(d(i, m(i)), |P_{m(i)|}\Big) \qquad (4)$$

As a final point, we enforce the following additional conditions on the $H()$ function,

1) $H(R, X) > 0, \forall R \geq 0, X \geq 1$.
2) $H(R, X) < \infty, \forall R \geq 0, X \geq 1$ (only for MTPA)

Notice that condition (2) is needed for the general MTPA problem as stated above to be well defined. Otherwise, any solution in which an MBN is placed on top of an RN could yield infinite aggregate throughput (i.e. artificially exploiting the so-called "near-field" effect). Since $K < N$, this is not an

[2]For simplicity we assume that both the RNs and MBNs are located in a 2 dimensional plane. While in practice, MBNs may be airborne, we assume that their relative elevation is sufficiently low to make the planar assumption both reasonable and practical. Moreover, for the purpose of the analysis and optimization presented in this paper, the projection of the MBNs' location on the plane is sufficient.

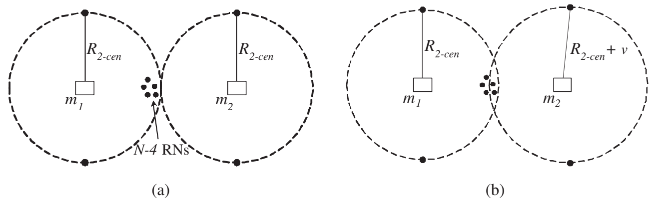

Fig. 2. $K = 2$ MFPA example. (a) 2-Center Solution. (b) Optimal Solution.

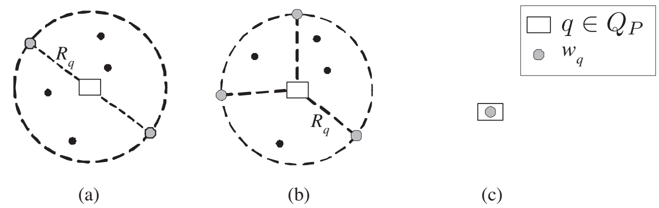

Fig. 3. Illustration of the forms of 1-center (location,radius) tuples. (a) Midpoint of a pair of points. (b) Circumcenter of a triplet of points. (c) On top of a single point.

issue for the MFPA problem, i.e. the worst case throughput RN cannot have an MBN on top of it.

IV. Illustrative Examples

In this section we attempt to give some additional intuition regarding the complexity of the joint placement and assignment problems addressed in this paper. To begin, consider a single MBN instance of the MFPA problem. With just one MBN, we immediately note that the assignment portion of the problem is trivial (i.e. all N RNs are assigned to the one MBN). Furthermore, the associated placement portion of the problem can be solved optimally by placing the single MBN so as to minimize the farthest distance from any RN. This is precisely the well known 1-center problem[3], for which several efficient polynomial time algorithms exist [1]. Applying one of these algorithms solves the 1 MBN MFPA problem optimally.

Next, consider the 2 MBN example illustrated in Fig. 2. Fig. 2(a) shows the MFPA solution if we simply apply a 2-center algorithm, and assign RNs to their nearest MBN. As shown, the worst case RN attains a throughput of $H(R_{2-cen}, N-2)$ in this case, where R_{2-cen} is the 2-center radius. However, by increasing the radius of the second cluster by a small amount, i.e. enough to enclose half of the $N-4$ RNs clustered together, the optimal solution can potentially increase the worst case RNs' throughput to $H(R_{2-cen}+\epsilon, \frac{N}{2})$; this is shown in Fig. 2(b). Clearly depending on the exact form of $H()$, this improvement can be quite significant. As demonstrated in this simple example, even if we are given a placement of the MBNs, the assignment problem is non-trivial, as it may potentially be beneficial to assign RNs to farther away MBNs.

Thus the main difficulty of the MFPA and MTPA problems for $K > 1$ can be summarized as follows. First, there are an infinite number of potential locations for the MBNs (i.e. anywhere on the plane). Second, for any particular placement of K MBNs, there are K^N different assignments of RNs to MBNs (i.e. each RN can be assigned to one of K MBNs).

V. MFPA Solution

The key to our approach in solving the MFPA problem is to decouple the placement and assignment problems in a way that does not affect the optimality of the resulting decoupled solution. We start with the following observation and lemma. The observation applies to any feasible MFPA solution, and follows from the fact that the overall minimum throughput RN must be the minimum throughput RN in its own cluster.

Observation 1: Let RN i have minimum throughput among all RNs, and let $m(i)$ be its assigned MBN. Then, the throughput of i can be expressed as a function of its cluster's radius and size, i.e. $TP(i) = H(R_{m(i)}, |P_{m(i)}|)$.

Lemma 1: Let $P_1^*, P_2^*, \ldots, P_K^*$ represent the optimal MFPA assignments of RNs to MBNs $m_1, m_2, \ldots, m_K$ respectively. Then, there exists an optimal solution to the overall MFPA problem in which the MBNs are placed at the *1-center locations* of $P_1^*, P_2^*, \ldots, P_K^*$.

Proof: Consider an optimal solution to the MFPA problem in which the MBNs are *not* placed at the 1-center locations of $P_1^*, \ldots, P_K^*$. Next, consider the solution obtained by moving all of the MBNs to their respective 1-center locations. By definition of the 1-center, doing this never increases the radius of any of the K clusters. Therefore, since the cluster sizes $|P_1^*|, \ldots, |P_K^*|$ are fixed, then by observation 1 the throughput of the worst case throughput RN does not decrease. ■

The consequence of the above lemma is that for the placement problem, the finite space of 1-center locations contains at least one solution of optimal cost. Additionally, the associated cluster radii of each of the K clusters are by definition 1-center radii. Thus as a first step, we have reduced the search space from an infinite number of locations on the plane, to a finite set of 1-center locations (with associated 1-center radii).

At first glance, the total number of 1-center locations/radii might seem prohibitively large and thus our reduction of limited use. For example, every subset of RNs has an associated 1-center location and radius, and there are 2^N subsets. However, it turns out that all of these locations/radii come from a relatively small (i.e. polynomial in N) set of candidates. To show this, we need the following fact, illustrated in Fig. 3, regarding the 1-center of a set of RNs P [16],

Fact 1: The unique 1-center location and radius of a set of RNs P, denoted $1C(P)$ and $R(P)$, is *defined* by either:

1) A *pair* of RNs $i, j \in P$. If this is the case, then $1C(P)$ is situated at the *midpoint* of i,j, and $R(P) = d(i,j)/2$.
2) A *triplet* of RNs $i, j, k \in P$ that form an *acute triangle*. If this is the case, then $1C(P)$ is situated at the *circumcenter*[4]of $\{i, j, k\}$ and $R(P)$ is the *circumradius*.
3) A *single* RN $i \in P$. This is the degenerate case where $P = \{i\}$ is a singleton set, and $1C(P)$ is situated on i itself, and $R(P) = 0$.

Indeed, the actual 1-center $(1C(P), R(P))$ tuple has minimum $R(P)$ such that all RNs are within distance $R(P)$ of the

[3]In general, the K-center problem places K MBNs such that the farthest distance from any RN to its nearest MBN is minimized.

[4]For a triplet of RNs, the *circumcenter* is the center of the circle that has all three RNs on its boundary. The radius of this circle is the *circumradius*.

location $1C(P)$. Let Q_P denote the full set of candidate 1-center locations, as described in fact 1 with respect to the original set of RNs P. Note that since each $q \in Q_P$ is defined by either 1,2 or 3 RNs in P, it follows that Q_P has cardinality at most $\binom{N}{1} + \binom{N}{2} + \binom{N}{3}$. Additionally, as described in Fact 1 and shown in Fig. 3, for each $q \in Q_P$, we associate R_q to denote the *1-center radius* of a cluster whose 1-center location is q, and the set w_q to denote the set of *defining RNs* for q. Note that though several locations in the set Q_P may be coincident, all w_q's are distinct. We now state the following lemma, which follows by construction of Q_P and fact 1.

Lemma 2: The 1-center (location, radius) tuple of *any subset* $T \subseteq P$ corresponds to some (q, R_q) tuple, $q \in Q_P$.

Proof: Consider some set of RNs T. By fact 1, the location of $1C(T)$ is uniquely defined by some subset of size 1,2 or 3 RNs in T; call the set of all such locations Q_T. However, since $T \subseteq P$, by definition of Q_P it must be the case that $Q_T \subseteq Q_P$. ■

Combining lemmas 1 and 2 and Fact 1, we can conclude that restricting our placements of MBNs to the set Q_P still allows us to find the optimal solution to the overall MFPA problem. Moreover, we can restrict ourselves to solutions whereby if an MBN m_k is placed at location $q \in Q_P$, all of the RNs assigned to it must be within distance R_q, i.e. $d(i, m_k) \leq R_q$, $\forall i \in P_k$. Otherwise, by Fact 1 q cannot be the unique 1-center location of P_k, i.e. there must exist some other location $q' \in Q_P$ that is the actual 1-center location of P_k, with corresponding 1-center radius $R_{q'}$. As per lemma 1, moving m_k to location q' cannot decrease the MFPA objective.

For clarity, we illustrate the exhaustive search over all placements among locations in Q_P as the high-level framework shown below. Let $m_1^*, \ldots, m_k^*$ denote the optimal locations of the K MBNs, $m^*(1), \ldots, m^*(N)$ the optimal RN to MBN assignments, and U^* the associated optimal cost.

Algorithm 1 High-Level Optimal MFPA Framework

1: **initialize** $U^* = -\infty$
2: **create** the set Q_P by enumerating over all defining subsets of size $1, 2$ and 3 of P.
3: **for** all $\binom{|Q_P|}{K}$ placements of K MBNs $m_1, \ldots, m_K$ **do**
4:   **if** all RNs are within R_j of at least 1 MBN m_j in current MBN placement **then**
5:     **calculate** the optimal MFPA assignments $m(i), \forall i \in P$, given the current MBN placement and subject to the constraint that $m(i) = k$ only if $d(i, m_k) \leq R_k$. Let U represent the corresponding worst case RN throughput.
6:     **if** $U > U^*$ **then**
7:       **set** $U^* \leftarrow U$, **update** $m^*(i), m_k^*, \forall i \in P, k \in K$
8: **return** U^*, $m_1^*, \ldots, m_K^*$ and $m^*(1), \ldots, m^*(N)$

In the above framework, step 2 describes constructing the restricted set of candidate locations for the MBNs, Q_P, that still preserve optimality. The key part is the loop of steps 3-7 in which we iterate over all possible placements of the K MBNs among the locations specified by Q_P. The goal of this loop is to find the placement (and corresponding RN-to-MBN assignment) that yields the optimal objective function value. Thus for each such placement, the optimal RN-to-MBN assignments must be computed in step 5 in order to determine the corresponding objective function value. Up to this point, this assignment subproblem has not been discussed. It turns out that the specific methodologies used to solve this problem for $K = 2$ and $K \geq 2$ are quite different, as we describe below.

A. $K = 2$ MFPA Assignment Subproblem

With the placement locations and radii fixed, for $K = 2$ the resulting MFPA assignment subproblem turns out to be easy to solve. In this situation, as depicted in Fig. 4(a), we define $C(1)$ and $C(2)$ as the sets of RNs that lie *exclusively* within radius R_1 and R_2 of MBNs m_1 and m_2 respectively. Similarly, let $C(1, 2)$ denote the "common set" of RNs that lie within the radii of both m_1 and m_2. The main idea is that since the radii are fixed, RNs in $C(1), C(2)$ *must* be assigned to m_1, m_2 respectively. Moreover, in assigning the remaining RNs in $C(1, 2)$, it is only the *number* assigned to each MBN that affects the MFPA objective. Thus the $|C(1, 2)| + 1$ different possibilities can be searched over, and the optimal one picked. Specifically, each possibility corresponds to w RNs assigned to m_1 and the remaining $|C(1, 2)| - w$ RNs assigned to m_2, $w = 0, \ldots, |C(1, 2)|$. Essentially, this procedure results in a "balancing" of the two MBNs with respect to the MFPA objective function, e.g. by assigning a greater number of the RNs in $C(1, 2)$ to the MBN with a smaller radius.

The worst case computational complexity of the overall MFPA algorithm for $K = 2$ is therefore $O(N^7)$. This follows from the fact that $|Q_P| \leq N^3$ and we need to solve $\binom{|Q_P|}{2}$ assignment problems, each of which takes $O(N)$ time.

B. General K MFPA Assignment Subproblem

The MFPA assignment subproblem for $K > 2$ is significantly more difficult than for $K \leq 2$. To get a sense of the additional complexity, consider the 2 vs. 3 MBN example illustrated in Fig. 4. For 2 MBNs m_1, m_2, there is only one type of "common set" of RNs, i.e. $C(1, 2)$, yielding at most $O(N)$ ways to assign different numbers of RNs to each MBN.

For $K > 2$ MBNs, the number of ways to divide different numbers of RNs *within a single common set* generalizes to $O(N^{K-1})$. Yet, the real difficulty is that for $K > 2$, there can potentially be many types of common sets. For example, in Fig. 4(b), RNs in the set $C(1, 2, 3)$ can be assigned to any of the 3 MBNs, whereas RNs in $C(2, 3)$ can only be assigned to either m_2 or m_3. Thus, the total number of ways the RNs within all of these different common sets can be divided among K MBNs is $O((N^{K-1})^I)$, where I represents the number of distinct common sets. Observing that each MBN location and radius represents a circular region, I can be bounded by K^2 [2]. This results in a total complexity of $O(N^{K^3})$ to enumerate all possible assignments. While still polynomial in N, incurring this complexity for *each* of the $O(N^{3K})$ assignment subproblems yields an overall algorithm definitely outside the realm of practicality (e.g. even for small values of K).

With a more practical solution desired, we now develop an optimal algorithm for the general K MFPA assignment subproblem that is polynomial in *both* K and N. Recall that the MFPA assignment problem assumes that the MBN's locations and radii are fixed, and aims to find the optimal

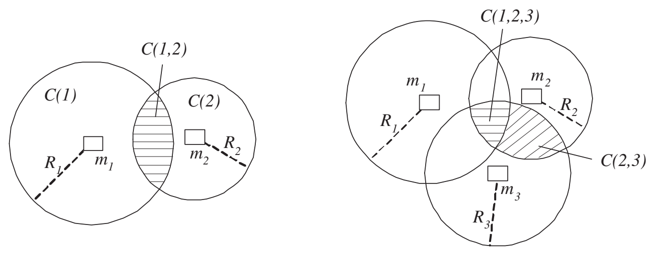

Fig. 4. (a) $K = 2$ vs. (b) $K = 3$ examples of assignment subproblem.

assignment of RNs to MBNs such that the MFPA objective is maximized. Thus to this end, we start by formulating the MFPA assignment subproblem using a mathematical programming notation. Define indicator variables x_{ij} to equal to 1 if RN i is assigned to MBN m_j. Next, define indicator constants z_{ij} to be equal to 1 if $d(i, m_j) \leq R_j$. The resulting formulation can be written as,

$$\max \quad \min_{j \in M} \quad H\big(R_j, \sum_{i \in P} x_{ij}\big) \tag{5}$$

$$s.t. \qquad \sum_{j \in M} x_{ij} = 1, \forall i \in P \tag{6}$$

$$x_{ij} \leq z_{ij}, \forall i \in P, j \in M \tag{7}$$

$$x_{ij} \in \{0, 1\} \tag{8}$$

where constraints (6) ensure that every RN is assigned to exactly one MBN, constraints (7) that we only make valid assignments, and constraints (8) integrality of the final assignment. Also recall that the objective function (5) is written as a minimization over the clusters j (as opposed to the RNs i) as a direct consequence of Observation 1. Defining the *increasing* function $F() = 1/H()$, since $H() > 0$, we can re-write the objective function in (5) as,

$$\min \quad \max_{j \in M} \quad F\big(R_j, \textstyle\sum_i x_{ij}\big) \tag{9}$$

Applying another transformation, we have (avoiding re-writing constraints (6)-(8) for brevity),

$$\min \qquad W \tag{10}$$

$$s.t. \qquad F(R_j, \sum_i x_{ij}) \leq W, \forall j \in M \tag{11}$$

where we have used the common trick of converting a *minimax* objective function into a simple *min* objective function by introducing an extra real valued variable W and moving the *max* part of the objective function into the constraints. A final transformation is applied to isolate $\sum_i x_{ij}$ on the left-hand side of (11). Define $g(W; R_j)$ to be the *inverse* with respect to $\sum_i x_{ij}$ of $F(R_j, \sum_i x_{ij})$, i.e.,

$$g\Big(F(R_j, \textstyle\sum_i x_{ij}) \ ; \ R_j\Big) = \textstyle\sum_i x_{ij} \tag{12}$$

which we assume exists. This assumption is justified since $F()$ is monotonically increasing, and therefore constitutes a one-to-one (in $\sum_i x_{ij}$) function. As an example, for the Slotted Aloha $H()$ given in (1) we have that $g(W; R_j) = W/(e \cdot R_j^{\alpha})$. Thus the overall transformed problem formulation can be written as,

$$\min \qquad W \tag{13}$$

$$s.t. \qquad \sum_{i \in P} x_{ij} \leq g(W; R_j), \forall j \in M \tag{14}$$

$$\sum_{j \in M} x_{ij} = 1, \forall i \in P \tag{15}$$

$$x_{ij} \leq z_{ij}, \forall i \in P, j \in M \tag{16}$$

$$x_{ij} \in \{0, 1\} \tag{17}$$

At this point, we note that the above *optimization* problem can be solved by way of solving a series of *feasibility* problems (e.g. fix W, and see if there exist x_{ij}'s that satisfy constraints (14)-(17). One way of doing this is by performing a binary search over the space of all possible values of W. Specifically, if the problem is feasible for a given W, we can conclude the optimal value of W, denoted W^*, is such that $W^* \leq W$. Otherwise, $W^* \geq W$ can be concluded. Alternatively, using the following Observation leads to an exact solution for W^*.

Lemma 3: The optimal W^* must satisfy $g(W^*; R_j) \in \mathbb{Z}$. That is, $g(W^*; R_j)$ must be integral.

Proof: Since $g(W; R_j)$ is the inverse of the increasing function $F(\sum_i x_{ij}; R_j)$, it too must also be increasing (i.e. in W). Next, suppose the optimal W^* did not satisfy $g(W^*; R_j) \in \mathbb{Z}$. Since the x_{ij}'s are integral, this implies that the left hand side of constraint (14) must also be integral. Thus since $g(W; R_j)$ is increasing in W, it follows that we could have further reduced W^* until $g(W^*; R_j)$ reached $\lfloor g(W^*; R_j) \rfloor$, while still satisfying constraint (14). This contradicts the minimality of W^*. ■

Combining the above lemma with the fact that there are at most $K \cdot N$ distinct integer feasible values for $g(W^*; R_j)$ yields an algorithm for finding the optimal W^* by solving $K \cdot N$ feasibility problems. Specifically, for each R_j (of which there are K), W^* can be one of $F(R_j, b), b = 1, \ldots, N$.

Given a value for W, the feasibility question is the existence of an assignment of x_{ij}'s that satisfies (14). This feasibility problem can be transformed into a classical graph problem, *Integer Max-Flow*, for which several efficient *polynomial time* algorithms exist [3]. At a high level, the Integer-Max-Flow problem aims to maximize the amount of integer-valued "flow" between a source vertex and destination vertex in a graph. The maximum amount of flow each edge can carry is an integer value, given by its "capacity". Therefore, the problem is to find the optimal amount of flow that should be sent through each edge (i.e. less than the maximum value for that edge) such that the sum flow between the source and destination is maximized. While this problem may seem somewhat unrelated to the MFPA assignment problem, a closer relationship between the edge flow values and MBN-to-RN assignments can be seen when one delves deeper into the mathematics. Indeed, the Integer-Max-Flow problem is a powerful tool that has been used to solve many assignment-type problems, such as bipartite matching and machine scheduling [13],[7]. A mathematical description of the Integer Max-Flow problem is given below.

Problem Integer Max-Flow: Given a *flow graph* $G = (V, E, C)$, where C defines an integer set of *capacities* c_{ij} on

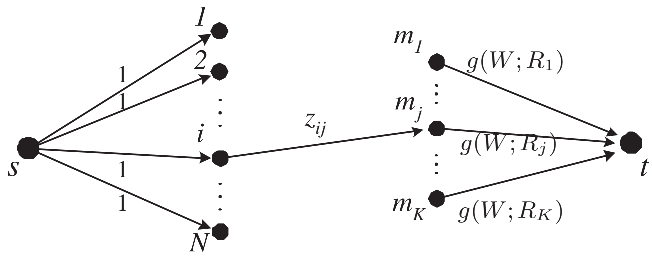

Fig. 5. Construction of the Flow Graph $G = (V, E, C)$ for a given W.

each edge $(i, j) \in E$, and a *source* vertex s and a *sink* vertex t, $s, t \in V$. The objective is to assign positive integer *flows* f_{ij} on each each edge $(i, j) \in E$ such that the aggregate flow from s to t, equal to $\sum_j f_{sj}$, is maximized. The f_{ij}'s must obey the following constraints:

1) $f_{ij} \leq c_{ij}, \forall (i, j) \in E$ (capacity constraints)
2) $\sum_i f_{ij} - \sum_k f_{jk} = 0, \forall j \in V \backslash \{s, t\}$ (flow conservation)
3) $\sum_i f_{is} = \sum_j f_{tj} = 0$ (source and sink property)

We start by constructing the flow graph $G = (V, E, C)$ depicted in Fig. 5, by defining the following graph elements:

- Nodes: Let $P \in V$ represent a set of vertices corresponding to each RN, $M \in V$ for the MBNs, and let $s, t \in V$ be the source and sink vertices in the flow graph depicted in Fig.5.
- Source Edges: N source edges (s, i) with capacities $c(s, i) = 1$, $\forall i \in P$.
- RN-to-MBN edges: edges between nodes (i, j) with capacities $c(i, j) = z_{ij}$, $\forall i \in P, j \in M$;
- Sink Edges: K sink edges (j, t) with capacities $c(j, t) = g(W; R_j)$, $\forall j \in M$.

Now, a Max-Flow algorithm can be used to find the maximum (integral) flow between s and t in G, in the above graph. Given the Max-Flow solution, a non-zero flow $f_{ij} = 1$ on an edge of type $(i, j), i \in P, j \in M$ implies that in the assignment solution RN i should be assigned to MBN j. The following lemma confirms that the above max-flow problem solves the feasibility problem.

Lemma 4: For a given W, the MFPA assignment subproblem is feasible if and only if the Max-Flow from s to t has value equal to N.

Proof: Assume an integer max-flow of value N is found. To show this corresponds to a feasible solution to the MFPA assignment subproblem, it suffices to show that all of the constraints (11), (5), (6) are satisfied. Constraints (5) are satisfied since if the max-flow is equal to N, it must mean that all source edges carry a flow of 1. Thus by flow conservation, each RN (at the endpoint of each of the source edges) must be assigned to exactly 1 MBN. Next, note that constraints (6) are satisfied since if edge $(i, j), i \in P, j \in M$ has non-zero flow across it, then by construction its capacity, which is equal to z_{ij} must be equal to 1. Finally, constraints (11) are satisfied since if more than $g(W; R_j)$ RNs are assigned to any MBN m_j, this would correspond to edge (j, t) having a greater flow than its assigned capacity. Moreover, the converse can be easily shown by construction. ■

The preceding lemma gives us the final piece of the puzzle needed in order to construct an efficient algorithm for the MFPA assignment subproblem. The algorithm is given below.

Algorithm 2 Fixed K MFPA assignment algorithm

1: **initialize** $W^* \leftarrow \infty$
2: **for** $k = 1$ to K **do**
3:   **for** $b = 1$ to N **do**
4:     **set** $W \leftarrow F(R_k, b)$
5:     **if** $W < W^*$ **then**
6:       **construct** flow graph $G = (V, E, C)$ as follows:
7:       **set** $V \leftarrow P \bigcup M \bigcup \{s, t\}$
8:       **set** $E \leftarrow E \bigcup \{(s, i)\}$, $c(s, i) \leftarrow 1, \forall i \in P$
9:       **set** $E \leftarrow E \bigcup \{(i, j)\}$, $c(i, j) \leftarrow z_{ij}, \forall i \in P, j \in M$
10:       **set** $E \leftarrow E \bigcup \{(j, t)\}$, $c(j, i) \leftarrow \lfloor g(W; R_j) \rfloor, \forall j \in M$
11:       **solve** $s - t$ Max-Flow on G. Let f_{ij} be the flows on each edge (i, j) and F_{max} the max-flow value.
12:       **if** $F_{max} = N$ **then**
13:         **set** $m(i) \leftarrow j$ if $f_{ij} = 1, \forall i \in P, j \in M$
14:         **set** $W^* \leftarrow W$
15: **return** W^*, $m(1), \ldots, m(N)$

The above algorithm operates by iterating over all possible integral values for the optimal MFPA objective function value, W, as per lemma 3. The goal is find the optimal such value for which there is a feasible solution to the assignment subproblem. To this end, the loops of steps 2 and 3 iterate all possible MBNs, 1 through k, and for each MBN the number of RNs that can be assigned to it, 1 through N. For each of these combinations, a candidate value of $W*$ is computed in step 4 as per (11). The feasibility of this candidate value is checked in steps 6 to 11, in which a flow graph is created and integer max flow evaluated in accordance with lemma 4. Finally, the minimum value of W is kept track of in step 14.

We conclude the section by noting that the best Integer Max-Flow algorithm has running time $O(KN^2 \log N)$ [9]. Therefore, the algorithm depicted above has $O(K^2 N^3 \log N)$ complexity. The result is a *worst case* complexity $O(N^{3K+3} \log N)$ algorithm for the *fixed* K MFPA problem. As will be shown in section VIII, this algorithm can be applied to solve instances with relatively small K and N.

VI. MTPA Solution

It turns out the general MTPA problem as formulated in (4) is significantly more difficult to optimally solve than the MFPA problem. For example, consider the MTPA problem for $K = 1$ MBN (i.e. ignore the assignment subproblem). At first glance it would seem like the MTPA problem looks like the well known 1-median/Fermat-Weber problem (numerically solvable in polynomial time [1]), in which one seeks to place the MBN in the location that minimizes the sum of the distances to each RN. However, the general MTPA objective is actually to maximize the sum of *arbitrary decreasing functions* of each of the distances; the difference is quite substantial. For example, consider a very simple decreasing function $H(d_i) = 1/(d_i + \gamma)$, where d_i represents the distance from RN i to the placed MBN and γ some positive constant. Clearly minimizing $\sum_i d_i$ achieves a significantly different objective from maximizing $\sum_i 1/(d_i + \gamma)$ (for which to our knowledge no optimal algorithm exists).

Thus we consider a restriction on the general MTPA problem, in which we enforce the condition on the $H()$ function that all RNs within a cluster get the same throughput, which is a function of the cluster radius and size, i.e.,

$$TP'(i) = H\Big(R_{m(i)}, |P_{m(i)}|\Big), \forall i \in P \tag{18}$$

The reasoning behind this particular restriction is two-fold. First, the above expression yields a lower bound on the general MTPA objective, i.e. since $H(d(i, m(i)), |P_{m(i)}|) \geq H(R_{m(i)}, |P_{m(i)}|), \forall i \in P$. It is therefore still useful to optimize. Second, this approach allows us to heavily leverage the discussion we have evolved through this paper for the MFPA problem. To start, for $K = 1$ the 1-center algorithm optimally solves the restricted version of the MTPA problem.

For $K > 1$, we note that Observation 1 along with lemmas 1-2 all apply to the restricted MTPA problem. Therefore, the high-level framework in section V solves the placement portion of the problem. Additionally, for $K = 2$ the simple MFPA assignment algorithm in section V-A also solves the restricted MTPA assignment subproblem, as long as the appropriate (i.e. MTPA) objective function is used.

For $K > 2$, the brute force approach discussed in the beginning of section V-B applies to the restricted MTPA assignment subproblem. However, recall that that this approach involved the impractical method of enumerating every possible RN-to-MBN assignment (i.e. given fixed MBN locations and radii), resulting in a total complexity of $O(N^{K^3})$. Furthermore, note that the fixed K MFPA assignment algorithm described in the previous section does not solve the fixed K restricted MTPA assignment problem. The main reason for this is that the type of mathematical transformation performed in (11) can only be applied to a minimax or maximin objective function, and does not translate to a "sum" objective function such as the MTPA objective function.

VII. Lower Complexity Heuristics

Although the algorithms developed so far in this paper find optimal solutions in polynomial time, their complexity is still prohibitively high unless both K and N are quite small. For example for $K = 3$, $N = 35$, the running time of the optimal MFPA algorithm was 3 hours on a Pentium 2.4GHz computer.

Thus in this section, our goal is to develop *suboptimal* approaches that have significantly less running time than the optimal approach, but still perform comparably well. We will discuss 2 such approaches: (i) An approximation algorithm that is based on cutting down the number of candidate MBN placements, and (ii) A simple and fast heuristic algorithm, but with no worst case performance guarantee. For the most part, the discussion applies to both the MFPA and restricted MTPA problems. For brevity, we will describe the algorithms in the context of the MFPA problem, noting any key issues specific to the restricted MTPA when appropriate.

A. Extended Diameter Algorithm (EDA)

As discussed in section V, the complexity of the optimal MFPA algorithm is dominated by the number of (optimality-preserving) possible placements, $\binom{|Q_P|}{K} = O(N^{3K})$. Indeed, the set Q_P is of size $O(N^3)$ due to having to consider all possible locations/radii corresponding to circumcenters/circumradii of triplets of RNs (see Fact 1). If we did not consider such "circumcenter-type" locations, but instead only looked at locations defined by (i) pairs of RNs (i.e. "diameter-type") and (ii) single RNs (i.e. "singular type"), the number of possible placements would immediately reduce to $O(N^{2K})$. This is the main idea behind the approach in this section.

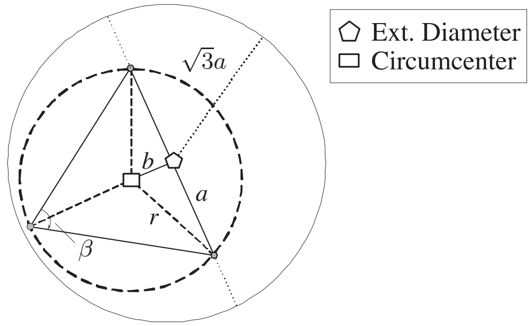

Fig. 6. Extended Diameter-type vs. Circumcenter-type placement.

Recall that in the high-level framework described in section V, we only considered placements at locations $q \in Q_P$, and assignments such that if an RN i is assigned to MBN m_k located at $q \in Q_P$, then $d(i, m_k) \leq R_q$. Such solutions were denoted as *valid*. However, an issue that comes up when circumcenter-type locations are removed from Q_P is that a valid solution may not even exist. An example of this is seen by considering 3 RNs that form a equilateral triangle. In this case, no "diameter-type" placement of an MBN with corresponding cluster radius will cover all three RNs.

To compensate for this, we define *extended-diameter* type locations, shown in Fig. 6, whose locations are the same as the original diameter-type locations, but whose associated radii are $\sqrt{3}$ times larger. Let Q'_P denote the set of all extended-diameter and singular-type locations with respect to a set of RNs P. Note that a direct analog with lemma 2 applies, i.e. Q'_P contains all extended-diameter and singular-type locations (with associated radii) with respect to any subset of RNs $T \subseteq P$. The next lemma ensures that placements among locations in Q'_P are guaranteed to contain a valid MFPA solution.

Lemma 5: For a set of RNs P, there exists a valid solution to the MFPA problem with placements at locations in Q'_P.

Proof: To prove the lemma, we just need to show that for every circumcenter-type location/radii tuple in Q_P, there exists an extended-diameter-type location/radii tuple in Q'_P that covers the same set of RNs. To this end, consider some circumcenter-type placement, and the extended-diameter location corresponding to the midpoint of the *longest* side (of length $2a$) of the acute triangle formed by the circumcenters' defining RNs. The situation is depicted in Fig. 6. Let b be the distance between the extended-diameter and circumcenter locations. Next, let r denote the circumradius. By the triangle inequality, we know that the distance between the extended diameter location and any RN covered by the circumcenter

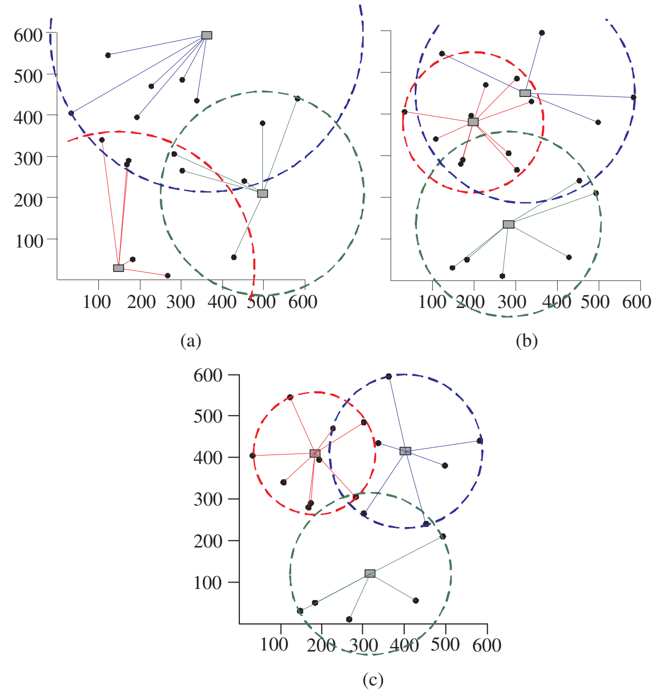

Fig. 7. Example of solutions found for a $K = 3$, $N = 20$ instance of the MFPA problem with the Slotted Aloha throughput function. (a) Unoptimized Farthest Point Heuristic (FPH) (b) Unoptimized Extended Diameter Algorithm (c) Optimal MFPA algorithm.

placement is at most $b + r$. Therefore, we have that,

$$\begin{aligned} b+r &= r+\sqrt{r^2-a^2} = \frac{a}{sin\beta} + \sqrt{\frac{a^2}{sin^2\beta} - a^2} \\ &\le \frac{2a}{\sqrt{3}} + \frac{a}{\sqrt{3}} = \sqrt{3}a \end{aligned} \tag{19}$$

where we have used a geometric property of circumcenters that $r = \frac{a}{sin\beta}$. Additionally, we have used the observation that since the defining triangle is acute and since the extended-diameter location under consideration is defined by the longest edge of the triangle, that $\pi/3 \le \beta \le \pi/2$. ■

Finally, define the *extended-diameter 1-center* of a set of RNs P as the location in Q'_P that minimizes the maximum distance from any RN in P. We now state the analog of lemma 1 applied to this context, whose proof follows from lemma 5.

Lemma 6: Let $P_1^*, P_2^*, \ldots, P_K^*$ represent the optimal assignments of RNs to MBNs $m_1, m_2, \ldots, m_K$ respectively. Then, there exists a solution to the overall MFPA problem in which MBNs are placed at the extended-diameter 1-centers of $P_1^*, P_2^*, \ldots, P_K^*$. Also, the objective value of this solution is at least $H(\sqrt{3}R^*, |P^*|)$, where R^* and $|P^*|$ represent the worst case cluster radius and size of the optimal solution.

We define the Extended-Diameter Algorithm (EDA) for the fixed K MFPA as well as the $K = 2$ MTPA problem, as basically the optimal algorithms described earlier, with Q'_P used in place of Q_P. The only difference is a final optimization step, in which after the suboptimal extended-diameter placement is decided, we move each of the MBNs to the actual 1-center location of their assigned RNs.

By the preceding discussion, the EDA algorithm is a $\frac{H(\sqrt{3}R^*,|P^*|)}{H(R^*,|P^*|)}$-approximation algorithm for the MFPA. For path loss exponent α=2 this ratio evaluates to $1/3$ for both

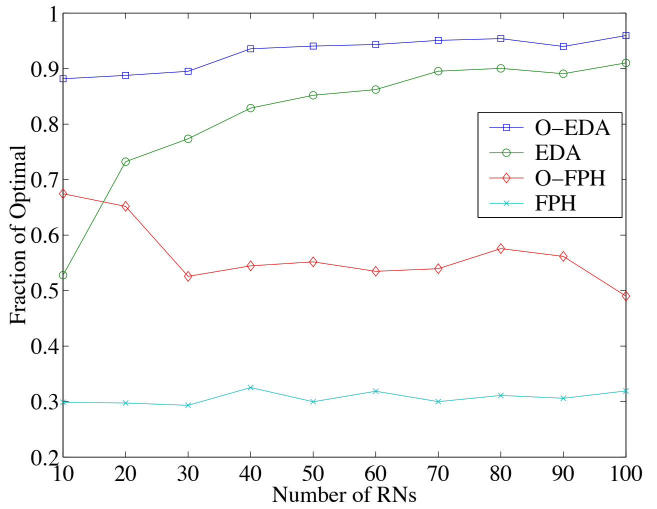

Fig. 8. Average case simulation for $K = 2$ for the MFPA problem with Slotted Aloha throughput function

the Slotted-Aloha and CDMA throughput functions. The worst case running time of the algorithm is $O(N^5)$ for K=2, and $O(N^{2K+3}\log N)$ for fixed K>2.

B. Farthest Point Heuristic (FPH)

This next algorithm is an adaptation of Gonzalez's Farthest Point Heuristic (FPH) algorithm, originally developed as an approximation algorithm for the K-center problem [10]. We apply the FPH algorithm to the MFPA problem, with a modification in which we optimize the placement of the backbone node within each cluster. Specifically, the modified algorithm works as follows: Initialize the algorithm by placing an MBN on top an arbitrary RN, and assign all RNs to this MBN. Place the next MBN on top of the RN farthest from its assigned MBN, and re-assign RNs to their nearest MBN. Repeat the previous step until all K MBNs are placed. The above placement can be "optimized" by moving each MBN to the 1-center location of its assigned RNs. The running time of the unoptimized version of this algorithm is $O(N\log K)$, and using a practical 1-center algorithm [1], the optimized version takes $O(KN\log N)$ time.

VIII. Simulation Results

In this section we compare the performance of the various algorithms presented in this paper via simulation. To this end, we begin with an example of running the algorithms on a single $K = 3$ MBNs, $N = 20$ RNs, MFPA instance, shown in Fig. 7. We assume the RNs are randomly distributed in a 600×600 plane, and we use the Slotted-Aloha $H()$ throughput function given in (1), with $\alpha = 2$. Note that in order to illustrate the fundamental differences between the algorithms, the optimization step of moving each placed MBN to the true 1-center location of its cluster is not performed for either the Extended Diameter Algorithm (EDA) or the Farthest Point Heuristic (FPH).

As can be seen, the optimal solution achieves the ideal balance between lightly loading clusters of large radii vs. heavily loading clusters of smaller radii. By contrast, the FPH

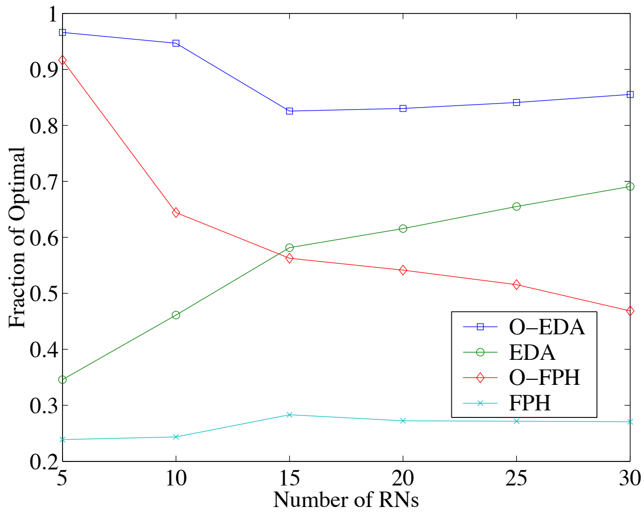

Fig. 9. Average case simulation for $K = 3$ for the MFPA problem with Slotted Aloha throughput function

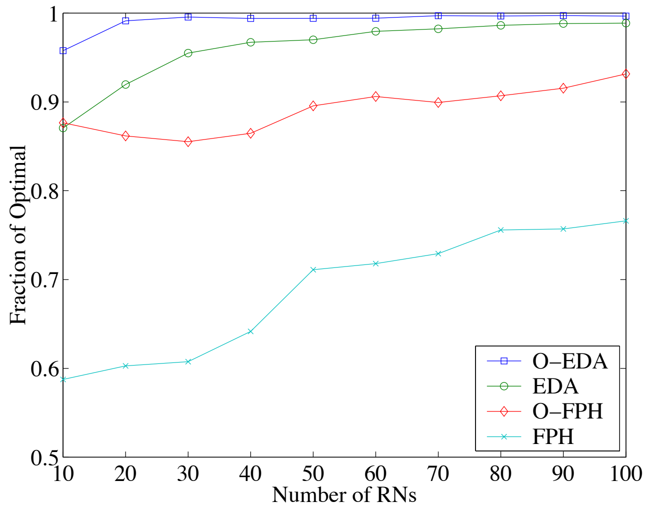

Fig. 10. Average case simulation for $K = 2$ for the MTPA problem with CDMA throughput function

solution potentially creates enormous radius clusters. Moreover, since nothing intelligent is done by the FPH regarding the assignment problem (i.e. just assign RNs to their closest MBN), the large radius clusters can also get heavily loaded. The EDA does better, in that even though its cluster radii are larger than optimal, it intelligently assigns RNs in a way that achieves optimal load balancing among the placed clusters.

Figs. 8 and 9 show an average case plot for varying numbers of RNs, and $K = 2$ and $K = 3$ MBNs. In these plots the optimized versions of the EDA and FPH are indicated by O-EDA and O-FPH respectively, to show the improvement this step results in. The parameters are the same as for the previous scenario, and we average each data point over 20 random instances. We present the average ratio of the throughput achieved by the suboptimal algorithms as compared to that of the optimal algorithms described in sections V. In both figures, we can notice that the optimization step significantly improves the performance of the heuristics. However, as exhibited by the poor performance of both the optimized and regular FPH, the optimization step can only help insofar as lowering the cluster radius if possible; it cannot make up for already-made poor assignment decisions.

Finally, Fig. 10 shows an average case simulation for the $K = 2$ MTPA problem with the CDMA throughput objective function from (2). We set $\eta = 10^{-4}$ in order to normalize the SNR somewhat, and add 1 to the denominator so as to maintain $H() < \infty$ as mentioned in section III. That these constants are set somewhat arbitrarily (i.e. "correct" values would come from a real system) should not effect the *relative* comparison of the algorithms. Note that the O-EDA achieves aggregate throughput very close to optimal. In fact, all of the algorithms perform significantly better (relative to optimal) for the MTPA objective than for the MFPA objective, albeit with different $H()$ functions. Nevertheless, this would seem to indicate that the max-sum (i.e. MTPA) objective is less *sensitive* to suboptimal MBN placement/assignment than the max-min (i.e. MFPA) objective.

IX. Conclusion

The recently studied Mobile Backbone Network architecture can significantly improve the performance, lifetime and reliability of MANETs and WSNs. In this paper, we have focused on the key problem of how to jointly place the Mobile Backbone Nodes (MBNs), and assign every Regular Node to exactly one MBN. To this end, we have formulated two problems under a general communications model. The first is the Maximum Fair Placement and Assignment (MFPA) problem in which the objective is to maximize the throughput of the minimum throughput RN. The second is the Maximum Throughput Placement and Assignment (MTPA) problem, in which the objective is to maximize the aggregate throughput of the RNs. Our main result is a novel optimal polynomial time algorithm for the MFPA problem for fixed K. We have also provided an optimal solution for a restricted version of the MTPA problem for $K \leq 2$. We have developed two heuristic algorithms for both problems, including an approximation algorithm with bounded worst case performance loss. Finally, we have presented simulation results to evaluate the performance of the various algorithms developed in the paper.

To our knowledge the problems presented in this paper have not been considered before. Thus for this paper, our primary goal has been to provide a theoretical framework, as well as basic optimal solutions. Future work involves the development of more efficient, distributed and mobility-handling algorithms for both the MFPA and MTPA problems.

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