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# Controlled mobility in stochastic and dynamic wireless networks

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**Abstract** We consider the use of controlled mobility in wireless networks where messages arriving randomly in time and space are collected by mobile receivers (collectors). The collectors are responsible for receiving these messages via wireless transmission by dynamically adjusting their position in the network. Our goal is to utilize a combination of *wireless transmission* and *controlled mobility* to improve the throughput and delay performance in such networks. First, we consider a system with a single collector. We show that the necessary and sufficient stability condition for such a system is given by $\rho < 1$ where $\rho$ is the expected system load. We derive lower bounds for the expected message waiting time in the system and develop policies that are stable for all loads $\rho < 1$ and have asymptotically optimal delay scaling. We show that the combination of mobility and wireless transmission results in a delay scaling of $\Theta(\frac{1}{1-\rho})$ with the system load $\rho$, in contrast to the $\Theta(\frac{1}{(1-\rho)^2})$ delay scaling in the corresponding system without wireless transmission, where the collector visits each message location. Next, we consider the system with multiple collectors. In the case where simultaneous transmissions to different collectors do not interfere with each other, we show that both the stability condition and the delay scaling extend from the single collector case. In the case where simultaneous transmissions to different collectors interfere with each other, we characterize the stability region of the system and show that a frame-based version of the well-known Max-Weight policy stabilizes the system asymptotically in the frame length.

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## 1 Introduction

There has been a significant amount of interest in performance analysis of mobility assisted wireless networks in the last decade (e.g., [24, 40, 43, 47, 48, 54]). Typically, throughput and delay performance of networks were analyzed where nodes moving according to a random mobility model are utilized for relaying data (e.g., [22, 24]). More recently, networks deploying nodes with controlled mobility have been considered focusing primarily on route design and ignoring the communication aspect of the problem (e.g., [28, 40, 48, 54]). In this paper, we explore the use of *controlled mobility* and *wireless transmission* in order to improve the throughput and delay performance of such networks. We consider a dynamic vehicle routing problem where a vehicle (collector) uses a combination of physical movement and wireless reception to receive randomly arriving data messages.

Our model consists of collectors that are responsible for gathering messages that arrive randomly in time at uniformly distributed geographical locations. The messages are transmitted when a collector is within their communication range and depart the system upon successful transmission. Collectors adjust their positions in order to successfully receive these messages in the least amount of time as shown in Fig. 1 for the case of a single collector. This setup is particularly applicable to networks deployed in a large area so that mobile elements are necessary to provide connectivity between spatially separated entities in the network [28, 40, 48]. Moreover, this model can be used to analyze the delay performance of a densely deployed sensor network where mobile base stations collect data from a large number of sensors inside the network [29, 48]. Our model is also applicable to a network where Unmanned Aerial Vehicles (UAVs) are used as data harvesting devices in a battlefield environment [28, 43], and to networks where data rate is relatively low so that data transmission time is comparable to the collector's travel time, for instance, in underwater sensor networks [1].

### 1.1 Related work

Vehicle Routing Problems (VRPs) have been extensively studied in the past (e.g., [2, 5, 7, 19, 41, 51, 54]). The common example of a VRP is the Euclidean Traveling Salesman Problem (TSP) in which a single server is to visit each member of a fixed set of locations on the plane such that the total travel cost is minimized. Several extensions of TSP have been considered in the past such as stochastic demand arrivals and the use of multiple servers [7, 8, 19, 54]. In particular, in the TSP with neighborhoods (TSPN) problem, a server is to visit a neighborhood of each demand location [5, 41], which can be used to model a mobile collector receiving messages from a

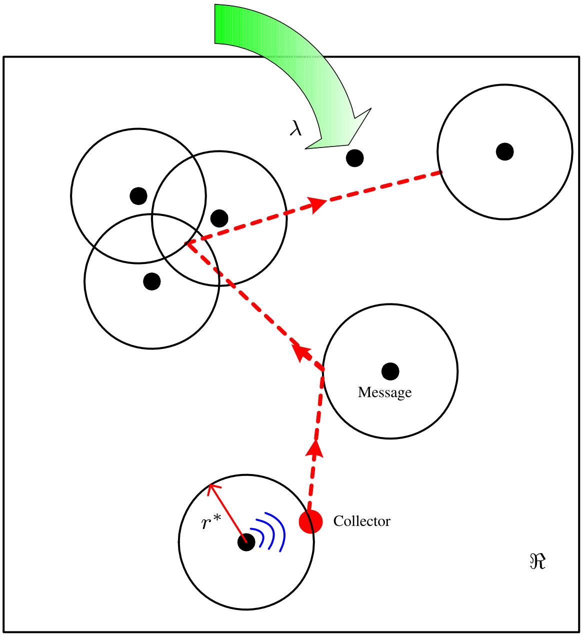



**Fig. 1** The system model for the case of a single collector. The collector adjusts its position in order to collect randomly arriving messages via wireless communication. The *circles* with radius $r^*$ represent the communication range and the *dashed line* segments represent the collector's path

communication distance. A more detailed review of the literature in this field can be found in [36, 41], and [57].

Of particular relevance to us among the VRPs is the Dynamic Traveling Repairman Problem (DTRP) due to Bertsimas and Van Ryzin [7–9]. DTRP is a stochastic and dynamic VRP in which a vehicle is to serve demands that arrive randomly in time and space. Fundamental lower bounds on delay were established and several vehicle routing policies were analyzed for DTRP for a single server in [7], for multiple servers in [8], and for general demand and interarrival time distributions in [9]. Later, [54] generalized the DTRP model to analyze Dynamic Pickup and Delivery Problem (DPDP), where fundamental bounds on delay were established. In this work, we apply the DTRP model to wireless networks where the demands are data messages to be transmitted to a collector which can receive messages from a distance using wireless transmission.[1]

Another closely related body of literature is the continuous or spatial polling models [2, 4, 18, 27, 34, 35, 37, 44]. Greedy or continuously polling server strategies were considered in [18, 27, 34, 35, 37], where necessary and, in some cases, sufficient stability conditions were derived. Excellent reviews on Greedy server systems in one dimensional regions can be found in [37] and [44]. In these works, the arrival locations and the server motion are constrained to a line segment or a circle and thus, they are fundamentally different from our model. Altman and Levy studied spatial polling models in two or higher-dimensional spaces in [2, 4], which are closely related to

[1]We refer to the DTRP model as the system without wireless transmission since in this model the collector needs to be at the message location in order to be able to serve it.

the work in this paper. In [2], they considered the *Queuing in Space* model which is similar to the DTRP model, however, it is more general in that it allows stochastic arrivals to queue up at particular locations in the Euclidean plane. They showed that $\rho < 1$ is a necessary and sufficient condition for stability and that a greedy version of the cyclic and Globally Gated policy, which serves the nearest message among the messages in the current cycle, stabilizes the system. Similar to DTRP, the delay scaling with load is shown to be $\Theta(1/(1-\rho)^2)$. This model was later extended to general independent arrival processes with only first-moment conditions on service and walking times in [4]. Tassiulas considered a similar model in [51], where they proposed adaptive routing policies that achieve maximum throughput independent of the statistical parameters of the system. Similar to DTRP, in these works, the collector must visit each message location without the possibility of wireless transmission.

The independent works in [29–31] considered utilizing mobile wireless servers as data relays on periodic routes. In [30] and [31], continuous polling systems where the collector's motion and the arrival distribution are confined to a one-dimensional space, such as a circle, were considered, and expected waiting time and workload in the system were analyzed. In [29], the authors considered the use of message ferries that are allowed to transmit/receive messages only when the ferry is at one of a finite number of predetermined stops. Namely, the ferry has a predetermined cyclic path, and the objective is to determine this cyclic path and the number of stops on it *offline*. This is a significantly different problem from the current paper in that in our model the collector updates its route online, and has unconstrained mobility in the two-dimensional network region. Furthermore, the works in [29–31] do not attempt to derive fundamental bounds on expected waiting time.

Data collection by a mobile server from a finite number of queues has been thoroughly analyzed under Polling models in Queuing Theory literature (e.g., [3, 11–13, 20, 23, 38, 39, 58]). Stability of Polling systems under Exhaustive, Gated, or Limited service disciplines with cyclic routing was studied in [3, 23], while stability of Exhaustive service policies under state-dependent routing was considered in [20]. In [11], steady-state queue length distribution of a general class of Polling models with and without switchover delays was analyzed, the impact of the switchover delay was studied, and a scheme to compute moments of message waiting time was proposed. In this context, [12] derived the pseudo conservation law for mean waiting times, while [13] and [58] derived mean waiting time formulas for Exhaustive, Gated, and Globally Gated service disciplines. Finally, optimal server routing and various dominance relationships were analyzed in [38] and [39]. For an extensive review of literature in the context of Polling Systems, please see [50] or [53].

Recently, [46] considered the problem of allocating data collection tasks among UAVs, where they proposed a Hedonic coalition formation algorithm which outperforms schemes that allocate tasks equally among UAVs. In a system where multiple mobile nodes with controlled mobility and communication capability relay, the messages of static nodes, [47] derived a lower bound on node travel times. Message sources and destinations were modeled as static nodes in [47] and queuing aspects were not considered. A mobile server harvesting data from two spatial queues in a wireless network was considered in [43] where the stability region of the system was characterized using a fluid model approximation.

### 1.2 Our contributions and outline

To the best of our knowledge, our work is the first attempt to develop fundamental lower and upper bounds on delay in a system where a collector is to gather data messages randomly arriving in time and space using *wireless communication* and *controlled mobility*. In the first part of the paper, we consider a system with a single collector and extend the results of [2] and [7] to the communication setting. In particular, we show that $\rho < 1$ is the necessary and sufficient stability condition where $\rho$ is the system load. We derive lower bounds on delay and develop algorithms that are asymptotically within a constant factor of the lower bounds. We show that the combination of mobility and wireless transmission results in a delay scaling of $\Theta(1/(1-\rho))$. This is in sharp contrast to the $\Theta(1/(1-\rho)^2)$ delay scaling in the system where the collector visits each message location analyzed in [2, 7]. In the second part of the paper, we consider the system with multiple collectors under the assumption that simultaneous transmissions to different collectors do not interfere with each other. We show that the necessary and sufficient stability condition is still given by $\rho < 1$, where $\rho$ is the load on multiple collectors. We develop lower bounds on delay and generalize the single-collector policies, analyzed in the first part, to the multiple-collectors case. Finally, we consider a multiple-collector system subject to interference constraints on simultaneous transmissions to different collectors. We formulate a scheduling problem and characterize the stability region of the system in terms of interference constraints. We show that a frame-based version of the Max-Weight scheduling policy can stabilize the system whenever it is feasible to do so at all.

This paper is organized as follows. In Sect. 2, we consider the single-collector case. We present the model in Sect. 2.1, characterize the necessary and sufficient stability condition in Sect. 2.2, derive a delay lower bound in Sect. 2.3, and analyze single-collector policies in Sect. 2.4. In Sect. 3, we extend the results for the single-collector to systems with multiple collectors whose transmissions do not interfere with each other. Finally, in Sect. 4, we consider the system with interference constraints on simultaneous transmissions.

## 2 The single collector case

In this section, we consider the case of a single collector and develop fundamental insights into the problem. We extend the stability and the delay results of [2] and [7], established for the system where the collector visits each message location, to systems with wireless transmission capability. We show that the combination of mobility and wireless transmission results in a delay scaling of $\Theta(\frac{1}{1-\rho})$ with the system load $\rho$, which is in contrast to the $\Theta(\frac{1}{(1-\rho)^2})$ delay scaling in the corresponding system without wireless transmission in [2] and [7].

### 2.1 Model

Consider a square region $\mathcal{R}$ of area $A$ and messages arriving into $\mathcal{R}$ according to a Poisson process (in time) of intensity $\lambda$. Upon arrival, the messages are distributed independently and uniformly in $\mathcal{R}$ and they are to be gathered by a collector via wireless reception. An arriving message is transmitted to the collector when the collector comes within the *communication range* of the message location and grants access for the message's transmission. Therefore, there is no interference power from the neighboring nodes during message receptions.

We assume a disk model [17, 25] for determining successful message receptions. Let $r^*$ be the *communication range* of the collector. Under the disk model, a transmission can be received only if it is within a disk of radius $r^*$ around the collector. Note that this model is similar to the Signal to Noise Ratio (SNR) packet reception model [17, 24, 25], under which a transmission is successfully decoded at the collector if its received SNR is above a threshold. Under this model, if the location of the next message to be received is within $r^*$, the collector stops and attempts to receive the message. Otherwise, the collector travels towards the message location until it is within a distance $r^*$ from the message. Under the disk model, transmissions are assumed to be at a constant rate taking a fixed amount of time denoted by $s$.

The collector travels from the current message reception point to the next message reception point at a constant speed $v$. We assume that at a given time the collector knows the locations and the arrival times of messages that arrived before this time. The knowledge of the service locations is a standard assumption in vehicle routing literature [2, 7, 19, 41]. Location information can be obtained from GPS devices or Inertial Measurement Units (IMUs), and distributed using a low-rate, but long-range, control channel. In the context of sensor networks, location information can also be obtained via distributed localization schemes using wide-band signaling [56].

Let $N(t)$ denote the total number of messages in the system at time $t$. The system is said to be *stable* under a given control policy $\pi$ if the number of messages in the system $N(t)$ converges in distribution to a stationary process with a finite mean. Let $\rho = \lambda s$ denote the load arriving into the system per unit time. For stable systems, $\rho$ denotes the fraction of time the collector spends receiving messages. *The stability region* $\mathbf{\Lambda}$ is the set of all loads $\rho$ such that there exists a control algorithm that stabilizes the system. A policy is said to be *stabilizing* if it stabilizes the system for all loads strictly inside $\mathbf{\Lambda}$.

We define $T_i$ as the time between the arrival of message $i$ and its successful reception. $T_i$ has three components: $W_{d,i}$, the waiting time due to collector's travel distance from the time message $i$ arrives until it gets served, $W_{s,i}$, the waiting time due to the reception times of messages served from the time message $i$ arrives until it gets served, and $s$, reception time of the message. The total waiting time of message $i$ is given by $W_i = W_{d,i} + W_{s,i}$, hence $W_i = T_i - s$. The expected waiting time $W$ is defined as $W \doteq \lim_{i\to\infty} \mathbb{E}[W_i]$ whenever the limit exists. $T$, $W$, $W_d$, and $W_s$ are defined similarly, and $T = W_d + W_s + s$, whenever the limits exist. Finally, $T^*$ is defined to be the optimal system time which is given by the policy that minimizes $T$.

### 2.2 Stability

Next, we show that $\rho < 1$ is a necessary and sufficient condition for stability of the system. Note that this condition is also necessary and sufficient for stability of the corresponding system without wireless transmission, as shown in [2], as well as for a

G/G/1 queue [33]. We first lower bound the number of messages in the system by that in the equivalent system in which travel times are zero (i.e., $v = \infty$). This idea was used in [2] to establish a necessary stability condition for the corresponding system without wireless transmission.

**Lemma 1** *A necessary condition for stability is* $\rho < 1$.

The proof is given in [16] and it is based on an induction argument that the total number of messages in the system dominates that in the corresponding infinite-speed system, i.e., the *M/D/1* queue, for which the stability condition is $\rho < 1$.

Next, we show that $\rho < 1$ is a sufficient condition for stability of the system under a policy based on Euclidean TSP with neighborhoods (TSPN). TSPN is a generalization of TSP in which the server is to visit a neighborhood of each demand location via the shortest path [5, 41], for which polynomial-time $(1 + \epsilon)$-approximation algorithms parameterized by $\epsilon > 0$ has been developed [41]. In our case, the neighborhoods are disks of radius $r^*$ around each message location.

Under the TSPN policy, the collector performs a cyclic service of the messages present in the system starting and ending the cycle at the center of the network region. Let time $t_k$ be the time that the collector returns to the center for the $k$th time, where $t_0 \doteq 0$. Suppose the system is initially empty at time $t_0$. The TSPN policy is described in detail in Algorithm 1. Let the total number of messages waiting for service at time $t_k$, $N(t_k)$, be the system state at time $t_k$. Note that $\{N(t_k)\};\ k \in \mathbb{N}$ is an irreducible Markov chain on countable state space $\mathbb{N}$.

**Theorem 1** *The system is stable under the TSPN policy for all loads* $\rho < 1$.

The proof is given in Appendix A. It follows techniques similar to those of [2] and [13] to first establish a bound on the waiting time of an arbitrary message. Then the proof utilizes the stationary version of Little's law to establish the finiteness of the expected number of messages in the system.

Theorem 1 establishes that $\rho < 1$ is also sufficient for stability. The travel time does not affect the stability region of the system as expected. Note that for the analysis above, we assumed that the computation time of the TSPN tour is negligible as compared to the travel time of the collector. In a real-world scenario, having to wait for the computation can potentially affect the stability region. For instance, if the computation time takes $\epsilon$-fraction of the expected cycle duration, then the TSPN policy cannot stabilize the system for arrival rates in the outer $\epsilon$-strip of the stability

**Algorithm 1** *TSPN Policy*

1: Wait at the center of $\mathcal{R}$ until the first message arrival, move to serve this message and return to the center.
2: If the system is empty at time $t_k, k = 1, 2, \ldots$, repeat the above process.
3: If there are messages waiting for service at time $t_k, k = 1, 2, \ldots$, compute the TSPN tour through all the messages that are present in the system at time $t_k$, receive these messages in that tour and return to the center. Repeat 2 and 3.

region. Note that the partitioning policy proposed in Sect. 2.4.2 is a simpler policy in that it does not require the knowledge of the message locations or arrival times, nor it needs to compute a tour for each cycle. The advantage of the TSPN policy is that it leads to shorter travel times in each cycle resulting in delay savings. Finally, simple greedy and cyclic policies based on receiving the closest message in the current cycle were considered in [2] and [37]. These policies do not need any tour computation, however, the analysis of such policies in the context of wireless transmission does not appear to be tractable.

While the wireless transmission capability does not enlarge the stability region, it fundamentally affects the delay scaling in the system as we show in the next section.

## 2.3 Lower bound on delay

For wireless networks with a small area or very good channel quality such that $r^* \geq \sqrt{A/2}$, the collector can receive messages from the center of the network region. In that case, we have an *M/D/1* queue and the associated queuing delay is given by the P-K formula as $W = \lambda s^2/(2(1-\rho))$. However, when $r^* < \sqrt{A/2}$, the collector has to move in order to receive some of the messages. In this case, the reception time $s$ is still a constant, however, the travel time per message is a random variable. Next, we provide a delay lower bound, similar to a lower bound in [7], with the added complexity of communication capability in our system.

**Theorem 2** *The optimal expected message waiting time in steady-state $T^*$ is lower bounded by*

$$T^* \geq \frac{\mathbb{E}[\max(0, \|U\| - r^*)]}{v(1-\rho)} + \frac{\lambda s^2}{2(1-\rho)} + s, \tag{1}$$

*where $U$ is a random variable that has a uniform distribution over the network region $\mathcal{R}$, and $\|U\|$ is the distance of $U$ to the center of $\mathcal{R}$.*

Note that the $\mathbb{E}[\max(0, \|U\| - r^*)]$ term can be further lower bounded by $\mathbb{E}[\|U\|] - r^*$, where $\mathbb{E}[\|U\|] = 0.383\sqrt{A}$ [7].

*Proof* As outlined in Sect, 2.1, the expected delay of a message in steady state has three components:

$$T = W_d + W_s + s. \tag{2}$$

A lower bound on $W_d$ is found as follows: Note that $W_d.v$ is the expected *distance* the collector moves during the waiting time of a message. This distance is at least as large as the average distance between the location of the message and the collector's location at the time of the message's arrival less the reception distance $r^*$. The location of an arrival is determined according to the uniform distribution over the network region, while the collector's location distribution is in general unknown as it depends on the collector's policy. We can lower bound $W_d$ by characterizing the expected distance between a uniform arrival and the best a priori location in the network that minimizes the expected distance to a uniform arrival. Namely, we are after the location $\nu$ that minimizes $\mathbb{E}[\|U - \nu\|]$ where $U$ is a uniformly distributed random variable. The location $\nu$ that solves this optimization is called the *median* of the region and in our case the median is the center of the square shaped network region. Thus, we obtain the following bound:

$$W_d \geq \frac{\mathbb{E}[\max(0, \|U\| - r^*)]}{v}. \tag{3}$$

Let $N$ be the expected number of messages served in a waiting time and let $R$ be the average residual service time. Due to the PASTA property of Poisson arrivals [6, p. 171] a given arrival in steady state observes the steady state occupancy distribution. Therefore, the average residual time observed by an arrival is also $R$, and it is given by $\lambda s^2/2$, which gives [7]

$$W_s = sN + R. \tag{4}$$

When the system is stable and in steady state, the expected number of messages served during a waiting time is equal to the expected number of arrivals during a waiting time, which in turn is equal to the expected number of messages in the system in steady state [7]. To see this, note that since the future arrivals are independent of the current number of messages in the system under Poisson arrivals, the steady state occupancy distribution observed by a Poisson arrival is the same as the time-stationary distribution of the number of messages in the system [6, p. 172]. Furthermore, since the messages are served one at a time, every state $N(t) = n$ is visited infinitely often in a stable system, and the steady state occupancy distribution observed by a departing customer is also equal to the occupancy distribution observed by an arriving customer [21, p. 173]. Therefore, the expected number of messages served during a waiting time, $N$, must be equal to the expected number of messages that arrive during a waiting time, which is also equal to the expected number of messages in the system. Finally, the last quantity is given by the steady state version of Little's law to be $N = \lambda W = \lambda(W_d + W_s)$ [7, 54]. Substituting this in (4), we obtain

$$W_s = s\lambda(W_d + W_s) + \frac{\lambda s^2}{2}.$$

This implies

$$W_s = \frac{\rho}{1 - \rho} W_d + \frac{\lambda s^2}{2(1 - \rho)}. \tag{5}$$

Substituting (3) and (5) in (2) yields (1). □

Theorem 2 shows that, in addition to the expected waiting time of an *M/G/1* queue $\lambda s^2/(2(1 - \rho))$, the queueing delay has another component dependent on the travel time of the collector.

### 2.4 Collector policies

We derive upper bounds on expected delay by analyzing policies for the collector. From (18) it can be shown that the expected number of messages in steady state under the TSPN policy is $O(1/(1 - \rho))$. Therefore, the TSPN policy has optimal delay scaling. We consider the First Come First Serve (FCFS) and the Partitioning policies that are much simpler than the TSPN policy, and have good delay properties. In particular, the FCFS policy is delay-optimal at light loads and the Partitioning policy has delay performance that is very close to the lower bound when the travel and reception times are comparable.

#### *2.4.1 First come first serve (FCFS) policy*

A straightforward policy is the FCFS policy where the messages are served in the order of their arrival times. A version of the FCFS policy, call FCFS′, where the collector has to return to the center of the network region after each message reception was shown to be delay optimal at light loads for the DTRP problem [7], i.e., $T_{\text{FCFS}'} \to T^*$ as $\rho \to 0$. This is because the center of the network region is the location that minimizes the expected distance to a uniformly distributed arrival. Since in our system we can do at least as well as the DTRP by setting $r^* = 0$, FCFS′ is delay optimal also for our system at light loads. Furthermore, the FCFS′ policy is not stable for all loads $\rho < 1$, namely, there exists a value $\hat{\rho}$ such that the system is unstable under FCFS′ policy for all $\rho > \hat{\rho}$. This is because under the FCFS′ policy, the average per-message travel component of the service time is fixed, which makes the average arrival rate greater than the average service rate as $\rho$ increases. Therefore, it is better for a policy to serve more messages in the same "neighborhood" in order to reduce the amount of time spent on mobility.

#### *2.4.2 Partitioning policy*

Next, we propose a policy based on partitioning the network region into subregions and the collector performing a cyclic service of the subregions. This policy is an adaptation of the Partitioning policy of [7] to the case of a system with wireless transmission and it implements a cyclic polling discipline with exhaustive service. An adaptive version of the Partitioning policy of [7] was considered in [51], where the subregion sizes are determined adaptively based on the number of service demands in the system, and the system is stabilized without requiring the knowledge of the arrival rates. We explicitly derive the delay expression for this policy and show that it scales with the load as $O(\frac{1}{1-\rho})$.

We divide the network region into $(\sqrt{2}r^* \times \sqrt{2}r^*)$ squares as shown in Fig. 2. This choice ensures that every location in the square is within the communication distance $r^*$ of the center of the square. The number of subregions in such a partitioning is given by[2] $n_s = A/(2(r^*)^2)$. The partitioning in Fig. 2 represents the case of $n_s = 16$ subregions. The collector services the subregions in a cyclic order, as shown in Fig. 2, by receiving the messages in each subregion from its center using an FCFS order. The messages within each subregion are served exhaustively, i.e., all the messages in a subregion are served before moving to the next subregion. The

[2]Note that such a partitioning requires $\sqrt{n_s} = \sqrt{A/(2(r^*)^2)}$ to be an integer. This may not hold for a given area $A$ and a particular choice of $r^*$. In that case one can partition the region using the largest reception distance $\underline{r}^* < r^*$ such that this integer condition is satisfied.

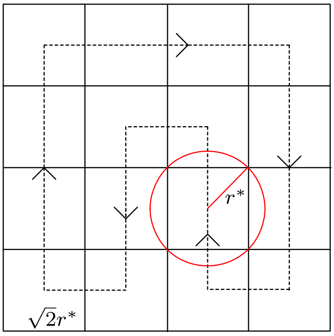



**Fig. 2** The partitioning of the network region into square subregions of side $\sqrt{2}r^*$. The *circle* with radius $r^*$ represents the communication range and the *dashed lines* represent the collector's path

collector then serves the messages in the next subregion exhaustively using FCFS order and repeats this process. The distance traveled by the collector between each subregion is a constant equal to $\sqrt{2}r^*$. It is easy to verify that the Partitioning policy behaves as a multiuser *M/G/1* system with reservations and exhaustive service (see [6, p. 198]), where the $n_s$ subregions correspond to users and the travel time between the subregions corresponds to the reservation interval. Using the delay expression for the multiuser *M/G/1* queue with reservations and exhaustive service in [6, p. 200], we obtain

$$T_{\text{part}} = \frac{\lambda s^2}{2(1-\rho)} + \frac{n_s - \rho}{2v(1-\rho)}\sqrt{2}r^* + s. \tag{6}$$

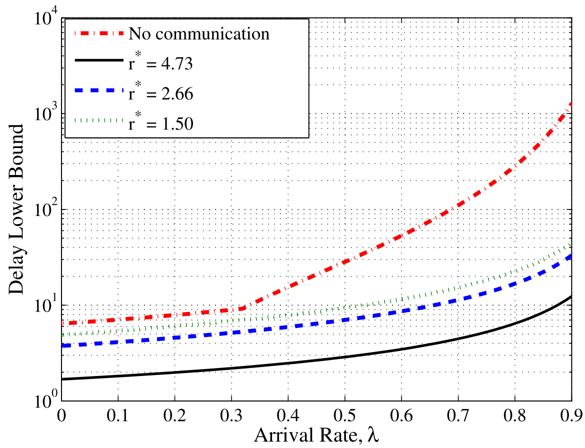
Combining this result with (1) and noting that the above expression is finite for all loads $\rho < 1$, we have established the following observation.

**Observation 1** *The expected message waiting time in steady state scales as $\Theta(\frac{1}{1-\rho})$ with the load $\rho$, and the Partitioning policy is stable for all $\rho < 1$.*

Despite the travel component of the service time, we can achieve $\Theta(\frac{1}{1-\rho})$ delay as in the *M/G/1* queue. This is the fundamental difference between this system and the corresponding system where wireless transmission is not used, where the delay scaling with load is $\Theta(\frac{1}{(1-\rho)^2})$ [2, 7]. This difference can be explained intuitively as follows. Denote by $N$ the average number of departures in a waiting time. The $W_s$ expression as a function of $W_d$ in (5) implies that $N$ can be lower bounded by $\frac{\lambda W_d}{1-\rho}$. For the system in [7], the minimum per-message distance traveled in the high load regime scales as $\Omega(\frac{\sqrt{A}}{\sqrt{N}})$ [7]. This is due to the fact that the nearest neighbor distance among $N$ uniformly distributed points on a square of area $A$ is $\Omega(\frac{\sqrt{A}}{\sqrt{N}})$. Therefore, for this system we have $W_d \approx N\Omega(\frac{\sqrt{A}}{\sqrt{N}}) \approx \Omega(\sqrt{NA})$ which gives $N \approx \Omega(\frac{\lambda^2 A}{(1-\rho)^2})$. In contrast, with the wireless reception capability, the collector does not need to move for messages that are inside a disk of radius $r^*$ around it. Since a finite (constant) number of such disks cover the network region, $W_d$ can be upper bounded by a constant independent of the system load.

It is interesting to note that [15] considered the case where messages were transmitted to the collector according to a random access scheme, i.e., transmissions occur

**Fig. 3** Delay lower bound vs. network load using different communication ranges for $A = 200$, $v = 1$, and $s = 1$




with probability $p$ in each time slot. There the delay scaling of $\Omega(\frac{1}{(1-\rho)^2})$ was observed, which is similar to the system without wireless transmission. The reason for this is that in order to have successful transmissions under the random access interference of neighboring nodes, the reception distance should be of the same order as the nearest neighbor distances [15, 24].

### 2.4.3 Numerical results-single collector

We present numerical results corresponding to the analysis in the previous sections. We lower bound the delay expression in (1) using $\mathbb{E}[\max(0, \|U\| - r^*)] \geq \mathbb{E}[\|U\|] - r^*$, where $[\|U\|] = 0.383\sqrt{A}$ is the expected distance of a uniform arrival to the center of square region of area $A$ [7]. Figure 3 shows the delay lower bound as a function of the network load for increased values of the communication range $r^*$.[3] As the communication range increases, the message delay decreases as expected. For heavy loads, the delay in the system is significantly less than the delay in the corresponding system without wireless transmission in [7], demonstrating the difference in the delay scaling between the two systems. For light loads and small communication ranges, the delay performance of the wireless network tends to the delay performance of [7].

Figure 4 compares the delay in the Partitioning Policy to the delay lower bound for two different cases. When the travel time dominates the reception time, the delay in the Partitioning policy is about 10 times the delay lower bound. For a more balanced case, i.e., when the reception time is comparable to the travel time, the delay ratio drops to 2.4.

[3] For the delay plot of the system without wireless transmission, the point that is not smooth arises since the plot is the maximum of two delay lower bounds proposed in [7].

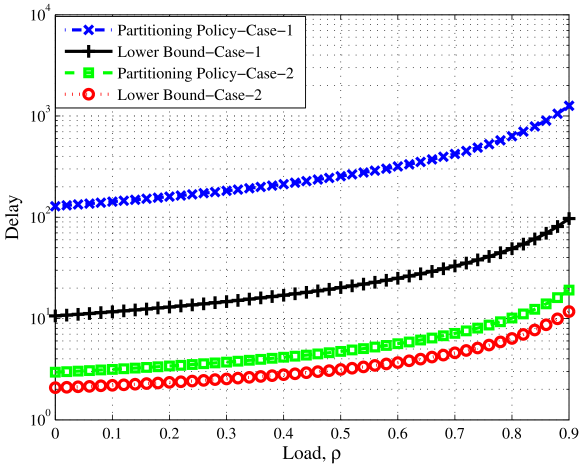



**Fig. 4** Delay in the Partitioning policy vs the delay lower bound for $r^* = 2.2$. Case-1: Dominant travel time ($A = 800$, $v = 1$, $s = 2$). Case-2: Comparable travel and reception times ($A = 60$, $v = 10$, $s = 2$)

## 3 Multiple collectors—interference-free networks

The analysis in the previous section can be extended to a system with $m > 1$ identical collectors that do not interfere with one another. This can be done, for example, by partitioning the network region into $m$ subregions and performing independent single-collector policies within each subregion. We call the class of such policies the *network partitioning policies*. For this class of policies, the interference-free assumption is satisfied if transmissions in different subregions use different frequency bands. The main difference in analysis as compared to the single-collector case is that we utilize the $m$-median problem in order to bound the travel times of the collectors and a load balancing argument in order to derive a delay lower bound. We show that similar to the single-collector case, the stability region is the set of loads such that $\rho < 1$ and the delay scaling is $\Theta(1/(1-\rho))$, where $\rho = \lambda s/m$.

Similarly to Sect. 2, the number of messages in the system dominates that in the corresponding system with zero travel time, i.e., an *M/D/m* queue. Therefore, $\rho = \lambda s/m < 1$ is necessary for stability, and average delay is lower bounded by that for the *M/D/m* queue [33] (please refer to the technical report [16] for details):

$$T^* \geq \frac{\lambda s^2}{2m^2(1-\rho)} - \left(\frac{m-1}{m}\right)\frac{s^2}{2s} + s. \tag{7}$$

Generalizing the single-collector TSPN policy to the case of multiple collectors through network partitioning shows that $\rho < 1$ is also sufficient for stability.

Next, we derive a lower bound on $W_d$, the average waiting time due to the collectors' travel, using a result from [26] for the $m$-median problem.

**Lemma 2**

$$W_d \geq \frac{\max(0, \frac{2}{3}\sqrt{\frac{A}{m\pi}} - r^*)}{v}. \tag{8}$$

*Proof* Let $\Omega$ be *any* set of points in $\Re$ with $|\Omega| = m$. Let $U$ be a uniformly distributed location in $\Re$ independent of $\Omega$ and define $Z^* \triangleq \min_{\nu \in \Omega} \parallel U - \nu \parallel$. Let the random variable $Y$ be the distance from the center of a disk of area $A/m$ to a uniformly distributed point within the disk. It is shown in [26] that

$$\mathbb{E}\big[f\big(Z^*\big)\big] \geq \mathbb{E}\big[f(Y)\big] \tag{9}$$

for any nondecreasing function $f(.)$. Using this result, we obtain $\mathbb{E}[\max(0, Z^* - r^*)] \geq \mathbb{E}[\max(0, Y - r^*)]$. Note that $W_d$ can be lower bounded by the expected distance of an arrival whose location is uniformly distributed on the network region to the closest collector at the time of arrival less $r^*$. Because the travel distance is nonnegative, we have

$$W_d \geq \mathbb{E}\big[\max\big(0, Y - r^*\big)\big]/v \geq \max\big(0, \mathbb{E}[Y] - r^*\big)/v,$$

where the second bound is due to Jensen's inequality. Substituting $\mathbb{E}[Y] = \frac{2}{3}\sqrt{\frac{A}{m\pi}}$ into the above expression completes the proof. □

When $W_d$ is lower bounded by a constant, a simple convexity argument shows that the equal area partitioning of the network region minimizes the resulting delay expression over all area partitionings [16]. Using this result, and similar steps to the proof of Theorem 2, yields the following lower bound on average delay for the class of network partitioning policies:

$$T^* \geq \frac{\max(0, \frac{2}{3}\sqrt{\frac{A}{m\pi}} - r^*)}{v(1-\rho)} + s. \tag{10}$$

This lower bound is more useful than (7) since it takes travel time into account.

Finally, by partitioning the region into $m$ subregions and then applying the single-collector Partitioning policy in each subregion shows that the average delay of this generalized Partitioning policy is given by the average delay of the single-collector Partitioning policy applied to a system with arrival rate $\lambda/m$ and area $A/m$:

$$T_{\text{part}} = \frac{\lambda s^2}{2m(1-\rho)} + \frac{\frac{A}{2m(r^*)^2} - \rho}{2v(1-\rho)}\sqrt{2}r^* + s. \tag{11}$$

From (10) and (11), we have that the delay scaling in the system is $\Theta(\frac{1}{1-\rho})$, in contrast to the $\Theta(\frac{1}{(1-\rho)^2})$ delay scaling for the multicollector DTRP [8].

## 4 Multiple collectors—systems with interference constraints

In this section, we consider systems in which simultaneous transmissions to different collectors interfere with each other. The problem is to dynamically determine a subset of collectors to route and schedule for transmission based on the present collector configuration and the number of messages in the system. The objective is to minimize the expected message waiting time in the system. This is a joint scheduling and Euclidean vehicle routing problem which has not been considered previously. Here,

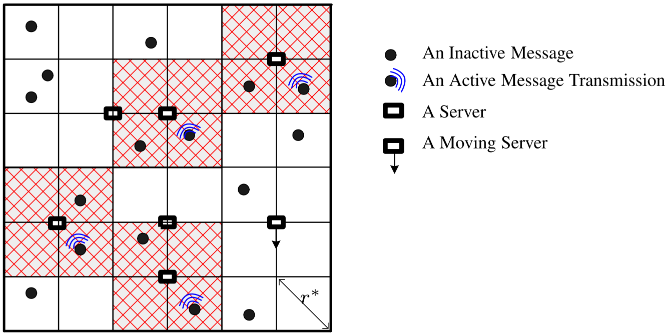



**Fig. 5** The network model. *Red regions* are the exclusion zones for the servers currently in service. Two servers are forced to be inactive since the messages in their vicinity are in the exclusion zones of other servers

we obtain preliminary results for this problem by emphasizing the scheduling aspect through fairly general interference constraints and simplifying the mobility aspect by discretizing the collectors' motion. We characterize the stability region of the system in terms of interference constraints and show that a frame-based version of the Max-Weight scheduling policy [10, 52], can stabilize the system whenever it is feasible to do so at all.

First, we explain the mobility and the interference models. We assume that time is slotted, $t = 0, 1, 2, \dots,$ where the slot length is equal to one message transmission time $s$. The collectors are confined to move on a grid $\mathcal{G}$ of $(\frac{r^*}{\sqrt{2}} \times \frac{r^*}{\sqrt{2}})$ squares, i.e., $K \doteq \frac{A}{(r^*)^2/2}$ square cells of diameter $r^*$ as shown in Fig. 5. Assuming a fixed ordering of the $K$ cells, where each cell $i = 1, 2, \dots, K$, receives Poisson arrivals with rate $\lambda_i \doteq \lambda/K$. Let $A_i(t)$ denote the number of messages that arrive into cell $i$ at time slot $t$. The expected *load* entering cell $i$ per time slot is given by $\rho_i = \lambda_i s$. Let $N_i(t)$ be the number of messages in cell $i$ at the beginning of time slot $t$, let $\mathbf{N}(t) \doteq [N_1(t), \dots, N_K(t)]$ denote the vector of queue sizes, and let $N(t) \doteq N_1(t) + \cdots + N_K(t)$ denote the total number of messages in the system at time slot $t$.

**Definition 1** (Cell interference model) Given a collector that is at the common corner of multiple adjacent cells, a transmission to the collector from one of the cells is successfully received if there is no other transmission within any adjacent cell.

The Cell Interference Model essentially creates an *exclusion region* of up to 4 cells around a collector receiving a message. Similar interference models have been considered in the literature. For example, the Protocol Model considered in [25, 47], assumes successful transmission if a disk region around the receiver has no other transmission. We characterize the interference constraints of the system in terms of *activation vectors*. We call a cell *active* if at least 1 message in the cell is scheduled for transmission, and we assume that each cell $k = 1, 2, \dots, K$ is associated with exactly one pick up location on the grid $\mathcal{G}$.[4] For instance, the pick up location for each cell could be the upper left corner of the cell as shown in Fig. 5. Therefore, specifying the set of cells to activate also specifies the locations of the collectors. A feasible activation vector $\mathbf{I} \in \mathcal{I}$ is one under which transmissions from a set of active cells do not interfere with each other, where $\mathcal{I}$ is the set of all feasible activation vectors. The set $\mathcal{I}$ consists of $K$-dimensional vectors of at most $m$ nonzero entries, where $\mathbf{I}_k = 1$ if cell $k$ is active under $\mathbf{I}$, and $\mathbf{I}_k = 0$ otherwise. Note that we include the zero vector $\mathbf{I} = \mathbf{0}$ in $\mathcal{I}$ for convenience.

Let $T_r$ denote the maximum *reconfiguration time*, i.e., the number of time slots required for a collector to move from the lower right corner of $\mathcal{G}$ to it's upper left corner. The *corresponding system with infinite speed*, i.e., $T_r = 0$, is a parallel queueing system with multiple servers and interference constraints, which is a special case of [42] or [52]. When $T_r = 0$, the stability region of this system, $\mathbf{\Lambda}^0$, consists of all load vectors $\boldsymbol{\rho} = [\rho_1, \rho_2, \dots, \rho_K] = s[\lambda_1, \lambda_2, \dots, \lambda_K]$ in the convex hull of $\mathcal{I}$ [10]:

$$\mathbf{\Lambda}^0 = \{\boldsymbol{\rho} | \boldsymbol{\rho} \in \operatorname{Conv}\{\mathcal{I}\}\}. \tag{12}$$

When $T_r > 0$, we have a significantly different system for which previously proposed algorithms are not stabilizing. Since we lose service opportunities during the reconfiguration times, we have

$$\mathbf{\Lambda} \subseteq \mathbf{\Lambda}^0. \tag{13}$$

We will show that $\mathbf{\Lambda} = \mathbf{\Lambda}^0$. The celebrated Max-Weight scheduling algorithm was introduced in [52] and was shown to stabilize the system for all $\boldsymbol{\rho} \in \mathbf{\Lambda}^0$ when $T_r = 0$. Specifically, the Max-Weight policy activates the set of users in $\mathbf{I}^*(t)$ where

$$\mathbf{I}^*(t) = \arg\max_{\mathbf{I} \in \mathcal{I}} \mathbf{N}(t) \cdot \mathbf{I}, \tag{14}$$

where $\mathbf{a} \cdot \mathbf{b} \doteq a_1 b_1 + \cdots + a_K b_K$.

### 4.1 Framed-Max-Weight policy

For systems with nonzero reconfiguration times, the Max-Weight policy is not stabilizing [10]. The intuitive reason behind this is that the Max-Weight policy makes frequent reconfiguration decisions, resulting in throughput loss during reconfiguration intervals. A frame based version of the Max-Weight policy where the same schedule is used throughout the frame incurs less throughput loss. A similar frame-based approach was used in [10] in the context of optical networks. We show that the Framed-Max-Weight (FMW) policy defined below stabilizes the system considered in this section. We prove this result using a quadratic Lyapunov drift technique.

Under the FMW policy, time is divided into intervals of length $T$ slots. The FMW policy employs the activation vector corresponding to the Max-Weight configuration at the beginning of each frame for $T - T_r$ slots, where the first $T_r$ slots of each frame are reserved for the servers to travel to their assigned locations. The policy requires $T > T_r/\epsilon(\rho)$ where $\epsilon(\rho)$ is determined by solving the linear program below [10].

[4] Of course, such an assumption may reduce the stability region. Here, we make this assumption in order to present preliminary results for the general problem.

$$\epsilon(\boldsymbol{\rho}) \triangleq \max\left(1 - \sum_{\mathbf{I}\in\mathcal{I}} \alpha_{\mathbf{I}}\right)$$

$$\text{subject to} \quad \rho_i = \lambda_i s \le \sum_{\mathbf{I}\in\mathcal{I}} \alpha_{\mathbf{I}} \mathbf{I}_i, \quad i \in \{1, \dots, K\} \tag{15}$$

$$\sum_{\mathbf{I}\in\mathcal{I}} \alpha_{\mathbf{I}} \le 1, \quad \text{and} \quad \alpha_{\mathbf{I}} \ge 0, \quad \forall \mathbf{I} \in \mathcal{I}.$$

Note that $\epsilon(\rho)$ is a measure of distance of the load vector to the boundary of the stability region [10]. The FMW policy is described in Algorithm 2 below.

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**Algorithm 2** Framed-Max-Weight Policy

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1: Assuming the system is at the $j$th frame, find

$$\mathbf{I}^* = \arg\max_{\mathbf{I}\in\mathcal{I}} \mathbf{N}(jT) \cdot \mathbf{I}. \tag{16}$$

2: Reconfigure the collectors to their new locations for the next $T_r$ slots.
3: Apply the activation vector $\mathbf{I}^*$ for $T - T_r$ slots.

---

**Theorem 3** *For any $\boldsymbol{\rho} = [\rho_1, \dots, \rho_K]$ strictly inside $\boldsymbol{\Lambda}_0$, the FMW policy stabilizes the system as long as $T > T_r/\epsilon(\rho)$.*

The proof is given in Appendix B. The reason that the FMW policy stabilizes the system is that as the load approaches the boundary of the stability region, the policy employs maximum-weight schedules over longer frames, decreasing the fraction of time spent on reconfiguration. The proof in Appendix B is based on a quadratic Lyapunov drift argument over frames of duration $T$. The proof establishes that the $T$-step expected drift of the queue lengths satisfies

$$\mathbb{E}\big[L\big(\mathbf{N}(jT+T)\big) - L\big(\mathbf{N}(jT)\big)|\mathbf{N}(jT)\big] \le KBT^2 - \frac{2T}{K}\left(\epsilon - \frac{T_r}{T}\right)\sum_i N_i(jT), \tag{17}$$

where $B = 1 + \frac{\lambda s}{K} + \frac{\lambda^2 s^2}{K^2}$ is a constant. From (17), we see that the drift becomes negative when the queue size is sufficiently large. Stability follows from this condition, similar to the proof of Theorem 1. Combining Theorem 3 with (13), we have the following corollary.

**Corollary 1**

$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_0.$$

Similarly to the case of collectors whose transmissions do not interfere, the stability region is not affected by the collector travel times.

## 5 Conclusion

In this paper, we considered the use of dynamic vehicle routing in order to improve the throughput and delay performance of wireless networks where messages arriving randomly in time and space are gathered by mobile collectors via wireless communications. For the case of a single collector, we characterized the stability region of this system. We developed a fundamental lower bound on expected message waiting time as well as matching upper bounds. For the case of multiple collectors whose communications do not interfere with each other, we extended the stability and delay scaling results of the single collector case. Our results show that combining controlled mobility and wireless transmission results in $\Theta(\frac{1}{1-\rho})$ delay scaling with load $\rho$. This is the fundamental difference between our system and the system without wireless transmission (DTRP) analyzed in [7] and [8] where the delay scaling with the load is $\Theta(\frac{1}{(1-\rho)^2})$. Finally, for the case where simultaneous transmissions to different collectors interfere with each other, we formulated a scheduling problem and characterized the stability region of the system in terms of interference constraints. We show that a frame-based version of the Max-Weight policy is stabilizing asymptotically in the frame length.

We have utilized a simple wireless communication model based on a communication range. Possible future directions include more sophisticated communication and interference models that take into account the signal to interference and noise ratio (SINR).

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## Appendix A: Proof of Theorem 1

Let $N(t)$ denote the number of messages in the system at time $t$, and let $W_j$ denote the delay experienced by the $j$th message. Recall the definition of time $t_k$, $k \geq 1$, the time at which the collector returns to the center of the network region for the $k$th time, where $t_0 \doteq 0$. Let $N(t_k)$ denote the total number of messages waiting for service at time $t_k$. We will denote $N(t_k)$ by $N_k$ for notational simplicity. The duration of time between $t_{k-1}$ and $t_k$ is called the $k$th cycle, and is denoted by $C_k$, $k \in \mathbb{Z}_+$. Note that $\{N_k : k \in \mathbb{N}\}$ is an irreducible Markov chain on countable state space $\mathbb{N}$, termed the cycle Markov chain. Given the system state $N_k$ at time $t_k$, we find the TSPN tour of length $L_k$ through the $N_k$ neighborhoods.[5] We prove Theorem 1 by establishing the following properties:

1. We first prove that the discrete-time Markov chain $\{N_k\}$ is positive recurrent and has a steady state distribution with a finite first moment.

[5] $L_k$ can be upper bounded by a constant $L$ for all $N_k$. This is because the collector does not have to move for messages within its communication range, and a finite number of such disks can cover the network region for any $r^* > 0$. The collector then can serve the messages in each disk from its center incurring a tour of constant length $L$. An example of such a tour is shown in Fig. 2.

2. Using this steady state distribution, we derive bounds on the first and second moments of the cycle duration, as well as the residual and past cycle durations under the TSPN policy.
3. Next, we show that the message delays $\{W_j\colon\ j \in \mathbb{Z}_+\}$ and the queue length process $\{N(t)\colon\ t \geq 0\}$ form positive recurrent regenerative processes and, therefore, converge in distribution to stationary processes.
4. Using the bounds on the residual and the past cycle times, we show that the stationary process of message delays has a finite expectation.
5. Finally, we utilize the stationary version of Little's law to show that the stationary process of number of messages in the system has finite expectation.

*Cycle Markov chain* $\{N_k\}$ First, we will use the Foster–Lyapunov criterion to show that the Markov chain described by the states $N_k$ is positive recurrent. We use the linear Lyapunov function $V(N_k) = sN_k$, the total load served during the $k$th cycle. Note that $V(0) = 0$, $S_k = \{x : V(x) \leq B\}$ is a bounded set for all finite $B$ and $V(.)$ is a nondecreasing function. Since the arrival process is Poisson, the expected number of arrivals during a cycle can be upper bounded as follows:

$$\mathbb{E}[N_{k+1}|N_k] \leq \lambda(L/v + sN_k). \tag{18}$$

Hence, we obtain the following drift expression for the load during a cycle:

$$\mathbb{E}[sN_{k+1} - sN_k|N_k] \leq \rho L/v - (1-\rho)sN_k. \tag{19}$$

Since $\rho < 1$, there exist a $\delta > 0$ such that $\rho + \delta < 1$:

$$\mathbb{E}[sN_{k+1} - sN_k|N_k] \leq \rho L/v - \delta sN_k.$$

Fix $\epsilon \in (0, \delta)$. A simple derivation shows that when $N_k$ is outside the finite and bounded set $S = \{N \in \mathbb{N} : N \leq \frac{\rho L/v+\epsilon}{s(\delta-\epsilon)}\}$ the drift expression is given by

$$\mathbb{E}[sN_{k+1} - sN_k|N_k] \leq -\epsilon(1 + sN_k).$$

For $N_k \in S$, using $\rho < 1 - \delta$ and the definition of the set $S$, we have from (18),

$$\mathbb{E}[sN_{k+1}|N_k] \leq \rho L/v + (1-\delta)sN_k < \rho L/v + \frac{(1-\delta)(\rho L/v + \epsilon)}{(\delta - \epsilon)} < \infty.$$

Moreover, since the state space is countable, the set $S$ is finite, and since the states in the Markov chain $\{N_k\}$ have nonzero probability of self transition, the Markov chain is strongly aperiodic. Therefore, all the conditions of Lemma 4.2 in [2] are satisfied (by the choice of the function $g(N_k) = 1 + sN_k$), and we have that the Markov chain $\{N_k\}$ is positive (Harris) recurrent, $N_k$ has a steady state distribution, where we let the random variable $N^c$ denote this steady state distribution. Moreover, $\mathbb{E}[N_k]$ converges to $\mathbb{E}[N^c]$, and the expected number of messages in steady state, $\mathbb{E}[N^c]$, is finite [2].

*Moments of cycle duration* Next, we derive bounds on the first and second moments of the cycle duration, and the expected residual and past cycle durations. These bounds will be necessary in order to obtain an upper bound for the expected message delay and the number of messages in the system. We will prove the finiteness of the expected number of messages in the system by first establishing that the expected message delay in the system is finite, and then utilizing the stationary version of Little's law [55]. The analysis in this section is similar to that in [2]. Let $C_k$ denote the duration of the $k$th cycle, $C_k = sN_k + L_k/v$. The location distributions of messages in different cycles are independent and uniformly distributed and the TSPN policy obtains the travel paths, $L_k$, using a stationary algorithm [41]. Therefore, $C_k$ is a function of $N_k$ and the location distribution of these $N_k$ messages. Note that the lengths of the travel paths $L_k$ are uniformly bounded from above by $L$ for all $k \in \mathbb{Z}_+$. Let $\mathbb{E}^0$ denote expectation at the time corresponding to the beginning of a cycle, in steady state. We let $N^c$ and $C$ denote the steady state versions of $N_k$ and $C_k$. Taking the expectation of (18) with respect to the steady state distribution at the beginning of the cycles we have

$$\mathbb{E}^0\big[N^c\big] \le \frac{\lambda L}{v} + \rho\mathbb{E}^0\big[N^c\big],$$

which implies that

$$\mathbb{E}^0\big[N^c\big] \le \frac{\lambda L}{v(1-\rho)}. \tag{20}$$

Using the bound on the cycle time $C_k = sN_k + L_k/v \le sN_k + L/v$, we have[6]

$$\mathbb{E}^0[C] \le \frac{\rho L}{v(1-\rho)} + \frac{L}{v}. \tag{21}$$

In order to lower bound the expected cycle duration, we lower bound the expected travel distance per cycle. This distance is at least as large as the expected distance between a uniformly distributed point (message location) in the network region and the center of the region less $r^*$. For a square shaped region of area $A$, this distance can be lower bounded by $\underline{d} \doteq 0.383\sqrt{A} - r^*$ [7]. Therefore, we have

$$\mathbb{E}[N_{k+1}|N_k] \ge \lambda(\underline{d}/v + sN_k).$$

Upon taking expectations, we have

$$\mathbb{E}^0\big[N^c\big] \ge \frac{\lambda\underline{d}}{v(1-\rho)}, \tag{22}$$

and

$$\mathbb{E}^0[C] \ge \frac{\underline{d}}{v(1-\rho)}. \tag{23}$$

Next, we characterize the second moment of the cycle duration. Let $T_s$ denote the time it takes to serve Poisson arrivals arriving in a time interval of random duration $D$. If the interarrival times, service times $s$, and the duration of time $D$ are independent, the second moment of $T_s$ is given by [32, p. 238] or [2, p. 1107]

$$\mathbb{E}\big[T_s^2\big] = \lambda^2\mathbb{E}[s]^2\mathbb{E}\big[D^2\big] + \lambda\mathbb{E}\big[s^2\big]\mathbb{E}[D].$$

This result can be applied to the workload in our system, where the workload for the $(k+1)$th cycle in our system is given by $sN_{k+1}$, and the random duration of interest

[6]Note that letting $A(t_1, t_2)$ denote the number of Poisson arrivals in the time interval $(t_1, t_2)$, we have $A(t_1, t_2) = A(t_2 - t_1)$, and $N_{k+1} = A(C_k)$. Taking expectations gives $\mathbb{E}[N_{k+1}] = \lambda\mathbb{E}[C_k]$. Finally, taking the limit as $k \to \infty$ yields $\mathbb{E}^0[C] = \mathbb{E}^0[N^c]/\lambda$, which gives the same relationship as (21).

is the $k$th cycle duration $C_k$. The reason we can use the result from [32] in our system is that the duration of the $k$th cycle is a function of the arrivals in the previous cycle, and it is independent of the interarrival times during the $k$th cycle. Therefore,

$$\mathbb{E}\big[s^2 N_{k+1}^2\big] = \lambda^2 s^2 \mathbb{E}\big[C_k^2\big] + \lambda s^2 \mathbb{E}[C_k]. \tag{24}$$

Thus, we have

$$\begin{aligned}\mathbb{E}^0\big[s^2\big(N^c\big)^2\big] &\le \rho^2 \mathbb{E}^0\big[\big(sN^c + L/v\big)^2\big] + \lambda s^2\left(\frac{\rho L}{v(1-\rho)} + \frac{L}{v}\right) \\ &\le \rho^2\left(\mathbb{E}^0\big[s^2\big(N^c\big)^2\big] + \frac{2sL}{v}\mathbb{E}^0\big[N^c\big] + \frac{L^2}{v^2}\right) + \lambda s^2\left(\frac{\rho L}{v(1-\rho)} + \frac{L}{v}\right),\end{aligned}$$

which upon utilizing the upper bounds on the first moments of $N^c$ and $C$ in (20) and (21) gives

$$\begin{aligned}\mathbb{E}^0\big[s^2\big(N^c\big)^2\big] &\le \frac{\frac{2\rho^3 L^2}{v^2(1-\rho)} + \frac{\rho^2 L^2}{v^2} + \frac{\rho^2 sL}{v(1-\rho)} + \frac{\rho sL}{v}}{1-\rho^2} \\ &= \frac{\frac{\rho^2 L^2}{v^2}(\frac{1+\rho}{1-\rho}) + \frac{\rho sL}{v}(\frac{1}{1-\rho})}{1-\rho^2} \doteq \overline{N^c},\end{aligned}$$

where we let $\overline{N^c}$ denote the finite constant on the right-hand side. Using the bound on $\mathbb{E}^0[(N^c)^2]$, we can upper bound the second moment of the cycle duration easily as follows:

$$\begin{aligned}\mathbb{E}^0\big[C^2\big] &\le \mathbb{E}^0\big[\big(sN^c + L/v\big)^2\big] = \mathbb{E}^0\big[s^2\big(N^c\big)^2 + 2sLN^c/v + L^2/v^2\big] \\ &\le \overline{N^c} + \frac{2sL}{v}\frac{\lambda L}{v(1-\rho)} + \frac{L^2}{v^2}.\end{aligned} \tag{25}$$

*Expected waiting time* Next, we bound the expected waiting time in order to bound the expected number of messages in the system via Little's law. For this, we first establish that the delay process $\{W_j\colon j \in \mathbb{Z}_+\}$, and the queue length process $\{N(t)\colon t \ge 0\}$ converge to stationary processes.

**Lemma 3** *The processes $\{W_j\colon j \in \mathbb{Z}_+\}$ and $\{N(t)\colon t \ge 0\}$ form positive recurrent regenerative processes under the TSPN policy.*

The proof is given at the end of the proof of Theorem 1. It establishes that the times when an arrival finds an empty system with the collector at the center of the network region constitute regeneration epochs for the system. Because the regeneration processes $\{N(t)\colon t \ge 0\}$ and $\{W_j\colon j \in \mathbb{Z}_+\}$ are positive recurrent and their regeneration periods are aperiodic, the sequences of message delays converge in distribution to a (customer)-stationary process, denoted by $\tilde{W}$, and the queue length process $\{N(t)\colon t \ge 0\}$ converge in distribution to a time-stationary process, denoted by $\tilde{N}$, see [49]. Now, we derive a bound on the expected waiting time, $\mathbb{E}[\tilde{W}]$, according to the stationary delay distribution. This bound is derived in a similar way to [2] or [13]. The delay of an arbitrary message is upper bounded by the sum of the residual cycle time $C_R$, plus the duration of the next cycle $C_N$. Note that the cycle during which the arrival occurs is a-typical and has expected duration $\mathbb{E}[C_R]+\mathbb{E}[C_P]$, where $\mathbb{E}[C_R]$ and $\mathbb{E}[C_P]$ denote the expected residual and past cycle times and are given by $\mathbb{E}^0[C^2]/2\mathbb{E}^0[C]$ [2, 13]. Therefore,

$$\mathbb{E}[\tilde{W}] \le \mathbb{E}[C_R] + \mathbb{E}[C_N] \le \frac{\mathbb{E}^0[C^2]}{2\mathbb{E}^0[C]} + \mathbb{E}[C_N]. \tag{26}$$

Note that $C_N$ is also a-typical and equal to the sum of the travel time plus the amount of workload that arrived during the previous cycle. Therefore, we have [2],

$$\mathbb{E}[C_N] \le \rho\big(\mathbb{E}[C_P] + \mathbb{E}[C_R]\big) + \frac{L}{v}. \tag{27}$$

Finally, combining (27) with the expression for the expected residual time, $\mathbb{E}[C_R] = \mathbb{E}^0[C^2]/2\mathbb{E}^0[C]$, we have from (26),

$$\mathbb{E}[\tilde{W}] \le \left(\rho + \frac{1}{2}\right)\frac{\mathbb{E}^0[C^2]}{2\mathbb{E}^0[C]} + \frac{L}{v} < \infty$$

where the last inequality holds due to (23) and (25),

Finally, the stationary version of Little's law gives a relationship between the first moment of the time-stationary process $\tilde{N}$, and the first moment of the customer-stationary process $\tilde{W}$ [55]. We have

$$\mathbb{E}[\tilde{N}] = \lambda\mathbb{E}[\tilde{W}] < \infty.$$

This establishes the stability of the TSPN policy for any load $\rho < 1$.

*Proof of Lemma 3* Let the arrival time of the $j$th message be $\tilde{t}_j$, and its delay $W_j$. We consider the Markov chain $\{N_k\colon k \in \mathbb{N}\}$ at the beginning of cycles which is positive recurrent, and therefore, hits the empty state infinitely often. Consecutive epochs and times at which an arrival finds the collector at the center of an empty system (i.e., start of a cycle) constitute an embedded renewal process for both processes $\{W_j\colon j \in \mathbb{Z}_+\}$, and $\{N(t)\colon t \ge 0\}$. Namely, let the sequence $\{\ell_n\colon n \in \mathbb{Z}_+\}$ denote the sequence of arrivals that find an empty system with the collector at the center. Because the arrival and the service processes are stationary, the discrete sequence $\{\ell_n\colon n \in \mathbb{Z}_+\}$ serve as an embedded renewal process for the delay process $\{W_n\colon n \in \mathbb{Z}_+\}$, and the continuous times $\tilde{t}_{\ell_n}$ serve as one for the queue length process $\{N(t)\colon t \ge 0\}$. More precisely, we have that the process $\{N(\tilde{t}_{\ell_1} + t)\colon t \ge 0\}$ is independent of $\{N(t)\colon t < \tilde{t}_{\ell_1}\}$ and of $\tilde{t}_{\ell_1}$, and the process $\{N(\tilde{t}_{\ell_1} + t)\colon t \ge 0\}$ is stochastically identical to $\{N(t)\colon t \ge 0\}$. Similarly, the process $\{W_{\ell_1+n}\colon n \in \mathbb{Z}_+\}$ is independent of $\{W_n\colon n < \ell_1\}$ and of $\ell_1$, and the process $\{W_{\ell_1+n}\colon n \in \mathbb{Z}_+\}$ is stochastically identical to $\{W_n\colon n \in \mathbb{Z}_+\}$; see [49].

Next, we show that these renewal processes are positive recurrent. Namely, we show that the expectation of the interrenewal periods, $\{\tilde{t}_{\ell_n} - \tilde{t}_{\ell_{n-1}}\colon n \in \mathbb{Z}_+\}$, are finite. Let $T_r$ be the duration of the $r$th renewal period, where the sequence $\{T_r\colon r \in \mathbb{Z}_+\}$ is i.i.d., and we need to show that $\mathbb{E}[T_1] < \infty$. Let $m_0$ be the mean recurrence time of the empty state (i.e., the state $N_k = 0$) in the Markov chain $\{N_k\}$, which is finite since the Markov chain is positive recurrent. Note that $m_0$ also denotes the expected number of cycles between renewals. Given $K$ let $M(K)$ be the number of renewals that have taken place up to and including cycle $K$. Since the last renewal might have taken place before cycle $K$, we have

$$\frac{\sum_{k=1}^{K} C_k}{\sum_{r=1}^{M(K)} T_r} \geq 1. \tag{28}$$

Furthermore, we have from Strong Law of Large Numbers (SLLN)

$$\lim_{K\to\infty} \frac{M(K)}{K} = \frac{1}{m_0}, \quad \text{a.s.} \tag{29}$$

The extended version of the Strong Law of Large Numbers (SLLN) for nonnegative valued random variables states that if the expectation of the random variables involved is infinite, then their average converges to infinity; see, for example [45, p. 370]. Now, applying the extended version of the SLLN to $T_r$ we have

$$\lim_{K\to\infty} \frac{1}{M(K)} \sum_{r=1}^{M(K)} T_r = \mathbb{E}[T_1], \quad \text{a.s.} \tag{30}$$

Note that we will establish that the above expectation is indeed finite. We utilize the upper bound on the cycle times $C_k \leq sN_k + L/v$ in (28) to have

$$\frac{\sum_{k=1}^{K} (sN_k + \frac{L}{v})}{\sum_{r=1}^{M(K)} T_r} \geq 1. \tag{31}$$

Since the Markov chain $\{N_k\colon k \in \mathbb{N}\}$ is ergodic, we have

$$\lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} N_k = \mathbb{E}^0\big[N^c\big], \quad \text{a.s.} \tag{32}$$

Finally, rewriting (31), taking the limit as $K$ tends to infinity, and applying (29), (30), and (32), we have

$$\lim_{K\to\infty} \frac{K}{M(K)} \frac{\frac{1}{K}\sum_{k=1}^{K} sN_k + \frac{L}{v}}{\frac{1}{M(K)}\sum_{r=1}^{M(K)} T_r} = m_0 \frac{s\mathbb{E}^0[N^c] + \frac{L}{v}}{\mathbb{E}[T_1]} \geq 1,$$

which implies that

$$\mathbb{E}[T_1] \leq m_0\left(s\mathbb{E}^0\big[N^c\big] + \frac{L}{v}\right) = m_0\left(\frac{\rho L}{v(1-\rho)} + \frac{L}{v}\right) < \infty,$$

where we used (20) for the last inequality. This establishes the fact that the regenerative processes $\{W_j\colon j \in \mathbb{Z}_+\}$ and $\{N(t)\colon t \geq 0\}$ are positive recurrent. □

## Appendix B: Proof of Theorem 3

We prove Theorem 3 for a broader class of arrival processes. We assume that each cell $i$ has an arrival process $A_i(t)$ that is i.i.d. over time and satisfies $\mathbb{E}[A_i(t)^2] \leq A_{\max}^2$ independent of the number of messages in the system, which is satisfied if the overall arrival process into the system is Poisson. Note that we have $\mathbb{E}[A_i(t)] = \lambda_i s$

independent of the number of messages in the system. Let $t_k$, $k = 0, 1, \dots,$ be the first time slot of the $k$th frame. Let $D_i(t)$, $t \in \{t_k + T_r, t_{k+1} - 1\}$, be 1 if cell $i$ is scheduled to be active during the $k$th frame and zero otherwise. Note that $D_i(t)$ is the *service opportunity* given to cell $i$ at time slot $t$ and not the actual departure process. Let $N_i(t)$ be the number of messages in cell $i$ at the beginning of time slot $t$. We assume that arrivals take place at the end of time slots. We have the following queue evolution relation:

$$N_i(t+1) = \max\bigl\{N_i(t) - D_i(t), 0\bigr\} + A_i(t).$$

Similarly, the following $T$-step queue evolution expression holds:

$$N_i(t_k + T) \le \max\left\{N_i(t_k) - \sum_{\tau=0}^{T-1} D_i(t_k+\tau), 0\right\} + \sum_{\tau=0}^{T-1} A_i(t_k+\tau).$$

The inequality is due to the fact that cell $i$ might become empty and that some arrivals depart during the frame. Squaring both sides we have

$$\begin{aligned}\bigl(N_i(t_k+T)\bigr)^2 - \bigl(N_i(t_k)\bigr)^2 \le{}& \left(\sum_{\tau=0}^{T-1} D_i(t_k+\tau)\right)^2 + \left(\sum_{\tau=0}^{T-1} A_i(t_k+\tau)\right)^2 \\ &- 2N_i(t_k)\left(\sum_{\tau=0}^{T-1} D_i(t_k+\tau) - \sum_{\tau=0}^{T-1} A_i(t_k+\tau)\right). \qquad (33)\end{aligned}$$

Define the quadratic Lyapunov function

$$L\bigl(\mathbf{N}(t_k)\bigr) = \sum_{i=1}^{K} N_i^2(t_k),$$

and the $T$-step conditional Lyapunov drift

$$\Delta_T(t_k) \triangleq \mathbb{E}\bigl\{L\bigl(\mathbf{N}(t_k+T)\bigr) - L\bigl(\mathbf{N}(t_k)\bigr) | \mathbf{N}(t_k)\bigr\}.$$

Summing (33) over the queues, taking conditional expectation, using $D_i(t) \le 1$ for all time slots $t$, $\mathbb{E}\{A_i(t)^2\} \le A_{\max}^2$ and $\mathbb{E}\{A_i(t_1)A_i(t_2)\} \le \sqrt{\mathbb{E}\{A_i(t_1)\}^2 \mathbb{E}\{A_i(t_2)\}^2} \le A_{\max}^2$ for all $t_1$ and $t_2$ we have

$$\begin{aligned}\Delta_T(t_k) &\le KBT^2 + 2\mathbb{E}\left\{\sum_i N_i(t_k) \sum_{\tau=0}^{T-1} \bigl[A_i(t_k+\tau) - D_i(t_k+\tau)\bigr] | \mathbf{N}(t_k)\right\} \\ &= KBT^2 + 2T\sum_i N_i(t_k)\lambda_i s - 2\sum_i N_i(t_k)\mathbb{E}\left\{\sum_{\tau=0}^{T-1} D_i(t_k+\tau) | \mathbf{N}(t_k)\right\}\end{aligned}$$

where $B = 1 + A_{\max}^2$ is a constant. Note that $D_i(t+\tau) = 0, \forall i \in \{1, \dots, K\}$ for $\tau \in \{0, 1, \dots, T_r - 1\}$ since the system is idle for the first $T_r$ slots of the frame under the FMW policy. Therefore,

$$\Delta_T(t_k) \le NBT^2 + 2T\sum_i N_i(t_k)\lambda_i s - 2\sum_i \sum_{\tau=T_r}^{T-1} N_i(t_k)\mathbb{E}\bigl\{D_i(t_k+\tau) | \mathbf{N}(t_k)\bigr\}.$$

Now using the fact that for any load vector $\boldsymbol{\rho} = \boldsymbol{\lambda} s$ that is strictly inside $\boldsymbol{\Lambda}^0$, there exist real numbers $\alpha_1, \ldots, \alpha_{|\mathcal{I}|}$ such that $\alpha_j > 0, \forall j \in 1, \ldots, |\mathcal{I}|$, $\sum_{j=1}^{|\mathcal{I}|} \alpha_j = 1 - \epsilon$ for some $\epsilon > 0$ and

$$\boldsymbol{\rho} = \sum_{j=1}^{|\mathcal{I}|} \alpha_j \mathbf{I}^j,$$

where $\mathbf{I}^j$ is a $K$-dimensional vector in $\mathcal{I}$. Over the time interval $[t + T_r, t + T - 1]$, the FMW policy applies the activation vector that has the property

$$\mathbf{I}^*(t_k) = \arg\max_{\mathbf{I} \in \mathcal{I}} \mathbf{N}(t_k) \cdot \mathbf{I}. \tag{34}$$

Therefore, $\sum_i N_i(t_k) D_i(t_k + \tau) = \mathbf{N}(t_k) \cdot \mathbf{I}^*(t_k)$. Hence, we have

$$\begin{aligned}
\Delta_T(t_k) &\le KBT^2 + 2T\mathbf{N}(t_k).\left(\sum_{j=1}^{|\mathcal{I}|} \alpha_j \mathbf{I}^j\right) - 2T\left(1 - \frac{T_r}{T}\right)\mathbf{N}(t_k).\mathbf{I}^*(t_k) \\
&= KBT^2 - 2T \sum_{j=1}^{|\mathcal{I}|} \alpha_j \left(\mathbf{N}(t_k).\mathbf{I}^*(t_k) - \mathbf{N}(t_k).\mathbf{I}^j\right) \\
&\quad - 2T\left(1 - \sum_{j=1}^{|\mathcal{I}|} \alpha_j\right)\mathbf{N}(t_k).\mathbf{I}^*(t_k) + 2T_r\mathbf{N}(t_k).\mathbf{I}^*(t_k) \\
&\le KBT^2 - 2T\epsilon\mathbf{N}(t_k).\mathbf{I}^*(t_k) + 2T_r\mathbf{N}(t_k).\mathbf{I}^*(t_k) \\
&= KBT^2 - 2T\left(\epsilon - \frac{T_r}{T}\right)\mathbf{N}(t_k).\mathbf{I}^*(t_k).
\end{aligned} \tag{35}$$

Note that we have $\mathbf{N}(t_k).\mathbf{I}^*(t_k) \ge \frac{1}{K}\sum_i N_i(t_k)$ since the maximum weight schedule has more weight than the average. Therefore, for $T > \frac{T_r}{\epsilon}$ we have

$$\Delta_T(t_k) \le KBT^2 - 2T\left(\epsilon - \frac{T_r}{T}\right)\frac{1}{K}\sum_i N_i(t_k). \tag{36}$$

Therefore, the $T$-step conditional Lyapunov drift is negative if $T > \frac{T_r}{\epsilon}$ and if the queue sizes are outside a bounded set. Therefore, the stability at the frame boundaries follows from Lemma 4.2 in [2] due to a similar reasoning to the proof of Theorem 1. This implies the stability of the system since the frame length $T$ is a constant.

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