Cooperative Routing in Wireless Networks*

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Abstract

The joint problem of transmission-side diversity and routing in wireless networks is studied. It is assumed that each node in the network is equipped with a single omni-directional antenna and multiple nodes are allowed to coordinate their transmissions to achieve transmission-side diversity. The problem of finding the minimum energy route under this setting is formulated. Analytical asymptotic results are obtained for lower bounds on the resulting energy savings for both a regular line network topology and a grid network topology. For a regular line topology, it is possible to achieve energy savings of 39%. For a grid topology, it is possible to achieve energy savings of 56%. For arbitrary networks, we develop heuristics with polynomial complexity which result in average energy savings of 30% – 50% on simulations.

1 Introduction

In this paper, we study the joint problem of route selection and physical layer space diversity in ad-hoc wireless networks for the sake of energy efficiency. It is known that in an ad-hoc network, nodes usually spend most of their energy in communication [1]. For this reason, the problem of energy efficiency and energy efficient communication in wireless networks has received a lot of attention. This problem, however, can be approached from two different angles: energy-efficient route selection algorithms at the network layer or efficient communication schemes at the physical layer. A combined cross-layer approach, that designs the network layer protocols to exploit the special properties of the wireless physical layer, may be beneficial in wireless networks.

Multi-path fading is one of the fundamental limiting factors in wireless communication, resulting in a higher likelihood of transmission errors than in a wired medium. Equalization, channel coding, and diversity are three techniques that are generally used,

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independently or in tandem, to improve the wireless link quality [2]. In diversity techniques, information is transmitted over channels that are affected by uncorrelated fading and noise processes. This effect may be achieved by separating the channels in frequency, time, or space. These techniques are reviewed in detail in [3]. Space diversity is usually achieved by employing multiple transmitting and/or multiple receiving antennas. Multiple antennas, on the transmitter or on the receiver side, must be about 0.4λ apart, a few inches at the typical carrier frequencies, to achieve the desired effect of uncorrelated channels (see [2]). However, in some cases, the use of multiple transmitters or receivers may be impractical, infeasible, or too costly. In this paper we propose a new way of achieving space diversity by allowing cooperation among nodes for routing purposes, in effect creating a virtual antenna array. The following simple example best illustrates the potential benefits of this approach.

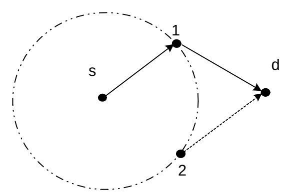

Figure 1: Cooperative Routing

Figure 1 depicts a simple 4-node wireless network, where s and d are the source and the destination nodes, respectively. We assume that the minimum energy path from s to d is through node 1, i.e. $s \rightarrow 1 \rightarrow d$. In this case, node 2, which is also located within the transmission radius of s to 1, receives the information transmitted from s at no additional cost. This property of wireless medium is usually referred to as *Wireless Broadcast Advantage* (WBA) (see [7]). Cooperation between nodes 1 and 2 in the second hop will create transmission-side diversity and may result in a lower energy route from s to d. Under this setting, each node can participate in cooperative transmission after it has completely received the information. For this reason, the problem of finding the optimal path is a multi-stage decision making problem, where at each stage a set of nodes may cooperate to relay the information to a chosen node. Thus the minimum energy cooperative route may be viewed as a sequence of sets of cooperating nodes along with an appropriate allocation of transmission powers. The tradeoff is between spending more energy in each transmission slot to reach a larger set of nodes, and the potential savings in energy in subsequent transmission slots due to cooperation.

In this paper, we develop a formulation that captures the benefit of this cooperation and develop an algorithm to find the optimal route under this setting. To our knowledge this problem has not been studied previously. The idea of wireless broadcast advantage was first introduced in [7]. The problem of finding the optimal multi-cast and broadcast tree in a wireless network and the added complexity due to WBA has been studied extensively in [7] and [8]. This problem is shown to be NP-Complete in [9] and [10]. The same problem, under the assumption that nodes can collect power in different transmission slots, was studied in [11]. The problem of transmission diversity is discussed in [4].

2 Problem Formulation

Consider a wireless ad-hoc network consisting of arbitrarily distributed nodes where each node has a single omni-directional antenna. We assume that each node can dynamically adjust its transmitted power and phase to control its transmission range and possibly synchronize with other nodes. Based on these two assumptions, The information is routed from the source node to the destination node during a sequence of transmission slots, where each transmission slot corresponds to one use of the wireless medium. In each transmission slot/stage, a node or group of nodes is selected to transmit the information to another single node (broadcast mode) or another group of nodes (cooperation mode). The routing problem can be viewed then as a multi-stage decision problem, where at each stage the decision is to pick the set of nodes S participating in relaying the information and the set of nodes T receiving the information. The objective is to get the information to the destination with minimum energy. The set of nodes that have the information at the k^{th} stage is referred to as the k^{th}-stage *Reliable Set*, S_k, and the routing solution may be expressed as a sequence of expanding reliable sets that starts with only the source node and terminates as soon as the reliable set contains the destination node. The single-stage cost, referred to as the Link Cost between S and T, $LC(S,T)$, is the minimum power needed for transmitting from S to T.

In this paper, we make several idealized assumptions about the physical layer model. The wireless channel between any transmitting node labeled s_i and any receiving node labeled t_j is modeled by two parameters, its magnitude attenuation factor α_{ij} and its phase delay θ_{ij}. We assume that the channel parameters are estimated by the receiver and fed back to the transmitter. This assumption is reasonable for slowly varying channels, where the channel coherence time is much longer than the block transmission time. We also assume a free space propagation model where the power attenuation α_{ij}^2 is proportional to the inverse of the square of the distance between the communicating nodes s_i and t_j. For the receiver model, we assume that the desired minimum transmission rate at the physical layer is fixed and nodes can only decode based on the signal energy collected in a single channel use. We also assume that the received information can be decoded with no errors if the received SNR level is above a minimum threshold SNR_{min}, and that no information is received otherwise. Without loss of generality, we assume that the information is encoded in a signal ϕ that has unit power $P_\phi = 1$ and that we are able to control the phase and magnitude of the signal arbitrarily by multiplying it by a complex scaling factor w_i before transmission. The noise at the receiver is assumed to be additive, and the noise signal and power are denoted by $\eta(t)$ and P_η respectively. This simple model allows us to find analytical results for achievable energy savings in some simple network topologies.

2.1 Link Cost Formulation

In this section, our objective is to understand the basic problem of optimal power allocation required for successful transmission of the same information from a set of source nodes $S = \{s_1, s_2, \cdots, s_n\}$ to a set of target nodes $T = \{t_1, t_2, \cdots, t_m\}$. In order to derive expressions for the link costs, we consider 4 distinct cases:

1. *Point-to-Point Link:* $n = 1, m = 1$: In this case, only one node is transmitting within a time slot to a single target node.

2. *Point-to-Multi-Point, Broadcast Link:* $n = 1, m > 1$: This is the broadcast mode, where a single node is transmitting to multiple target nodes.

3. *Multi-Point-to-Point, Cooperative Link:* $n > 1, m = 1$: This is the cooperative mode, where multiple nodes cooperate to transmit the same signal to a single node. We will assume that coherent reception, i.e. the transmitters are able to adjust their phases so that all signals arrive in phase at the receiver. In this case, the signals simply add up at the receiver and complete decoding as long as the received SNR is above the minimum threshold SNR_{min}. In this paper, we do not address the feasibility of precise phase synchronization. The reader is referred to [12] for a discussion of mechanisms for achieving this level of synchronization.

4. *Multi-Point-to-Multi-Point Link:* $n > 1, m > 1$: This is not a valid option under our assumptions, as synchronizing transmissions for coherent reception at multiple receivers is not feasible. Therefore, we will not be considering this case.

2.1.1 *Point-to-Point Link:* $n = 1, m = 1$

In this case, $S = \{s_1\}$ and $T = \{t_1\}$. The channel parameters may be simply denoted by α and θ, and the transmitted signal is controlled through the scaling factor w. The model assumptions made in Section 2 imply that the received signal is simply

$$r(t) = \alpha e^{j\theta} w \phi(t) + \eta(t).$$

The total transmitted power is $P_T = |w|^2$. Therefore the SNR at the receiver is $\frac{\alpha^2 |w|^2}{P_\eta}$. For complete decoding at the receiver, the SNR must be above the threshold value SNR_{min}. Therefore the minimum power required $\hat{P_T}$, and hence the point-to-point link cost $LC(s_1, t_1)$, is given by

$$LC(s_1, t_1) \equiv \hat{P_T} = \frac{SNR_{min} P_\eta}{\alpha^2} \tag{1}$$

In equation 1, the point-to-point link cost is proportional to $\frac{1}{\alpha^2}$, which is the power attenuation in the wireless channel between s_1 and t_1, and therefore is proportional to the square of the distance between s_1 and t_1 under our propagation model.

2.1.2 *Point-to-Multi-Point, Broadcast Link:* $n = 1, m > 1$

In this case, $S = \{s_1\}$ and $T = \{t_1, t_2, \cdots, t_m\}$, hence m simultaneous SNR constraints must be satisfied at the receiver. Assuming that omni-directional antennas are being used, the signal transmitted by the single node s_1 is received by all nodes within the transmission radius. Hence, a broadcast link can be treated as a set of point-to-point links and the cost of reaching a set of node is the maximum over the costs for reaching each of the nodes in the target set. Thus the minimum power required for the broadcast transmission, denoted by $LC(s_1, T)$, is given by

$$LC(s, T) = max\{LC(s_1 t_1), LC(s_1 t_2), \cdots, LC(s_1 t_n)\} \tag{2}$$

2.1.3 *Multi-Point-to-Point, Cooperative Link:* $n > 1, m = 1$

In this case $S = \{s_1, s_2, \cdots, s_n\}$ and $T = \{t_1\}$. We assume that the n transmitters are able to adjust their phases in such a way that the signal at the receiver is

$$r(t) = \sum_{i}^{n} \alpha_{i1} |w_i| \phi(t) + \eta(t).$$

The power allocation problem for this case is simply

$$\text{Minimize} \quad \sum_{i=1}^{n} |w_i|^2 \qquad \text{Subject to} \quad \frac{|\sum_{i=1}^{n} w_i \alpha_{i1}|^2}{P_\eta} \geq SNR_{min} \tag{3}$$

Lagrangian multiplier techniques may be used to solve the constrained optimization problem above, and the resulting optimal allocation for each node i is given by

$$|\hat{w}_i| = \frac{\alpha_{i1}}{\sum_i^n \alpha_{i1}^2} \sqrt{SNR_{min} P_\eta} \tag{4}$$

The resulting cooperative link cost $LC(S, t_1)$, defined as the optimal total power, is therefore given by

$$LC(S, t_1) \equiv \hat{P}_T = \frac{1}{\sum_{i=1}^{n} \frac{\alpha_{i1}^2}{SNR_{min} P_\eta}} \tag{5}$$

It is easy to see that it can be written in terms of the point-to-point link costs between all the source nodes and the target nodes (see Equation 1) as follows:

$$LC(S, t_1) = \frac{1}{\frac{1}{LC(s_1,t_1)} + \frac{1}{LC(s_2,t_2)} + \cdots + \frac{1}{LC(s_n,t_1)}} \tag{6}$$

2.2 Minimum Cost Cooperative Route

The problem of finding the optimal cooperative route from the source node s to the destination node d, formulated in Section 2, can be mapped to a Dynamic Programming (DP) problem. The state of the system at stage k is the reliable set S_k, i.e. the set of nodes that have completely received the information by the k^{th} transmission slot. The initial state S_0 is simply $\{s\}$, and the termination states are all sets that contain d. The decision variable at the k^{th} stage is U_k, the set of nodes that will be added to the reliable set in the next transmission slot. The dynamical system evolves as follows:

$$S_{k+1} = S_k \cup U_k \quad k = 1, 2, \cdots \tag{7}$$

The objective is to find a sequence $\{U_k\}$ or alternatively $\{S_k\}$ so as to minimize the total transmitted power P_T, where

$$P_T = \sum_k LC(S_k, S_{k+1} - S_k) \tag{8}$$

We will refer to the solution to this problem as the optimal transmission policy. This is a shortest path problem over a graph whose nodes are all the possible states and with arcs representing the possible transitions between states. As the network nodes are allowed only to either fully cooperate or broadcast, the graph has a special layered structure as illustrated by Figure 3. Arcs between nodes in adjacent layers correspond to cooperative links, whereas broadcast links are shown by cross layer arcs. The costs on the arcs are the link costs defined in Section 2.1. All terminal states are connected to a single artificial terminal state, denoted by D, by a zero-cost arc. The optimal transmission policy is basically the shortest path between nodes s and D. There are 2^n nodes in the graph for a network with $n+1$ nodes. Therefore standard shortest path algorithms will in general have a complexity of $O(2^{2n})$. We are able to take advantage of the special structure of this graph to come up with an algorithm with complexity reduced to $O(n2^n)$. However, the complexity is still exponential, which makes finding the optimal cooperative policy computationally intractable for large networks. For this reason, for arbitrary networks we will focus on developing computationally simpler and relatively efficient heuristics and on assessing their performance through simulation.

2.3 Simple Example

Having developed the necessary mathematical tools, we now present a simple example that illustrates the benefit of cooperative routing. Figure 2 shows a simple network with 4 nodes. The arcs represent links and the arc labels are point-to-point link costs. The diagrams below show the six possible routes, P_0 through P_5. P_0 corresponds to a simple 2-hop, non-cooperative minimum energy path between s and d. P_1, P_2, and P_3 are 2-hop cooperative routes, whereas P_4 and P_5 are 3-hop cooperative routes. Table 1 lists the costs of the six policies. The policy with the lowest cost is P_3, where nodes 1 and 2 receive the information in the first transmission slot due to the *Wireless Broadcast Advantage*, and nodes $s, 1, 2$ cooperate to transmit the information to d with minimum energy.

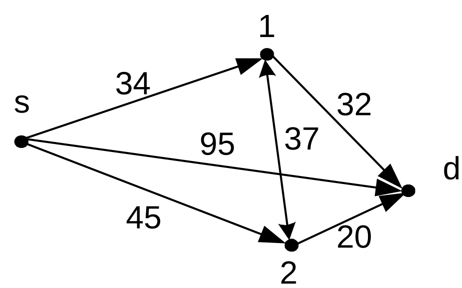

Figure 2: 4-Node Network Example

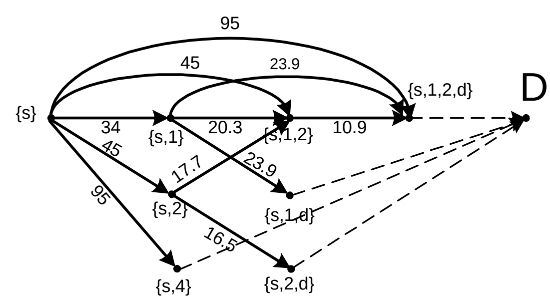

Figure 3: 4-Node Cooperation Graph

No.	Policy	Cost
P_0	$NonCooperative$	65
P_1	$(\{s\},\{s,2\},\{s,2,d\})$	≈ 61.5
P_2	$(\{s\},\{s,1\},\{s,1,d\})$	≈ 57.9
P_3	$(\{s\},\{s,1,2\},\{s,1,2,d\})$	≈ 55.9
P_4	$(\{s\},\{s,2\},\{s,1,2\},\{s,1,2,d\})$	≈ 73.6
P_5	$(\{s\},\{s,1\},\{s,1,2\},\{s,1,2,d\})$	≈ 65.2

Table 1: Transmission Policies for Figure 2

3 Analytical Results for Line and Grid Topologies

In this section, we develop analytical results for achievable energy savings in line and grid networks. In particular, we consider a *Regular Line* Topology (see Figure 4) and a *Regular Grid* Topology (see Figure 5) where nodes are equi-distant from each other. For each of these topologies, we derive the optimal non-cooperative route and obtain an lower bound on the optimal energy savings achievable by cooperative routing. The bound is obtained by deriving analytical expressions for energy savings for a sub-optimal cooperative route, where cooperation is restricted to nodes along the optimal non-cooperative route. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy non-cooperative route. This cooperation strategy is referred to as the CAN (*Cooperation Along the Minimum Energy Non-Cooperative Path*) strategy. Before proceeding further, let us define precisely what we mean by energy savings for a cooperative routing strategy relative to the optimal non-cooperative strategy:

$$Savings = \frac{P_T(Non-cooperative) - P_T(Cooperative)}{P_T(Non-cooperative)}, \tag{9}$$

where $P_T(strategy)$ denotes the total transmission power for the strategy.

Figure 4: Regular Line Topology

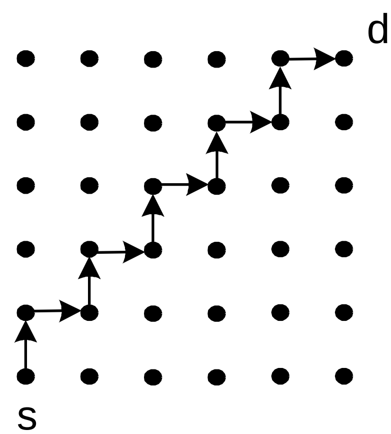

Figure 5: Regular Grid Topology

3.1 Line Network-Analysis

For the 3-node line network in Figure 4, it is easy to show that the optimal non-cooperative routing strategy is to relay the information through the middle node. Since a longer line network with can be broken down into short 2-hop components, it is clear that the optimal non-cooperative routing strategy is to always send the information to the next node until the destination node is reached. From Equation 1) the link cost for every stage is $\frac{SNR_{min}P_\eta}{\alpha^2}$, where α is the magnitude attenuation between two adjacent nodes 1-distance unit apart. Under our assumptions, α^2 is proportional to the inverse of the distance squared. Therefore,

$$P_T(Non-cooperative) = n\frac{SNR_{min}P_\eta}{\alpha^2} \tag{10}$$

With the CAN strategy, after the m^{th} transmission slot, the reliable set is $S_m = \{s, 1, \cdots, m\}$, and the link cost associated with the nodes in S_m cooperating to send the information to the next node $(m+1)$ follows from Equation 6 and is given by

$$LC(S_m, m+1) = \frac{SNR_{min} P_\eta}{\sum_{i=1}^{m+1} \frac{\alpha^2}{i^2}} \tag{11}$$

Therefore, the total transmission power for the CAN strategy is

$$\begin{aligned} P_T(CAN) &= \sum_{m=0}^{n-1} LC(S_m, m+1) && (12) \\ &= \frac{SNR_{min} P_\eta}{\alpha^2} \sum_{m=0}^{n-1} \frac{1}{C(m+1)} && (13) \end{aligned}$$

where $C(m) = \sum_{i=1}^{m} \frac{1}{i^2}$.

Theorem 1 *For a regular line network as shown in Figure 4, the CAN strategy results in energy savings of* $(1 - \frac{1}{n} \sum_{m=1}^{n} \frac{1}{C(m)})$*. As the number of nodes in the network grows, the energy savings value approaches* $(1 - \frac{6}{\pi^2}) \approx 39\%$*.*

Proof: The first statement of the theorem follows directly from the definition in Equation 9 and from equations 10 and 3.1. The second statement follows from the fact that $\lim_{m \to \infty} C(m) = \frac{\pi^2}{6}$.

3.2 Grid Network

Figure 5 shows a regular $N \times N$ grid topology. A minimum-energy non-cooperative route is obtained by a stair-like policy (illustrated in Figure 5), and its total power is $2N$. We are able to derive results similar to those of the line network for CAN strategy. We state the theorem and omit the proof for the sake of brevity.

Theorem 2 *For a regular grid network as shown in Figure 5, the energy savings value approaches approximately* 56% *for large networks.*

4 Heuristics & Simulation Results

We present two possible general heuristic schemes and related simulation results. The simulations are over a network generated by randomly placing nodes on an 100×100 grid and randomly choosing a pair of nodes to be the source and destination. The performance results reported are the energy savings of the resulting strategy with respect to the optimal non-cooperative path averaged over $100,000$ simulation runs.

CAN-L Heuristic This heuristic is based on the CAN strategy described Section 3. CAN-L is a variant of CAN as it limits the number of nodes allowed to participate in the cooperative transmission to L. In particular, these nodes are chosen to be the last L nodes along the minimum energy non-cooperative path. The complexity of this class of algorithms is $O(N^2)$.

PC-L Heuristic *Progressive Cooperation* Heuristic. This heuristic is described below:

Initialize Initialize Best Path to the optimal non-cooperative route. Initialize the Super Node to contain only the source node.

Repeat Send the information to the first node along the current Best Path. Update the Super Node to include all past L nodes along the current Best Path. Update the link costs accordingly. Compute the optimal non-cooperative route for the new network/graph and update the Best Path accordingly.

Stop Stop as soon as the destination node receives the information.

For example, with $L = 3$, this algorithm always combines the last 3 nodes along the current Best Route into a single node, finds the shortest path from that combined node to the destination and send the information to the next node along that route. This algorithm turns out to have a complexity of $O(N^3)$.

A variant of this algorithm keeps a window W of the most recent nodes, and in each step all subsets of size L among the last W nodes are examined and the path with the least cost is chosen. This variant has a complexity of $O(W \times N^3)$, where W is the window size. We refer to this variant as *Progressive Cooperation with Window*.

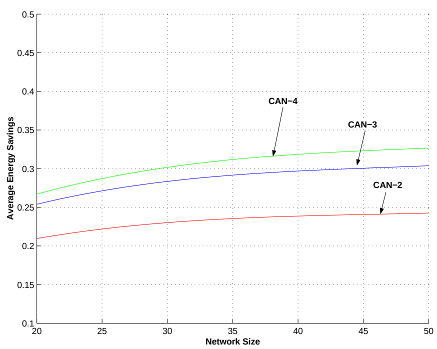

Figure 6: Performance of CAN

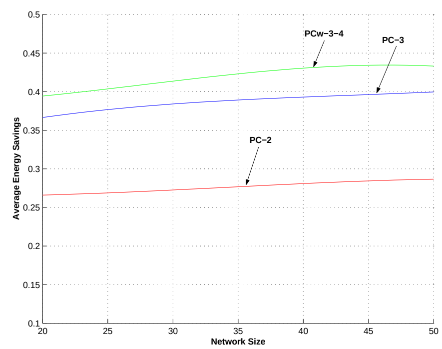

Figure 7: Performance of PC

Figures 6 and 7 show average energy savings ranging from 20% to 50%. It can be seen that PC-2 performs almost as well as CAN-3. Furthermore, PC-3-4 performs modestly better than PC-3. Both PC-3 and PC-3-4 perform substantially better than CAN-4. In general, it can be seen that the energy savings increase with L, and that improvements in savings are smaller for larger values of L. As there is a trade-off between the algorithm complexity and the algorithm performance, these simulation results indicate that it would be reasonable to chose L to be around 3 or 4 for both the CAN and PC heuristics.

5 Conclusion

In this paper we formulated the problem of finding the minimum energy cooperative route for a wireless network under idealized channel and receiver models. We focused on the optimal transmission of a single message from a source to destination through sets of nodes, that may act as cooperating relays. Fundamental to the understanding of the routing problem was the understanding of the optimal power allocation for a single message transmission from a set of source nodes to a set of destination nodes. We presented solutions to this problem, and used these as the basis for solving the minimum energy cooperative routing problem. We used Dynamic Programming (DP) to formulate the cooperative routing problem as a multi-stage decision problem. However, general shortest algorithms are not computationally tractable and are not appropriate for large networks. For a Regular Grid Topology and a Regular Grid Topology, we obtained good lower bounds on the energy savings, demonstrating the benefits of the proposed cooperative routing scheme. For general topologies, we proposed two heuristics and showed significant energy savings (close to 50%) on simulation results.

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