

The Legacy of George Zames



(January 7, 1934–August 10, 1997)

GEORGE ZAMES tragically passed away on August 10, 1997, after a brief illness. Some of George's friends have assembled this impressionistic overview of his seminal contributions to the field of Systems and Control.

George Zames was born on January 7, 1934, in Lodz, Poland. He was a child living with his parents in Warsaw, Poland, when the bombing of that city on September 1, 1939, marked the start of World War II. His family escaped Europe in an odyssey through Lithuania, whose occupation by Soviet tanks they witnessed, then through Russia, Siberia, followed by a triple crossing of the Sea of Japan, eventually reaching Kobe, Japan, early in 1941. (This episode, which involved the extraordinary help of the Japanese consul to Lithuania, Senpo Sugihara, is the subject of the book *The Fugu Plan* by M. Tokayer.) Later that year they moved to the Anglo-French International Settlement in Shanghai, China, where they were stranded by the outbreak of the war in the Pacific. Despite the war, their sojourn in Shanghai was a happy one, and George was able to attend school without interruption. The family moved to Canada in 1948. In spite of losing a year in the move, George entered McGill University at age 15. He graduated at the top of the Engineering Physics class and won an Athlone Fellowship for study in England. He gravitated to the Imperial College of London University, where his advisors included Colin Cherry, Denis Gabor, and J. H. Westcott. At the suggestion of Colin Cherry he decided to explore Europe and its skiing slopes, thereby taking two years to obtain his Master's degree.

In 1956, George began his doctoral studies at the Massachusetts Institute of Technology (MIT), Cambridge. He was briefly associated with Doug Ross at the Servomechanisms Laboratory, where he was assigned the task of developing computer graphics for Gordon Brown's recently developed computer-controlled milling machine which, at the time, was the only one in existence. Eventually this program became the APT programming language. He later switched to the

Communications Theory Group of Norbert Wiener, Y. W. Lee, and Amar Bose at MIT's Research Laboratory of Electronics. His doctoral thesis entitled, "Nonlinear Operations for System Analysis" submitted for the Sc.D. degree at MIT forms the foundation of much of his later work in Systems and Control.

In 1957, Niels Bohr arrived for a lecture tour of North America and asked for a "typical American" student to guide him around Cambridge. George was found to be appropriate for this task, and after being asked by Norbert Wiener for an introduction to Bohr, witnessed a remarkable argument between the two men on the merits of research into the natural sciences, such as Physics versus the sciences which focus on man-made phenomena, notably Cybernetics.

After receiving the Sc.D. degree, George was appointed Assistant Professor at MIT. The following summer he set out for a vacation in Greece. On his way he stopped in Israel and met Eva. He never got to Greece on that trip. They were married two years later. They have two sons, Ethan and Jonathan. Between 1961 and 1965, he moved back and forth between MIT and Harvard, continuing his work on nonlinear stability. In 1965, he won a Guggenheim Fellowship, which he spent at the NASA Electronics Research Center in Cambridge forming the nucleus of what was to become the Office of Control Theory and Application (OCTA), with which P. L. Falb, M. I. Freedman, G. Kovatch, H. J. Kushner, A. S. Morse, A. E. Pearson, O. H. Schuck, W. A. Wolovich, W. M. Wonham, and others later became associated.

In December of 1969 it was announced that NASA/ERC would be closed and that in its place the Department of Transportation (DOT) would open a new Transportation Research Center. During the ensuing transitional period from January to June in 1970, OCTA was asked to develop concepts and a long-range research agenda aimed at dealing with the growing air traffic problem in the United States. In response, OCTA's members, spearheaded by George, conceived the concept of a reconfigurable runway as a means of reducing congestion at airports. The idea was to employ a large, disk-

shaped slab with embedded lights which could be used to define runways in directions most suitable to existing weather conditions. Although the idea never got off the ground, the study of OCTA's "circular runway" has persisted and grown. For example, it is said that a report on the subject, prepared by George, was hand carried by NASA/ERC's director to Washington for presentation to Congress with the aim of defining the role of the new Transportation Research Center. Actually, there was such a report authored by George which was used in this manner, but it did not include the circular runway concept. In any event, the study was certainly one of George's favorites.

The DOT Transportation Research Center opened in the summer of 1970. For a year, George worked on transportation planning with George Kovatch, authoring the 1971 20-year Transportation Technology Forecast and the studies of Personalized Rapid Transit Systems (PPRT's). George then took an extended sabbatical at the Technion in Haifa, Israel. Here he hosted Claude Shannon during Shannon's visit to receive the Harvey Prize. George's interest in metric complexity theory was stimulated by this event as well as interactions with the Technion professors Jacob Ziv and Moshe Zakai.

In 1974, George returned to McGill where he was appointed Professor of Electrical Engineering. He remained at McGill until his untimely death this year. George was awarded the Macdonald Chair of Electrical Engineering at McGill in 1983. He has won several outstanding paper awards of the IEEE Control Systems Society and won the IEEE Field Award for Control Science in 1984. He is a Fellow of the Canadian Institute for Advanced Research and the Royal Society of Canada. He was awarded the Killam Prize, the most important scientific award in Canada, in 1995 and the Rufus Oldenburger Medal of the ASME in 1996.

The late 1950's and 1960's were a period of great creativity and ferment for the systems and control field. With the seminal contributions of Kolmogoroff and Wiener on Filtering and Prediction and of Shannon through his creation of the new science of Information Theory, the creative coupling between abstract mathematics and electrical engineering was firmly established. The influence of this research in the Systems and Control field was beginning to be visible, perhaps most significantly in the book by Newton *et al.* [1].

Thomas Kuhn [2] in writing about the distinction between normal science and scientific revolutions draws a parallel between political revolution and scientific revolutions and goes on to say "In much the same way, scientific revolutions are inaugurated by a growing sense, again often restricted to a narrow subdivision of the scientific community, that an existing paradigm has ceased to function adequately in the exploration of an aspect of nature to which the paradigm itself had previously led the way. In both political and scientific development the sense of malfunction that can lead to crisis is prerequisite to revolution." For the Systems and Control field the latter half of the 1950's was indeed a period of crisis in the sense of Kuhn. Attempts to extend the theory of single-input/single-output control systems to the multivariable situation were quite unsuccessful. Internal instability in feedback control systems was something that existing theories dealt with inadequately.

The extension of Wiener-Kolmogoroff theory to nonstationary and multivariable contexts proved to be extremely difficult and did not lead to computable solutions. The natural setting for the solution of problems of guidance and control of aerospace vehicles was certainly not the existing theory of control. It is in this historical context that the state-space theory of systems was born and dominated the field for many years. But there was another half of the revolution, less visible perhaps in those initial years, but equally creative, more enduring, and whose influence we see even today. The roots of this revolution go back to the work of Black, Nyquist, and Bode. It originated in their work on Feedback Amplifier Design where the concerns were the design of systems which are robust against uncertainty, and feedback of appropriate signals was shown to be most effective in guaranteeing such robustness. Indeed, uncertainty and feedback have become inseparable in viable control applications. The carrier of this tradition in sharpened and highly original forms was undoubtedly George Zames. His lifelong work was concerned with understanding and quantifying the fundamental limitations and capabilities to controlling systems in the presence of uncertainty by means of feedback.

For Zames, there existed two kinds of systems: systems that could be modeled precisely and those for which only imprecise models were available. It was the second kind on which he focused his attention. In his view, the external input-output or black-box view was the preferred framework for modeling systems where the models were necessarily uncertain because there was no choice but to make approximations. The distinction between model uncertainty and measurement uncertainty and the need for nonparametric nonprobabilistic models for system uncertainty is one of Zames's lasting contributions.

Zames understood that the idea of a state-space (internal) model was a highly model-sensitive concept and was only appropriate as initial models of systems when they were known *a priori* to be precise, for example, those that could be based on the availability of physical laws such as Newton's laws. On the other hand, for systems such as feedback amplifiers consisting of many stages and built from components which were imperfect, the input-output (external) view was necessarily the correct one. In such systems, models were approximate with possibly a coarse description of the input-output approximation error available. Such errors are difficult to reflect on a particular state-space description of the system, since the order of the state-space model is a function of these errors. State-space models come into the picture at the level of computation and at the level of hardware-software implementation of control systems. As George often said, the processes of approximation in model building and obtaining state-space models do not commute.

The idea that the deleterious effects of disturbances on imprecisely modeled systems can be successfully overcome by means of feedback led Zames to the question of understanding the fundamental limitations of system performance. He realized that the reduction of the deleterious effects of uncertainty in systems by means of feedback was a special case of a more general principle of complexity reduction via organization, of which feedback was only a special case. Moreover, this

complexity could be measured in terms of metric entropy as defined by Kolmogoroff and the process of complexity reduction led to organizing the system in the form of a hierarchy with multiple feedback loops. This circle of ideas was first articulated in George's plenary lecture "Feedback, Hierarchies and Complexity" at the 1976 IEEE Conference on Decision and Control and was to be a recurrent theme in his many subsequent lectures. Indeed, the founding of H^∞ -control theory was an expression of the intrinsic limitations to linear system performance under feedback, as well as a sharp, quantitative statement about the reduction of complexity by means of feedback.

The seeds of George's lifelong work were planted in his doctoral dissertation [3]. It is a remarkable document for several reasons. First, it is one of the most creative examples of the use of mathematics, in this case functional analysis, to the study of engineering systems, notably feedback control systems. George got the idea of using functional analysis (in particular the contraction mapping principle) from a graduate course in Analysis at MIT taught inspirationally by Isadore Singer. The use of this mathematics for the study of interconnection of systems, definition of the gain of a system, existence and uniqueness of solutions of feedback systems, and giving a precise definition of the physical realizability of systems is startlingly original. The ideas of Chapter VI of this thesis were completed in a later paper [4]. In this paper, he posed the following question: "What is a good enough mathematical model of a physical system—a model that does not lead to impossible results in feedback problems?" In this work he defined the concepts of "generalized delay" and "generalized attenuation," and showed that such properties are at the heart of "system realizability," and moreover, "that they are necessary in order to avoid paradoxical and perplexing behavior of models in feedback." This was the first fundamental treatise on the subject of what we now call well-posedness of feedback systems.

George's thesis contains other highly original material, such as the idea of global linearization of nonlinear systems, a theorem about the invertibility of an operator $B \circ N$, where B is a bandlimiter and N is a memoryless nonlinear operator with bounded slope, and the realization of this inverse by means of a feedback system. Part II of his thesis, "Statistically Orthogonal Operators Applied to Optimum Filtering," does not form part of the technical report cited above. A Bayes procedure for obtaining optimum nonlinear filters for processes admitting a Wiener expansion is presented here. This research, apparently not well known, was published in part as "Bayes Optimum Filters Derived Using Wiener Canonical Forms" [5]. This is apparently one of the earliest papers to embrace the Bayesian viewpoint in nonlinear filtering.

Zames' fundamental work on the stability of nonlinear time-varying feedback systems was first published in "On the Stability of Nonlinear Feedback Systems" [6] and in a more complete form in the two part-paper "The Input-Output Stability of Time-Varying Nonlinear Feedback Systems" [7]. The theory of input-output stability receives almost a definitive treatment in these two papers. The introduction of extended Normed Linear Spaces, the Small Gain Theorem, the Passivity

Theorem, the Circle Criterion in input-output form, and the use of multipliers can all be found in these two papers. At the end of Part I of this paper, Zames writes "One of the broader implications of the theory developed here concerns the use of functional analysis for the study of poorly defined systems. It seems possible, from only *coarse information about a system*, and perhaps *even without knowing details of internal structure*, to make assessments of qualitative behavior," a refrain which was to creatively obsess George throughout his scientific life. George was to continue his work on stability theory for the next few years, some of it in collaboration with P. L. Falb and M. Freedman, but the principal ideas were laid out in the above two papers [6], [7].

In the mid-1970's, George wrote two important papers with his doctoral student N. A. Schneypor on the subject of augmenting stability and quenching of jump phenomena by introducing dither (high frequency signal) into a nonlinear system where the nonlinearity satisfies a Lipschitz condition [8], [9]. In many ways, these two papers represent a culmination of George's work on input-output stability and well-posedness of nonlinear systems initiated in his thesis. In these papers, a notion of structural stability is introduced, and the feedback system is studied via an approximately equivalent nonlinear system with a smoothed nonlinearity. Zames gives a creative physical interpretation of the idea of mollifiers used in analysis and partial differential equations. He shows that the effect of stability augmentation and quenching can be quantitatively captured in terms of a narrowing of the nonlinear incremental sector. These ideas should be better known than they appear to be. The self-linearizing effect of introducing dither in a nonlinear system deserves further study.

With the publication of these papers on stability and qualitative behavior of nonlinear systems, a line of research that began in his thesis came to an end. It was characteristic of Zames's work that problems that he worked on were deep; they required long periods of gestation, after which almost definitive solutions to the problems were obtained. Publications for lengthening one's publication list was definitely not a priority for George.

How much information about a system's input-output behavior is needed to control it to a specified accuracy? How much identification is required if only rough bounds on time and frequency responses are available *a priori*? How does one model plant uncertainty? What are the limitations to controlling a system to arbitrary accuracy by means of feedback? How does one make precise the statement that feedback reduces complexity? These were the questions that were to dominate George's research from the mid-1970's to at least the mid-1980's. An early glimpse of George's thinking can be found in his paper "Feedback and Complexity" [10]. The concept of complexity of systems was made precise in the paper "On the Metric Complexity of Causal Linear Systems: ϵ -Entropy and ϵ -Dimension for Continuous Time" [11] by using Kolmogoroff's notion of metric entropy and obtaining bounds for metric entropy for linear systems satisfying an exponential bound. It is interesting to note that this paper is in the same spirit as that of work by Valiant, Vapnik, and others on the learnability of concept classes.

If Zames were to be known for one paper which established him as one of the most original and influential thinkers in the Systems and Control field, it would have to be his paper “Feedback and Optimal Sensitivity: Model Reference Transformations, Multiplicative Seminorms and Approximate Inverses” [12]. It introduced for the first time what was to be called H^∞ -control theory, by restricting attention to causal linear time-invariant bounded maps from L^2 to L^2 , which, by the Fours–Segal theorem, is in one-to-one correspondence with operators which are multiplications by H^∞ functions. This paper was many years in the making and was a result of George’s dissatisfaction with existing indicators of performance of feedback systems, especially as it relates to attributes of systems which impose fundamental limitations to feedback control performance. He chose to illuminate this by considering input–output stable systems and adopting sensitivity as an appropriate performance measure. As a matter of fact, George liked to separate the issue of stability from performance. He viewed stabilizing the system as a preproblem, almost an issue of well-posedness. He believed that no satisfactory theory of synthesis of control systems could be developed without better performance indicators. The deeper clarification of the concept “feedback reduces complexity” had to be substantiated in limited contexts first.

Besides the problem formation, this paper contains highly original results. Inspired by earlier work of Bode on the limitations to sensitivity reduction by feedback in the presence of R. H. P. zeros, Zames proved that for minimum phase plants the sensitivity could be reduced to zero over any finite bandwidth. Finally, there is the implicit statement in this paper that feedback compensation is essentially the issue of constructing an approximate inverse by feedback in a suitable topology. It seems that Zames was returning to a theme which he first articulated in his doctoral thesis. The origins of his later work on learning and adaptation were also to be found in this paper. If accurate information about R. H. P. zeros is not available *a priori*, how does one obtain this information through identification and learning? Is this a situation where adaptation is required?

The introduction by Zames of H^∞ -control theory opened a new field of study and has put robust control onto center stage for the past decade and a half. There are hundreds of papers on the subject, workshops, sessions at conferences, MATLAB toolboxes, and numerous books. The original problem posed by Zames was to minimize the sensitivity function with respect to the H^∞ norm. He realized that this was an interpolation problem, and this led to the introduction of powerful operator-theoretic methods into control based on interpolation theory of analytic functions on the unit disc. He collaborated with B. Francis, C. Foias, W. Helton, S. K. Mitter, and A. Tannenbaum in developing the subject further. The fact that prominent operator theorists and functional analysts became interested in systems and control has led to a wonderful cross-fertilization between mathematicians and control theorists, a process still very much in progress today. Since George’s initial insight, a number of solutions and extensions of H^∞ have been proposed and worked out. The whole question of the use of various norms in control problems came to the forefront from

this work, and because of this, today the systems engineer can choose to employ analysis and synthesis techniques with a variety of norms including L^1 , H^2 (classical LQG optimization), and of course H^∞ .

The H^∞ enterprise had another important consequence, which was quite unexpected—the solution to the problem of the computation of the gap metric on the space of systems. In his quest to understand systems, Zames viewed the graph as the primary object, with state-space realizations, integral representations, and transfer functions being but convenient tools. This was particularly important for unstable systems where many of the standard input–output concepts resulted in restrictive analysis and synthesis methods. The graph is the collection of all possible input–output pairs that can be generated at the system ports. Model uncertainty can then be understood via set-theoretic containment of the graph. From this vantage point George asked the questions “What types of uncertainty can be tolerated without destroying closed-loop stability?” and “In which sense should plant uncertainty be small in order to achieve a small uncertainty in the closed loop?” These questions turned the dominant viewpoint around, for until that time the prevalent view was to postulate a precise model for uncertainty and ask, e.g., for the maximal interval of parameter variation that can be tolerated. Instead, George asked for a suitable topology based on which robustness and well-posedness questions can be formulated.

At the Allerton Conference in October 1980, Zames, with his student Ahmed El-Sakkary, put forward a deeply original approach to uncertainty in feedback systems based on these ideas [13]. They identified the natural topology for studying questions of robustness in feedback systems. They went on to characterize the tolerable open-loop errors in terms of a metric defined as the “gap” or “aperture” between the graphs of operators in Hilbert space and use it to quantify distances between systems and to show that all metrics which are “continuously robust” in the above sense are equivalent to the gap metric. The introduction of a topology for robustness questions provided a unifying framework within which other approaches to robustness could be considered, and where questions of well-posedness and continuity of design techniques can be addressed. In fact, their approach solved a major puzzle of how to deal with possible instability of the open loop while avoiding an overly structured model for uncertainty.

It is striking that Zames was developing H^∞ theory and ideas using the gap metric independently at roughly the same time, since the two theories became inextricably linked in a profound manner that still has major ramifications for the whole subject of system uncertainty and feedback. Indeed, it turned out that the so-called H^∞ two-block problem was exactly what was needed to compute the gap, and that the proof was based on the operator theoretic tools that George loved, in this case the Sz.–Nagy–Foias Commutant Lifting theorem. The circle was closed, the two theories were united, and a completely general but computable theory for robust feedback control was at hand at last! The richness of this approach is now being extended to nonlinear systems, and the vision of George is still far from being completely worked out. This work of George’s has all of the hallmarks which defined his

career: incredible intuition and insight, a deep understanding of feedback, and the unfailing ability to perceive the correct mathematical tools necessary for the solution to the problem at hand.

One should note that the popular four-block formulation was not the solution of the optimal robustness problem in the H^∞ framework, but only a compromise solution. In fact, one of George's last main projects was to obtain a mathematical solution to this previously unsolved problem with his student James Owen [14].

In the last few years before his untimely death, George's main research preoccupation was to give a sharp definition of learning and adaptation in the nonparametric input-output (external) setting. Indeed, this constitutes a major open problem in the systems and control field. The question George asked himself was: when is feedback insufficient for robust reduction of uncertainty leading to satisfactory performance of the closed-loop control system? It is precisely in this situation that one has to "adapt" the controller by appropriate identification of the systems for which only an uncertain model is available. Is it possible to decide *a priori* when feedback is insufficient and one has to resort to adaptation? In light of his earlier work, one knows that for arbitrary sensitivity reduction in linear time-invariant systems if one does not have precise information about the right half-plane zeros, then these zeros have to be robustly identified on line in order to exercise effective control. This is a situation where adaptation is required. George wanted to quantify this by evaluating the best performance that can be achieved using feedback on the basis of *a priori* knowledge and then demonstrating that by learning about the plant through appropriate identification and using control based on this *a posteriori* knowledge the performance could be improved. This performance gain was a result of adaptation. In effect, adaptation is called for in reducing model uncertainty through proper identification. An important concept in this work is the principle that optimal (adaptive) feedback performance is an increasing function of information where information is represented by sets of uncertainty.

In the 1960's, a theory of stochastic adaptive control with parametric uncertainty was developed by Bellman, Feldbaum, Florentin, and others by using Bayesian analysis and dynamic programming. This approach leads to so-called "dual control" problems. For additive cost functions, one can show under reasonable hypotheses that the control is a nonlinear function of the conditional distribution of the states and parameters given the past observations and control. Updating the conditional distribution leads to a nonlinear filtering problem which in general is infinite dimensional. Even when the joint state-parameter process is Gaussian, not much can be said qualitatively about the behaviors of the optimally controlled system. Much subsequent work has been done in the parametric situation, and a well-developed asymptotic theory of stochastic adaptive control exists today [15]. George was, however, concerned with representations of model uncertainty which usually will have a parametric part and will also contain a residual unmodeled nonparametric part. This nonparametric part will not be naturally modeled probabilistically, and

George was interested in the question of robust identification of the parametric part in the presence of unmodeled nonparametric uncertainty. In his view, a resolution of this problem was a prerequisite for a satisfactory theory of adaptive control. George, together with some of his graduate students, did important preliminary work on this subject. Some of his ideas are outlined in "Toward a General Complexity-based Theory of Identification and Adaptive Control," [16] and in his forthcoming paper in a special issue of *Systems and Control Letters*. Sadly, he is not here to complete this line of research.

Besides his family, the search for fundamental knowledge and understanding, a passion for research, and a love for intellectual debate be it on feedback control, Chomskyan linguistics, or the Vietnam War, are the things that George cared for most in life. He lived a simple life, trying to shield himself from the unnecessary distractions and complexities of the modern world so that he could remain faithful to his beliefs and convictions. At a time when intellectual values are being systematically eroded in universities, we hope George Zames will be remembered as much for the values by which he lived his life as for his research contributions.

On a more personal note, for those of us who were fortunate enough to know George for a long time, certain memories often come to mind: his electrifying presentation of the Circle Criterion at the 1964 National Electronics Conference in Chicago, IL, and the astonishing gracefulness he demonstrated while dancing native style at a workshop in Lake Ochrid, Yugoslavia, and while skiing down the slopes of Mt. Mansfield in Stowe, VT. We will remember the spark-flying debates we had with him, his verbalized struggle in the early 1970's to crystallize his ideas about feedback, and his devilish use of a laser which he would point from his Back Bay apartment window to the street below to confound unsuspecting passersby.

We also know of his love for his family: his two sons Ethan and Jonathan, and of course his wife Eva. Eva Zames was truly George's best friend and intellectual partner who shared and delighted in their mutual life's adventure together. George was fond of saying that he lived a charmed life, and the foundation of that life was his family. Those of us who visited George during his last days in the hospital could only be deeply moved by the love and support shown to him by his family. George himself did not show any fear of death. Indeed, when he learned about his condition, his main concern was providing for Eva and his two sons. About himself, he said that he had lived a full life and could ask for nothing more.

Our dear friend and mentor, George Zames, Zichrono Lev-racha (of Blessed Memory), will be sorely missed.

SANJOY MITTER, *Fellow, IEEE*
Massachusetts Institute of Technology
Cambridge, MA, USA

ALLEN TANNENBAUM, *Member, IEEE*
University of Minnesota
Minneapolis, MN, USA

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