Network Coding Schemes for Underwater Networks

The benefits of implicit acknowledgement

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Abstract

Underwater acoustic communications are characterized by a long propagation delay and limited bandwidth. Power consumption is an additional constraint for underwater networks. Network layer schemes for that minimize both transmission time and power consumption are thus of interest. Conventional routing schemes have limitations in both power consumption and delay performance. Application of network coding schemes in rateless fashion results in better delay performance; however, power consumption is greater than for most routing schemes when the network is lightly loaded. This paper proposes a new method for network coding that relies on implicit acknowledgements to improve power consumption performance. Numerical results demonstrate superior performance of the scheme proposed. The method is also applicable to other wireless channels.

Categories and Subject Descriptors CR-number [*subcategory*]: third-level

General Terms Performance

Keywords Underwater Acoustic Networks, Network Coding, Routing.

1. Introduction

With the advances in acoustic communication technology the interest in study and experimental deployment of underwater networks has been growing. However, underwater channels impose many constraints that affect the design of wireless networks. They are characterized by a path loss that depends on both the transmission distance and the signal frequency. As a result, the useful bandwidth depends on the transmission distance[1]. In addition, underwater acoustic propagation speed is very low, typically 1500m/s, which introduces a significant delay in data transmission between the nodes. The low propagation speed also introduces a tradeoff between the probability of collision and packet delay in multiple-access scenarios. Finally, underwater acoustic networks are constrained in terms of energy supply.This is notably the case for fixed, battery-powered sensors [2]. As a consequence, important goals in the design of an underwater network are minimal power consumption and transmission delay. In particular, these goals affect the design of the network layer of a communication system. This layer should reduce energy consumption while maintaining a good delay performance [2].

In this paper, a comparison between different routing and network coding schemes is presented for an underwater acoustic channel, similar to the work of [3] for a wireless radio channel. In addition, a new technique to reduce power consumption in a network coding scenario is presented. This technique contrasts the rateless transmission mechanism, usually considered in network coding. It takes advantage of the broadcast nature of the channel for the nodes to get an implicit acknowledgment of previously transmitted packets. In Section 5, it will be shown that this technique has transmission delay equivalent to the rateless network coding scheme but a much better behaviour in terms of power consumption when the network is lightly loaded.

The different schemes are compared based on the time they take to complete the transmission of a given number of packets between a source and a sink in a network, and the power consumption to accomplish this transmission. In particular, a concatenated relay network is considered, in which relay nodes are located on a line between the source and the sink. This corresponds to the case of an underwater network with fixed sensors in which there exists one collecting node for data, one sensor transmitting information and all other sensors acting as relays.

The paper is organized as follows. In Section 2, a summary of previous research and basic concepts on network coding are presented. In Section 3, a model of an underwater channel is outlined. In Section 4, a model of the network, as well as the network layer schemes are presented. In Section 5, the concatenated relay network is analyzed. In section 6, numerical results for transmission delay and power con-

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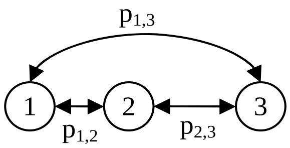

Figure 1. Concatenated relays network with probabilities of successful transmission

sumption are presented for the cases studied. Conclusions are summarized in the last section.

2. Network Coding

The concept of network coding was introduced by Ahlswede *et al*[4], and is also known as coded packet networks. Network coding considers the nodes to have a set of functions to operate upon received or generated data packets. Today's networks would represent a subset of the coded packet networks, in which each node has two main functions: forwarding and replicating a packet. These functions refer to taking an incoming packet and transmitting it on an outgoing link, or several outgoing links, respectively[5]. In this case, the network's task is to transport information provided by the source nodes unmodified. In contrast, network coding considers information as a mathematical entity that can be operated upon. Work in [6] and [7] showed that linear codes over a network are sufficient to implement any feasible multicast connection. Also, [7] provides an algebraic framework for studying this subset of coded networks. In both of these cases, the nodes are considered to transmit a linear combination of the packets previously received.

Let us consider the following example in a wireless scenario to illustrate the advantages of network coding. Let us assume a network with three nodes (Figure 1). Since nodes communicate using a lossy wireless medium, there exists a certain probability $p_{i,j}$ that a transmission will be successfully received at node j when i transmits. For simplicity, let us consider symmetry in the transmission links, i.e. $p_{i,j} = p_{j,i}$. If a common routing scheme is chosen, a path is selected to transmit a packet from node 1 to 3. Assuming that the optimal path is to go from node 1 to 2 and then from 2 to 3, and that a packet A_1 has arrived at node 1, this node will try to transmit the packet to node 2. If this transmission is unsuccessful (Figure 2(a)), node 1 will try again. Let us assume that packet A_2 has arrived at node 1 before the first retransmission. Figures 2(b)-(c) show transmission of A_1 once the packet has been successfully received at nodes 2 and 3, respectively. Once packet A_1 has arrived to node 3, the transmission process will be repeated for A_2 (Figure 2(d)).

When network coding is used, there is no path selection. Packet A_1 might thus arrive at node 3 even if the link between 1 and 2 is down during that transmission (Figure

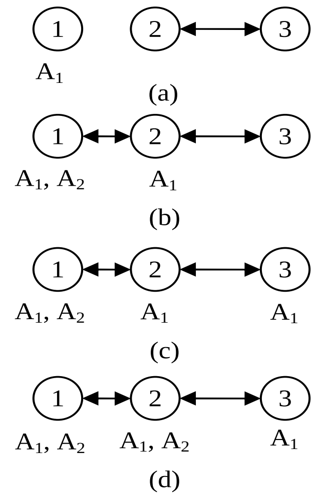

Figure 2. Routing example: wireless scenario. Arrows indicate the availability of links. The four diagrams correspond to four instants in time

3(a)). Also, when packet A_2 arrives at node 1, as in the previous case, a coded packet $\alpha_2 A_1 + \beta_2 A_2$ is transmitted (Figure 3(b)). This coded packet will have certain probability of getting from 1 to 3 directly or going through 2. If the link between node 1 and 3 is down, under the same conditions as in the routing case, both A_1 and A_2 will be received (Figure 3(c)) at node 3, while in routing only packet A_1 has been received. A worst case scenario for network coding would be to have $p_{1,3} = 0$ and packet A_2 received at node 1 after A_1 has been received at node 3. But this will yield the same delay as in routing. Thus, under the same conditions, network coding will have at most the same delay as routing. This improved performance is explained by the ability of network coding to send data through links different from the optimal path selected by routing, and transmission of coded packets.

3. Channel Model

An underwater acoustic channel is characterized by a path loss that depends on both the distance *l* and signal frequency *f* as:

$$A(l,f) = l^k a(f)^l \tag{1}$$

where k is the spreading factor and $a(f)$ is the absorption coefficient [1]. The spreading factor describes the geometry of propagation, e.g. $k = 2$ corresponds to spherical spreading, $k = 1$ to cylindrical spreading, and $k = 1.5$ to practical spreading. The absorption coefficient can be expressed in dB/km using Thorp's empirical formula for f in kHz:

$$\begin{aligned} 10 \log a(f) = \; & 0.11 \tfrac{f^2}{1+f^2} + 44 \tfrac{f^2}{4100+f^2} \\ & + 2.75 \cdot 10^{-4} f^2 + 0.003 \end{aligned} \tag{2}$$

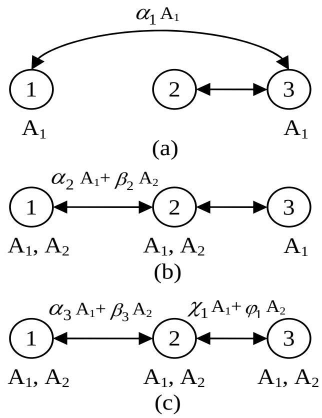

Figure 3. Network coding example: wireless scenario. Arrows indicate the availability of links. The diagrams correspond to three instants in time

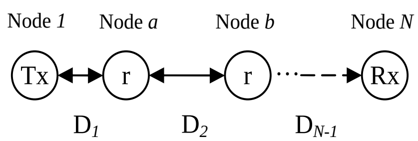

Figure 4. Concatenated relays network

for frequencies above a few hundred Hz. The noise in an acoustic channel can be modeled through four basic sources: turbulence, shipping, waves, and thermal noise, which results in a probability sprectral density (p.s.d) that decays with frequency at approximately $18dB/dec$ [1]. The fact that the signal attenuation and the noise power depend on the frequency causes a frequency dependent SNR. Assuming a constant signal p.s.d, the SNR observed over a distance l is

$$SNR(l,B(l)) = \frac{P(l)}{B(l)} \frac{\int_{B(l)} A^{-1}(l,f)\,\mathrm{d}f}{\int_{B(l)} N(f)\,\mathrm{d}f} \tag{3}$$

where $P(l)$ and $B(l)$ are the power and the bandwidth chosen for the distance l. If $B(l)$ and $P(l)$ are fixed the SNR at a different distance l' in terms of $SNR(l,B(l))$ is

$$SNR'(l',B(l)) = SNR(l,B(l)) \frac{\int_{B(l)} A^{-1}(l',f)\,\mathrm{d}f}{\int_{B(l)} A^{-1}(l,f)\,\mathrm{d}f} \tag{4}$$

Finally, the equivalent bit SNR E_b/N_0 is defined as

$$\frac{E_b}{N_0} = SNR'(l',B(l)) \frac{B(l)}{C(l',B(l))} \tag{5}$$

where

$$C(l',B(l)) = \int_{B(l)} \log_2 \left[1 + \frac{P(l)}{A(l',f)N(f)B(l)}\right] \mathrm{d}f \tag{6}$$

Note that l' could be different from l. This corresponds to a situation in which the bandwidth is calculated for a particular distance, but the current node is at a different distance.

4. Network Model

The network considered in this paper is shown in Figure 4. There is a receiving or data collecting node, and all the information is destined to it. If a node b is closer than node a to the collecting node, a is said to be upstream with respect to b, and b is said to be downstream with respect to a.

Every node in a routing scheme will have data to be transmitted downstream and upstream, which corresponds to data packets and acknowledgment packets, respectively. In network coding, each node is transmitting a linear combination of the packets in its queue. If a node has m data packets in its queue, it will transmit a coded packet with m degrees of freedom (DOF). In a unicast scenario with transmission of a known number of packets, any coded packet with m DOF contains the first m data packets coming from the source node. For this study, six approaches to unicast have been considered. The first four approaches consider different routing schemes. The remaining two approaches consider network coding in a rateless fashion and with the implicit acknowledgment scheme:

1)Routing using end-to-end acknowledgement: The source node transmits the same packet until it receives an acknowledgement from the sink. All other nodes behave as relays. These relays will also transmit the acknowledgement from the receiving node to the source node. In this case, a relay node will stop transmitting the packet once the end acknowledgment reaches it. It will transmit the acknowledgement packet until it overhears an upstream node transmitting the acknowledgement or when a new data packet is received.

2)Routing using windowing: If the source node has more than one unacknowledged packet to transmit, it can transmit as many as a window size W cyclically, i.e. in every transmission it will send a different packet from the first unacknowledged packet i with the following sequence $i, i+1, ..., i+W-1, i, i+1....$ When a packet reaches the receiving node, this packet will be stored and an acknowledgement of the last ordered packet will be send. For example, if the collecting point has received packets 1,2,3,5 and 7, it will send an acknowledgement of packet 3 until it receives packet 4. If packet 4 is received but packet 6 has not arrived, it will send acknowledge of packet 5.

3)Routing using link-by-link acknowledgement: Every time a node receives a packet, it will retransmit the packet and send an acknowledgement to the previous node. Once a packet has been acknowledged, the node can start transmitting a new data packet in its queue. If it has no new packets to transmit, it will only transmit if a node upstream sends new information, or sends a previous packet, in which case the node will acknowledge this packet.

4)Routing using link-by-link acknowledgement, opportunistic: This scheme is similar to (3). However, all the nodes eavesdrop on the packets of nodes farther downstream. For example, if node i sends an acknowledgement of packet k to node $i-1$, and node $i-2$ has not received an acknowledgement of that packet from node $i-1$, and has overheard this transmission it will also consider packet k acknowledged.

5)Network coding in rateless fashion: Once a relay node gets its first coded packet, which means at least one degree of freedom, it will transmit until the receiving node sends a confirmation that all the information has been received, i.e. all degrees of freedom have arrived. The same happens at the source node. This strategy assumes that there is a mechanism that informs the collecting node about the number of degrees of freedom that constitute the total message or that this number is fixed *a priori*. The receiving node in this scheme will not transmit until all packets have been received.

6)Network coding with implicit acknowledgement: This scheme is similar to (5). However, nodes eavesdrop on other transmissions if they are in range. If a node receives from a node further downstream the same, or greater number of degrees of freedom than what the node has, it will stop transmitting and update its information, if necessary. It will resume transmitting if an innovative packet is received from a node upstream. The sink (data collecting) node will retransmit its degrees of freedom when a coded packet is received.

5. Analysis

The network can be modeled as a set of states $\S = S_1 S_2 ...$; a set of rules that govern the transition between those states, and a set of probabilities to transit from a state to the next. The state $S(t)$ is given by the information on the packets or DOF that each node of the network has at any given time t. The rules that govern the state transitions are directly related to the network layer scheme used. Finally, the transition probabilities are related to the rules and the probabilities of successful packet transmission through the underwater channel.

For routing, the state at any time t is of the form $S(t) = (\vec{r}(t), \vec{a}(t))$ where $\vec{r}(t) = (r_1, r_2, ..., r_N)^T(t)$ and $r_i(t)$ shows the number of packets or the number of the packet, depending on the scheme, received by the $i-th$ node in the network; $\vec{a}(t) = (a_1, a_2, ..., a_N)^T(t)$ is similar but refers to the acknowledgement information received by the nodes; N is the number of nodes in the network.

For network coding, the state at any time t is of the form $S(t) = (\vec{dof}(t))$ where $\vec{dof}(t) = (dof_1, dof_2, ..., dof_N)(t)$ and $dof_i(t)$ shows the number of DOF present in the $i-th$ node of the network. Again, N is the number of nodes in the network.

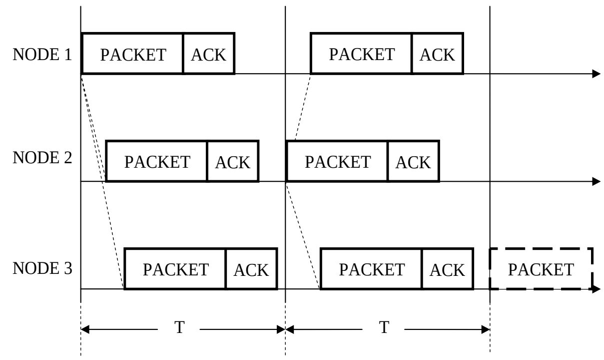

Figure 5. MAC layer.

As an example of a set of rules, let us study the routing using link-by-link acknowledgements and the network coding in rateless fashion schemes. Let us enumerate the nodes in an increasing order from the source to the sink node, i.e. source is node 1 and the collecting point is node N. For routing with link-by-link acknowledgement the rules are:

1)$r_i(t) \geq r_j(t)$, for $i < j$
2)$r_i(t) \leq r_i(t + t_0)$, for $t_0 > 0$
3)$r_i(t) \geq a_i(t)$
4)$a_i(t) \leq a_i(t + t_0)$, for $t_0 > 0$
5)$r_{i+1}(t) \geq a_i(t)$ and $r_{i+1}(t) \leq a_i(t) + 1$
6)While $r_i(t) > a_i(t)$, node i transmits
7)If i receives successful ACK packet from $i+1$,
then $a_i(t) = a_i(t) + 1$
8)If i receives successful data packet from $i-1$,
then $r_i(t) = r_i(t) + 1$
9)If $r_i(t) = a_i(t),\ i$ transmits if $i-1$ sends old packet

For network coding in rateless fashion:

1)$dof_i(t) \geq dof_j(t), \quad for \quad i < j$
2)$dof_i(t) \leq dof_i(t + t_0), \quad for \quad t_0 > 0$
3)If $dof_i(t) > 0$, node i transmits
4)If i receives successful coded packet from j, and
$dof_i(t) < dof_j(t)$, then $dof_i(t) = dof_j(t)$

6. Numerical Results

The MAC layer of the system is assumed to implement polling with equal opportunities for each node to have access to the medium with equal priority. In each transmission slot, a packet is transmitted that includes new data to downstream nodes and acknowledgement data to upstream nodes (Figure 5). In the case of network coding the downstream (data) and upstream (acknowledgement) information is embedded in a

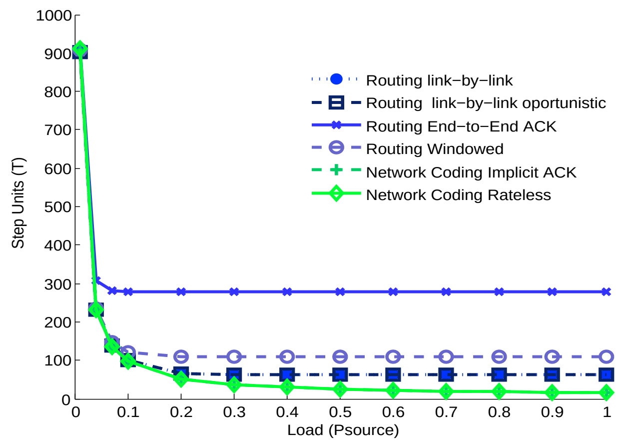

Figure 6. Average number of steps vs load for a 6 node network, transmitting 10 packets using optimal power calculation

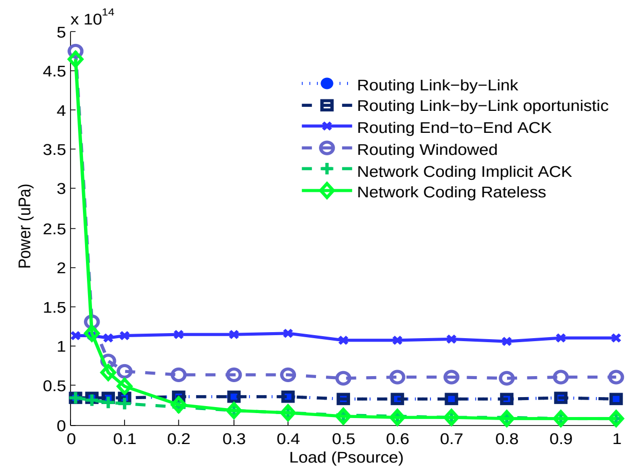

Figure 7. Average power consumed by node vs load for a 6 node network, transmitting 10 packets using optimal power calculation

single packet. This transmission packet will contain a linear combination of all the packets in the queue of the transmitting node (DOF). The polling order assumes knowledge of the position of the nodes. It assigns the channel going from the source node to the sink node, and then starts over, as seen in Figure 5. For numerical computations, the initial node with slot assignment is uniformly chosen between all nodes in the network.

In these computations, it is assumed that the maximum distance between nodes is 20 km. Distances between nodes in the concatenated relay network are uniformly distributed between zero and the maximal distance. It is assumed that: time T (Figure 5) is large enough to avoid collisions; packet generation at the source node is a Bernoulli process with P_{source} as the probability of generating a new packet every T; packets have 1000 bits; bit errors occur independently; PSK modulation is used; the minimum SNR required for correct transmission is $SNR_0 = 20dB$. Two approaches are considered for the computations. The first approach assumes that nodes have prior knowledge of the distance to their closest neighbors. Therefore, the power is calculated to reach both the closest upstream and the downstream neighbor, i.e. the farthest of the closest neighbors, with SNR_0. This ensures that there is connectivity between the source to the collecting node. Also, the transmission band is selected to be optimal for this case. The second approach uses a fixed power for all nodes. In this case, bandwidth is optimized for the maximum distance and the fixed power value is computed to reach that maximum. As in the previous approach, power is computed to achieve SNR_0. For convenience, power consumption is calculated using the approximation proposed in [1], i.e. $P(l) = pl^{2.22}$ with $p = 106.78dB$ re μPa. Probability of successful transmission over the link from node i to j is obtained from the probability of bit error by $P_{Success}(i,j) = (1 - P_{\text{bit error}})^n$ given the assumptions, where n is the number of bits in the packet. $P_{\text{bit error}}$ is calculated using the standard PSK bit error probability and the equivalent E_b/N_0defined in equations (4)-(6). Measurements considered in these computations are 1) average number of steps T until the last packet of the transmission reaches the receiving node, and 2) average power consumed by a node in the network to complete this transmission. These computations have been carried out for different loads (P_{source}), number of nodes in the network, and number of packets to be transmitted.

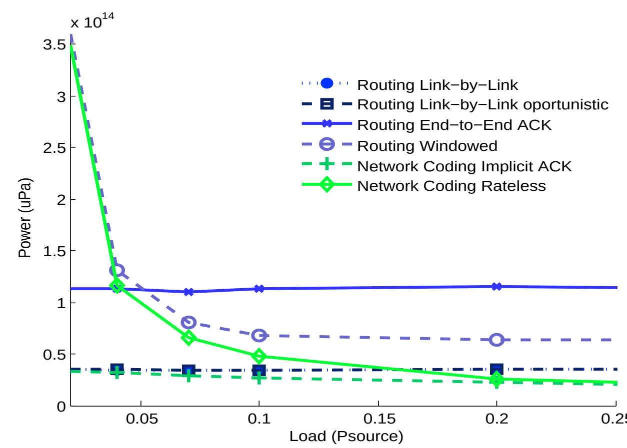

Figure 8. Average power consumed by node vs load for a 6 node network, transmitting 10 packets using optimal power calculation: Detail for P_{source} between 0.02 and 0.25

Figure 6 shows that both network coding schemes have less or equal delay, measured in time slots T, than the studied routing schemes for different packet generating loads P_{source}. For small P_{source} (light load) there is little difference between any of the schemes. However, when P_{source}

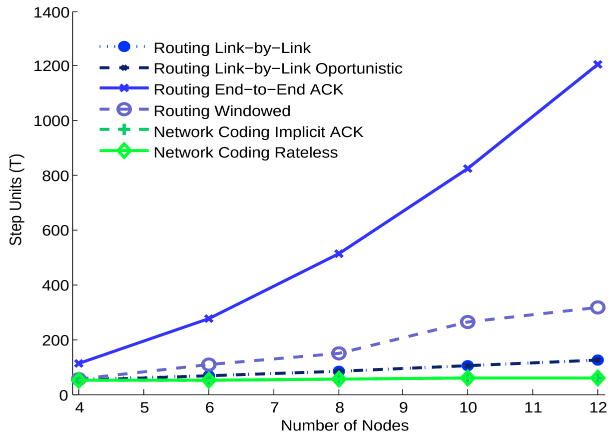

Figure 9. Average number of steps vs total number of nodes in the network, transmitting 10 packets generated with $P_{source} = 0.2$ using optimal power calculation

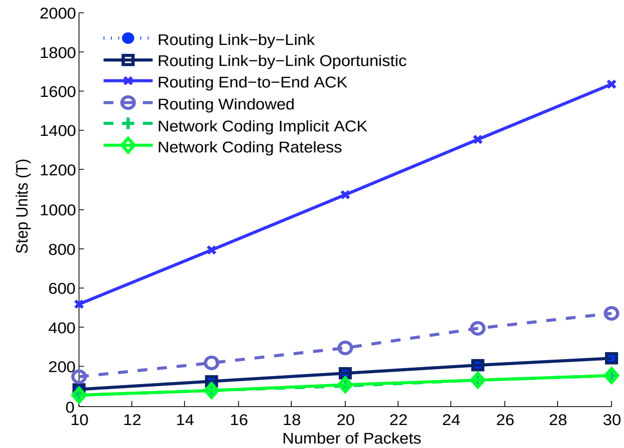

Figure 11. Average number of steps vs number of packets in an 8-node network, transmitting 10 packets using optimal power calculation and $P_{source} = 0.2$

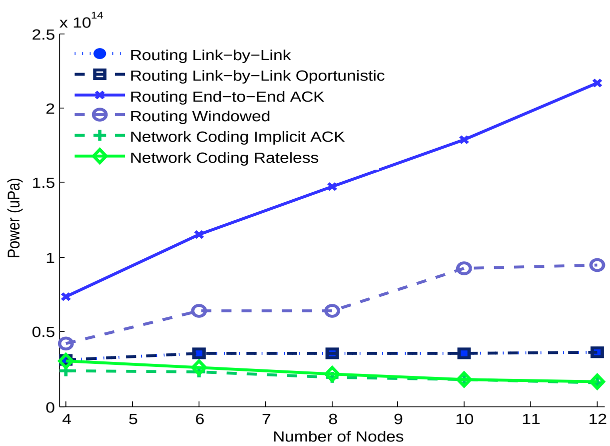

Figure 10. Average power consumed by node vs total number of nodes in the network, transmitting 10 packets generated with $P_{source} = 0.2$ using optimal power calculation

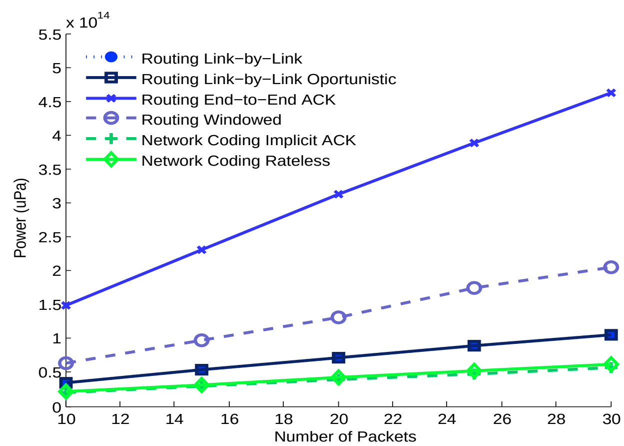

Figure 12. Average power consumed by node vs number of packets in an 8-node network, transmitting 10 packets using optimal power calculation and $P_{source} = 0.2$

increases, i.e. there is more load to the network, routing schemes settle to a minimum fix delay, whilst network coding schemes decrease for all $P_{source} \in (0, 1]$. This means that routing schemes have a minimum transmission delay for a file of a fixed number of packets to be transmitted. This minimum transmission delay is related to the scheme used and number of nodes that form the network. Network coding has a smaller delay for large P_{source} because it can exploit the ability to send coded packets. Note that both network coding schemes have very similar behavior in mean steps T for every value of P_{source} considered.

Figure 7 shows that for small P_{source} the proposed scheme with implicit acknowledgement has a better performance than any of the routing schemes, and a much better performance than network coding in rateless fashion (See Figure 8 for more detail). Given that the delay performance was found to be equivalent in both network coding approaches, implicit acknowledgement will have an overall better performance. For larger P_{source}, the rateless scheme requires less power than the scheme proposed, although both decay similarly. This is explained by the fact that the last node in rateless network coding does not transmit until it has all packets, whilst with implicit acknowledgement, the last node will transmit a coded packet every time it receives a new packet. This difference will be negligible when the number of nodes increases.

In Figures 7 and 6 there is little difference between the routing using link-by-link acknowledgement without and with opportunistic behavior. In fact, in all Figures both curves overlap producing a black filled-squares curve. This

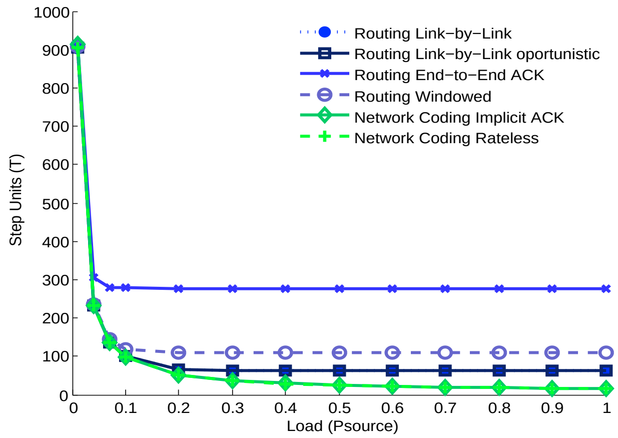

Figure 13. Average number of steps vs load for a 6 node network, transmitting 10 packets using fixed power in all nodes

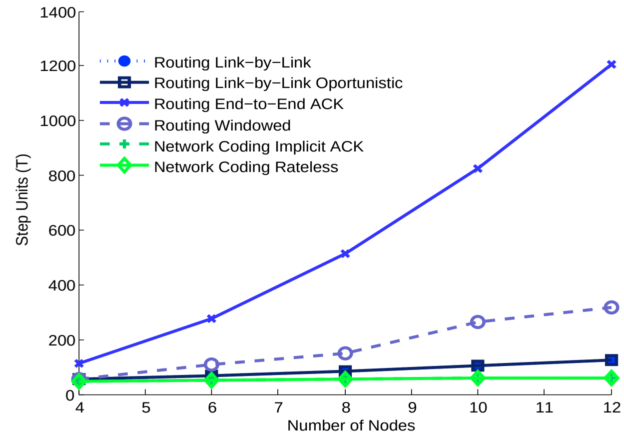

Figure 15. Average number of steps vs number of nodes in network, transmitting 10 packets using fixed power in all nodes and $P_{source} = 0.2$

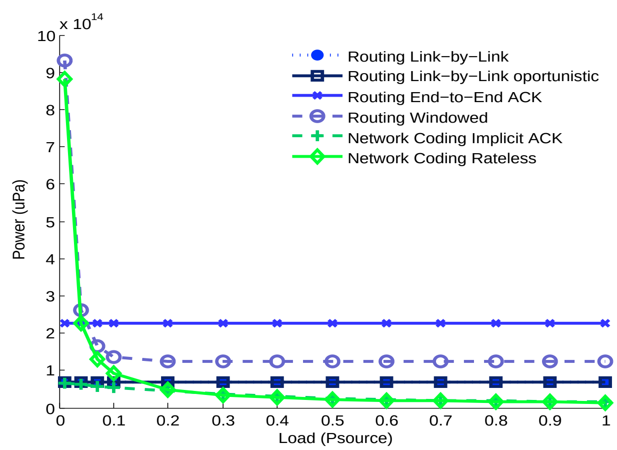

Figure 14. Average power consumed by node vs load for a 6 node network, transmitting 10 packets using fixed power in all nodes

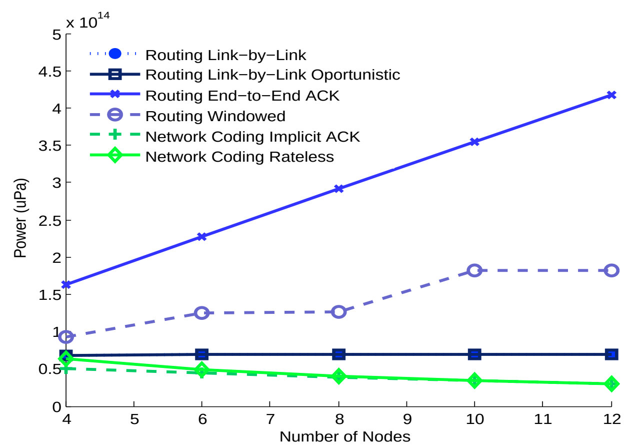

Figure 16. Average power consumed by node vs number of nodes in network, transmitting 10 packets using fixed power in all nodes and $P_{source} = 0.2$

can be explained by the high SNR_0 considered in the computations. A similar overlapping occurs for network coding schemes in some regions. In this case, the overlapping produces a curve with filled diamonds. Relation between delay and power consumption, with respect to the size of the network is shown in Figures 9 and 10, respectively, for $P_{source} = 0.2$. Note that average delay increases for all schemes, but it increases with a far lower rate for network coding. Among the routing schemes, routing with link-by-link acknowledgement has the best performance. Also, average power consumption increases with the number of nodes in any routing scheme. However, network coding diminishes power requirement per node when the number of nodes increases. Figures 11 and 12 show the relation between delay and power consumption with respect to the number of packets transmitted. Note that for network coding both of these figures increase at a much slower rate than for any of the routing schemes.

Similar results have been obtained for the case of nodes transmitting with fixed power (Figure 13, 14, 15, 16). The main difference between this results and the ones of the previous approach is the increased power consumption.

7. Discusion

Implicit acknowledgement implementation for a straight-line concatenated relay network is straigth forward. However, when nodes are distributed in more complex structures or with no particular structure, implicit acknowledgement will require more carefull considerations. For example, should a node wait until all downstream or descendant nodes have achieved the same rank (same number of DOF) as that node for it to stop transmission or would a subset of descendant nodes (e.g. one downstream node) suffice to obtain good performance and connectivity? Should the network try to acquire some knowledge of its topology to implement implicit acknowledgement? if so, at what cost?

Subgraph analysis and distributed algorithms in [5] could provide means to determine the time a node should stop transmiting, e.g. if a cut in the network has achieved a certain rank, all previous nodes stop transmiting and allow the cut to continue the forwarding process. Although a rateless transmission is used, the implementation could make use of knowledge of the rates of the subgraph connections to improve performance through implicit acknowledgement.

8. Conclusions

Several network layer schemes have been compared for an acoustic concatenated relay network. Network coding schemes were shown to have a significant advantage in transmission delay at high traffic loads. At light loads (low P_{source}) average transmission delay for a specified number of packets is equivalent for both routing and network coding, because it is influenced by the packet generation process more than by the network topology. Although at light loads transmission delay difference between the schemes considered is small, average power consumption by node is not. A conventional rateless network coding scheme for this unicast scenario yields a higher power consumption. Network coding with implicit acknowledgement has the lowest power consumption per node for all the schemes studied. For high loads (high P_{source}), rateless network coding has a smaller power consumption because the receiving node does not transmit until all expected packets have been received. Network coding with implicit acknowledgement constitutes a good approach for maintaining a low transmission delay and a low power consumption per node for the studied range of P_{source}.

Numerical results show that increasing the number of nodes increases average power consumption per node in all routing schemes under this setup. Note that increasing the number of nodes increases the physical coverage of the network. If a fixed distance between source and sink is maintained, average power consumption per node diminishes if optimal power is used. For both network coding approaches increasing the number of nodes produces a reduction in average power consumption per node.

For numerical results, comparison between the different schemes was carried out considering a fixed T. This unique parameter favors routing in the performance comparison. If a downstream and upstream transmission time is considered for routing(i.e. having T_d and T_u, possibly of different length in which each node could transmit downstream and upstream data, respectively) it would introduce more delay in transmissions. Network coding does not require this additional complexity in the MAC layer, thus having even better performance.

The work presented in this paper is not confined to the case in which the nodes are on a straight line. Given the work in [8], a wireless network can be organized as a concatenated relay network with the nodes aware of their upstream and downstream nodes, without physically being distributed in a straight line. Therefore, implicit acknowledgement is a valid and implementable method for more complicated networks. Also, implicit acknowledgement provides a possible extension to CodeCast [8] to reduce power consumption. Furthermore, implicit acknowledgement allows to save resources, e.g. memory required in the nodes, and rate adaptation following a similar analysis as in [9]. Finally, note that network coding with implicit acknowledgement is not limited to underwater channels and will have similar performance for any other wireless channel.

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