Feedback Effects of Credit Ratings

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Abstract

Rating agencies are often criticized for being biased in favor of borrowers, for being too slow to downgrade following credit quality deterioration, and for being oligopolists. Based on a model that takes into account the feedback effects of credit ratings, I show that: (i) rating agencies should focus not only on the accuracy of their ratings but also on the effects of their ratings on the probability of survival of the borrower; (ii) even when rating agencies pursue an accurate rating policy, multi-notch downgrades or immediate default may occur in response to small shocks to fundamentals; (iii) increased competition between rating agencies can lead to rating downgrades, increasing default frequency and reducing welfare.

Keywords: Credit rating agencies; performance-sensitive debt; financial regulation; credit-cliff dynamic;

1. Introduction

Rating agencies are supposed to provide an independent opinion on the credit quality of issuers. However, if market participants rely on credit ratings for investment decisions, then credit ratings themselves affect the credit quality of issuers. For example, a rating downgrade may lead to higher cost

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of capital for the borrowing firm because it induces a deterioration in investors' perceptions about the credit quality of the borrowing firm, because of regulations that restrict investors' holdings of lower rated bonds, or because of rating triggers in financial contracts.[1] Rating agencies thus face the problem of setting credit ratings that accurately represent the credit quality of a particular issuer, taking into account the effect of these ratings on the credit quality of the issuer.

Based on a model that incorporates the feedback effects of credit ratings, I show that: (i) rating agencies should focus not only on the accuracy of their ratings but also on the effects of their ratings on the probability of survival of the borrower; (ii) even when rating agencies pursue an accurate rating policy, multi-notch downgrades or immediate default may occur in response to small shocks to fundamentals; (iii) increased competition between rating agencies can lead to rating downgrades, increasing default frequency and reducing welfare. These findings call into question the recent criticism directed at rating agencies for being biased in favor of borrowers, for being too slow to downgrade following credit quality deterioration, and for being oligopolists.

The model is based on the *performance-sensitive-debt* (PSD) model introduced by Manso, Strulovici and Tchistyi (2010). Cash flows of the firm follow a general diffusion process. The firm has debt in place in the form of a ratings-based PSD obligation, which promises a non-negative interest payment rate that decreases with the credit rating of the firm.[2] Equityholders

[1] Kisgen (2007) describes in more detail the channels through which credit ratings affect the cost of capital for a borrower.

[2] As discussed in Manso, Strulovici and Tchistyi (2010), PSD obligations can be explicit, as in bank loans with performance pricing provisions. In a survey Moody's conducted in 2002, 87.5% of firms reported exposure to explicit rating triggers in their financial contracts (see "Moody's Analysis of US Corporate Rating Triggers Heightens Need for Financial Disclosure," Moody's July 2002). PSD obligations can also be implicit, as in the rollover of short-term debt. If the firm is performing well and has high credit ratings it will pay lower interest rates when rolling over its maturing debt. If the firm is performing poorly

choose the default time that maximizes the equity value of the firm. The rating agency's objective is to set accurate ratings that inform investors about the probability of default over a given time horizon. In this setting, the interaction between the borrowing firm and the rating agency produces feedback effects. With a ratings-based PSD obligation, the rating determines the interest rate, which affects the optimal default decision of the issuer. This, in turn, influences the rating.

The interaction between the rating agency and the borrowing firm is a game of strategic complementarity (Topkis (1979), Vives (1990), Milgrom and Roberts (1990)). Typically, games of strategic complementarity exhibit multiple equilibria. In the smallest equilibrium, which I call the *soft-rating-agency equilibrium*, the rating agency assigns high credit ratings, leading to lower interest rates for the borrowing firm, and consequently, a lower default probability. In the largest equilibrium, which I call the *tough-rating-agency equilibrium*, the rating agency assigns low credit ratings, leading to higher interest rates for the borrowing firm, and consequently, a higher default probability. The soft-rating-agency equilibrium is associated with the lowest bankruptcy costs and consequently the highest welfare among all equilibria.

Given the welfare implications of the different equilibria, it is important to understand how rating agencies set their rating policies in practice. To deal with the feedback effects introduced by rating triggers, rating agencies have proposed the use of stress tests.[3] In such tests, the company with exposure to rating triggers needs to be able to survive stress-case scenarios in which the triggers are set off. When the tough-rating-agency equilibrium involves immediate default, the borrowing firm will fail the stress test, potentially

and has low credit ratings it will pay higher interests rate when rolling over its maturing debt.

[3] "Moody's Analysis of US Corporate Rating Triggers Heightens Need for Increased Disclosure," Moody's, July 2002.

inducing rating agencies to select the tough-rating-agency equilibrium, the worst equilibrium in terms of welfare.

The best equilibrium in terms of welfare is the soft-rating-agency equilibrium, since it is the equilibrium with the lowest probability of default over any given time horizon. To implement such equilibrium, a credit rating agency should be concerned not only with the accuracy of its ratings, but also with the survival of the borrowing firm. One way in which this can be achieved is by having rating agencies collect a small fee from the firms being rated. Under this scheme, rating agencies become interested in the survival of the borrowing firm, inducing them to select the soft-rating-agency equilibrium.

The fact that rating agencies are paid by issuers has received intense criticism. The concern is that this practice may induce bias in favor of issuers. While this is a valid concern, the results of this paper suggest that if the fee is small relative to the reputational concerns of rating agencies, it only introduces small distortions while inducing rating agencies to select the Pareto-preferred soft-rating-agency equilibrium.

Stability of an equilibrium may play an important role in equilibrium selection and in the dynamics of credit ratings. The paper shows that if equilibrium is unique, then it is globally stable, so that small shocks to fundamentals lead to gradual changes in credit ratings. If there are multiple equilibria, however, some of them may be unstable. Small shocks to fundamentals may thus lead to multi-notch downgrades or even immediate default, in what has been called a "credit-cliff dynamic."

The effect of competition between rating agencies on equilibrium outcomes depends crucially on how credit ratings from different agencies affect interest payments by the borrowing firm. If interest payments depend on the minimum (maximum) of the available ratings then only the equilibrium with the highest (lowest) probability of default survives.

The above result is a consequence of the feedback effects of credit ratings. When interest payments depend on the minimum of the available ratings, a rating agency can undermine the credit quality of the borrowing firm by reducing its credit rating. Therefore, when a rating agency is concerned about being more accurate than other rating agencies, competition creates downward pressure on ratings that only subsides in the tough-rating-agency equilibrium. Increased competition may thus lead to the selection of the tough-rating-agency equilibrium, reducing welfare.

The model specification is flexible to capture realistic cash-flow processes, potentially allowing rating agencies and other market participants to incorporate the feedback effects of credit ratings into debt valuation and rating policies. Because we have a game of strategic complementarity, we can use iterated best-response to compute the soft-rating-agency equilibrium and the tough-rating-agency equilibrium. To calculate best-responses in the case of a general diffusion process, we need to solve an ordinary differential equation and compute the first-passage-time distributions of a diffusion process through a constant threshold. I compute equilibria of the game for the case of mean-reverting cash flows. For the base-case example, the present value of losses due to bankruptcy costs is approximately 10% of asset value under the tough-rating-agency equilibrium and close to zero under the soft-rating-agency equilibrium.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model. Section 4 shows existence of equilibrium in Markov strategies. Section 5 discusses equilibrium selection and the role of stress tests and fee structures in the credit rating industry. Section 6 studies equilibrium stability and discusses the "credit-cliff dynamic." Section 7 studies competition between rating agencies. Section 8 provides some comparative statics results. Section 9 studies the numerical computation of equilibria. Section 10 contains additional discussion and Section 11 concludes. All proofs are in the appendix.

2. Related literature

Previous work on rating agencies has overlooked potential feedback effects of credit ratings, focusing instead on how the conflicts of interest between investors and information intermediaries affect the quality of the information disclosed to the market. Lizzeri (1999) considers the optimal disclosure policy of an information intermediary who can perfectly observe the type of the seller at zero cost, and finds that in equilibrium the information intermediary does not disclose any information. Doherty, Kartasheva and Phillips (2009) and Camanho, Deb and Liu (2010) study how competition between rating agencies affects information disclosed to investors. Bolton, Freixas and Shapiro (2009) and Skreta and Veldkamp (2009) develop models in which rating inflation emerges due to investors behavioral biases. Opp, Opp and Harris (2010) study rating inflation due to preferential-regulatory treatment of highly rated securities. Fulghieri, Strobl and Xia (2010) study the welfare effects of unsolicited credit ratings. One exception is Boot, Milbourn and Schmeits (2006) who consider a model in which credit ratings have a real impact on the firm's choice between a risky and a safe project. In their model, if some investors base their decisions on the announcements of rating agencies, then rating agencies can discipline the firm, inducing first-best project choice.

The paper contributes to the credit risk literature, which can be divided into two classes. In some models, such as Black and Cox (1976), Fischer, Heinkel and Zechner (1989), Leland (1994), Leland and Toft (1996), Duffie and Lando (2001), He and Xiong (2012), and Hugonnier, Malamud and Morellec (2011) default is an endogenous decision of the firm. In other models, default is exogenous. There is either an exogenously given default boundary for the firm's assets (Merton (1974), Longstaff and Schwartz (1995)), or an exogenous process for the timing of bankruptcy, as described in Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997) and Duffie and Singleton (1999).

The current paper belongs to the class of models with endogenous default, which is an important feature to capture the feedback effect of credit ratings. Das and Tufano (1996), Acharya, Das and Sundaram (2002), Houweling, Mentink and Vorst (2004), and Lando and Mortensen (2005) obtain pricing formulas for ratings-based step-up bonds using the second class of models of the valuation of risky debt. Since they examine only an exogenous default process, the effect of credit ratings on the default time is not apparent in their models.

The closest paper in the credit risk literature is Manso, Strulovici and Tchistyi (2010), who study performance-sensitive debt (PSD) with general performance measures. In contrast to Manso, Strulovici and Tchistyi (2010), I restrict attention to ratings-based PSD and focus on the strategic interaction between rating agencies and the borrowing firm. This allows me to study the existence of multiple equilibria and their implications for rating agencies policies and industry regulation.

The model is similar in spirit to models of self-fulfilling crises, such as the bank-run model of Diamond and Dybvig (1983), the currency crises models of Flood and Garber (1984) and Obstfeld (1986), and the debt crises model of Calvo (1988). In these models, there are potentially multiple equilibria and investors' expectations can become self-fulfilling leading to a crisis.[4]

At a broader level, the paper is also related to the literature linking financial markets to corporate finance and demonstrating the real effects of financial markets. Fishman and Hagerty (1989), Leland (1992), Holmstrom and Tirole (1993), Dow and Gorton (1997), Subrahmanyam and Titman (1999), Fulghieri and Lukin (2001), Khanna and Sonti (2004), and Goldstein and Guembel (2008) are examples of papers in this literature.

[4] The equilibrium outcome may result from sunspots (Azariadis (1981), Cass and Shell (1983)). Alternatively, Morris and Shin (1998), Morris and Shin (2004), and Goldstein and Pauzner (2005) apply global games techniques to study how higher-order beliefs may lead to particular equilibrium outcomes.

Several studies provide direct evidence that credit ratings affect the cost of capital for a borrower.[5] There is also indirect evidence on this link. Kisgen (2006) finds that credit ratings directly affect firms' capital structure decisions. Kraft (2010) finds that rating agencies are reluctant to downgrade borrowers whose debt contracts have rating triggers.

3. The model

The model is based on the performance-sensitive debt model introduced by Manso, Strulovici and Tchistyi (2010). A firm generates non-negative cash flows at the rate δ_t, at each time t. I assume that δ is a diffusion process governed by the equation

$$d\delta_t = \mu(\delta_t)dt + \sigma(\delta_t)dB_t, \tag{1}$$

where B is the standard Brownian motion. The drift μ and diffusion σ satisfy classic conditions for the existence of a unique strong solution to (1), which are provided in Appendix Appendix A.

Agents are risk neutral and discount future cash flows at the risk-free interest rate r. The expected discounted value of the firm at time t is:

$$A_t = E_t\left[\int_t^\infty e^{-r(s-t)}\,\delta_s\,ds\right]. \tag{2}$$

For the value of the firm to be finite, I assume that μ is uniformly less than $r - \epsilon$ for some constant $\epsilon > 0$.

The firm has debt in place in the form of a ratings-based performance-sensitive debt (PSD) obligation, which promises a non-negative payment rate that may vary with the credit rating of the firm. Credit ratings are represented by a stochastic process R taking values in $\mathcal{I} = \{1, \ldots, I\}$, with 1

[5] For example, Kliger and Sarig (2000), Kisgen and Strahan (2010), and Chen, Lookman, Schurhoff and Seppi (2010) provide evidence on this link using natural experiments.

the lowest ("C" in Moody's ranking) and I the highest ("Aaa" in Moody's ranking). Formally, a ratings-based PSD obligation $C(\cdot)$ is a function $C : \mathcal{I} \to \mathbb{R}^+$, such that the firm pays $C(R_t)$ to its debtholders at time t with $C(i) \geq C(i+1)$.[6]

Given a rating process R, the firm's optimal liquidation problem is to choose a default time $\widehat{\tau}$ to maximize its initial equity value W_0^C, given the debt structure C. That is,

$$W_0 \equiv \sup_{\widehat{\tau} \in \mathcal{T}} E\left[\int_0^{\widehat{\tau}} e^{-rt}[\delta_t - C(R_t)]\, dt\right] , \tag{3}$$

where $C(R_t)$ is the coupon rate, $\mathcal{T}$ is the set of $\mathcal{F}_t$-stopping times, and $(\mathcal{F}_t)$ is the filtration generated by the standard Brownian motion B. If τ^* is the optimal liquidation time, then the market value of the equity at time $t < \tau^*$ is

$$W_t = E_t\left[\int_t^{\tau^*} e^{-r(s-t)}[\delta_s - C(R_s)]\, ds\right] . \tag{4}$$

Analogously, the market value U_t^C of the ratings-based PSD obligation C at time t is

$$U_t \equiv E_t\left[\int_t^{\tau^*} e^{-r(s-t)} C(R_s)\ ds\right] + E_t\left[e^{-r(\tau^*-t)}\left(A_{\tau^*} - \rho(A_{\tau^*})\right)\right] , \tag{5}$$

where $\rho(A)$ is the bankruptcy cost, which is increasing in A and is less than the asset level at time of default.[7]

To focus on the feedback effects of credit ratings, I abstract away from any conflict of interest between the rating agency and investors. In particular, I

[6] The ratings-based PSD obligation C represents the total debt payment of the borrower. If the firm has a complex capital structure that includes various issues of ratings-based PSD obligations and also fixed-coupon debt, then $C(R_t)$ is the sum of the payments for each of the firm's obligations at time t given the rating R_t at time t. In other words, a complex capital structure consisting of a combination of ratings-based PSD obligations is a ratings-based PSD obligation.

[7] For simplicity, I assume that upon bankruptcy, bondholders take over the firm, which is financed entirely by equity thereafter.

assume that the rating agency prefers to assign accurate ratings to inaccurate ones (and is indifferent among all accurate ratings).[8] Given a default policy $\widehat{\tau}$, a rating process R is *accurate* if

$$R_t = i \text{ whenever } P(\widehat{\tau} - t \leq T \mid \mathcal{F}_t) \in [G_i, G_{i-1}), \tag{6}$$

for fixed time horizon T and target rating transition thresholds $\{G_i\}_{i=0}^{I}$ with $G_0 = 1$, $G_I = 0$, and $G_{i-1} > G_i$.[9] An accurate rating provides information to investors about the probability of default over a given time horizon. Higher ratings correspond to lower default probabilities.

In this setting, the interaction between the borrowing firm and the rating agency produces important feedback effects. With a ratings-based PSD obligation, the rating determines the coupon rate, which affects the optimal default decision of the issuer. This, in turn, influences the rating.

Definition 1. *An equilibrium (τ^*, R^*) is characterized by the following:*

1. *Given the rating process R^*, the default policy τ^* maximizes equity value, i.e. solves* (3)*.*
2. *Given the default policy τ^*, the rating process R^* is accurate, i.e. satisfies* (6)*.*

4. Equilibrium in Markov strategies

The cash flow process δ is a time-homogeneous Markov process. Therefore, the current level δ_t of cash flows is the only state variable in the model.

[8]This would hold if for example the rating agency is concerned exclusively about its reputation which is a function only of whether its rating process is accurate.

[9]The time horizon T and rating transition thresholds $\{G_i\}_{i=0}^{I}$ are exogenous in the model. They could be set to reflect historical default rates associated with each rating. These historical default rates can be obtained, for example, from rating transition matrices published by rating agencies.

I will thus focus on Markov Perfect Equilibria[10] of the game, in which strategies are a function of the current level δ_t of cash flows.

A Markov default policy takes the form $\tau(\delta_B) = \inf\{s : \delta_s \leq \delta_B\}$. Under such policy, default is triggered the first ("hitting") time that the cash flow level hits the threshold δ_B.

A Markov rating policy takes the form of rating transition thresholds $H = \{H_i\}_{i=0}^{I}$ such that $R_t = i$ if $\delta_t \in [H_{i-1}, H_i)$ with $H_i \geq H_{i-1}$, $H_0 = 0$, and $H_I = \infty$. Under such policy, rating transitions happen when the cash flow process crosses specific cash-flow thresholds.

Given a Markov rating policy H, a best-response default policy for the firm is a Markov strategy. Under a Markov rating policy H, the ratings-based PSD obligation C is equivalent to a step-up PSD obligation C^H promising coupon payment $C^H(\delta_t) = C(i)$ if $\delta_t \in [H_i, H_{i-1})$. Manso, Strulovici and Tchistyi (2010) show that, under a step-up PSD obligation C^H, the optimal default policy of the firm takes the form $\tau(\delta_B)$, and provides the following algorithm to compute the optimal default boundary δ_B:

Algorithm 1. *1. Determine the set of continuously differentiable functions that solve the following ODE*

$$\frac{1}{2}\sigma^2(x)W''(x) + \mu(x)W'(x) - rW(x) + x - C^H(x) = 0. \tag{7}$$

at each of the intervals $[H_i, H_{i-1})$. It can be shown that any element of this set can be represented with two parameters, say L_1^i and L_2^i.

2. Determine δ_B, L_1^i, and L_2^i using the following conditions:

a. $W(\delta_B) = 0$ and $W'(\delta_B) = 0$.
b. $W(H_i-) = W(H_i+)$ and $W'(H_i-) = W'(H_i+)$ for $i = 1, \ldots, I$.
c. W' is bounded.

The above conditions give rise to a system of $2I + 1$ equations with $2I + 1$ unknowns ($L_j^i, j \in \{1, 2\}, i \in \{1, \ldots I\}$ and δ_B).

[10] These are subgame perfect equilibria in Markov strategies (Maskin and Tirole (2001)).

On the other hand, for a fixed Markov default policy $\tau(\delta_B)$, an accurate ratings policy is also a Markov strategy. This is due to the fact that δ_t is a sufficient statistic for $P(\tau(\delta_B) - t \leq T \,|\, \mathcal{F}_t)$ for any $t \leq T$. Therefore, the best-response rating transition thresholds H are such that

$$P(\tau(\delta_B) - t \leq T \,|\, \delta_t = H_i) = G_i. \tag{8}$$

Because $P(\tau(\delta_B) - t \leq T \,|\, \delta_t)$ is strictly decreasing and continuous in δ_t, the thresholds H, as defined by (8), exist and are unique. Solving for rating transition thresholds H amounts to computing first-passage time $\tau(\delta_B)$ distributions, which is a classical problem in statistics.[11]

Since best responses to Markov strategies are also Markov strategies, when characterizing the Markov equilibria of the game, without loss of generality, I restrict attention to deviations that are Markov strategies. Therefore, from here on, I represent the default and ratings policies as Markov strategies. A default policy is thus given by some $\delta_B : \mathbb{R}^{I+1} \mapsto \mathbb{R}$ that maps rating transition thresholds into a default boundary $\delta_B(H)$. A rating policy is given by some $H : \mathbb{R} \mapsto \mathbb{R}^{I+1}$ that maps a default boundary into rating transition thresholds $H(\delta_B)$.

For given rating transition thresholds H, the equityholders' optimal problem is to choose the default threshold δ_B that maximizes:

$$\widetilde{W}(\delta_B, H) \equiv E\left[\int_0^{\tau(\delta_B)} e^{-rt}\left[\delta_t - C^H(\delta_t)\right] dt\right],$$

The function $\widetilde{W}(\delta_B, H)$ represents the equity value if the rating agency chooses rating transition thresholds H and equityholders default at the threshold δ_B.

[11]See, for example, Ricciardi, Sacerdote and Sato (1984) for a characterization of this distribution in terms of an integral equation, and Giraudo, Sacerdote and Zucca (2001) for a method to compute the distribution using Monte Carlo simulation.

The set $\mathcal{E}$ of Markov equilibria of the game is given by:

$$\mathcal{E} = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{I+1}; (x, y) = (\delta_B(y), H(x))\}. \tag{9}$$

I now prove existence of Markov equilibria in pure strategies. The key for existence is to establish that best-responses are increasing in the other player's strategy. The next two propositions establish these results.

Proposition 1. *The best-response default policy $\delta_B(H)$ is increasing in the rating transition thresholds H.*

Higher rating transition thresholds H imply lower credit ratings and consequently higher coupon payments. As a result, it is optimal for the firm to default earlier by setting a higher default threshold δ_B.

Proposition 2. *The best-response rating policy $H(\delta_B)$ is increasing in the default threshold δ_B.*

A higher default threshold δ_B translates into earlier default. To remain accurate, the rating agency needs to set higher rating transition thresholds H.

Propositions 1 and 2 show that the game between the rating agency and the borrowing firm is a game of strategic complementarity. The next theorem relies on the results of these two propositions to show existence of pure strategy equilibrium in Markov strategies.

Theorem 1. *The set $\mathcal{E}$ of Markov equilibria has a largest and a smallest equilibrium.*

Theorem 1 shows not only existence of equilibrium, but also that there exist a smallest and a largest equilibrium. Since the smallest equilibrium of the game has a low default boundary and low rating thresholds, I will call it the *soft-rating-agency equilibrium.* Since the largest equilibrium of the game has high rating thresholds and a high default boundary, I will call it

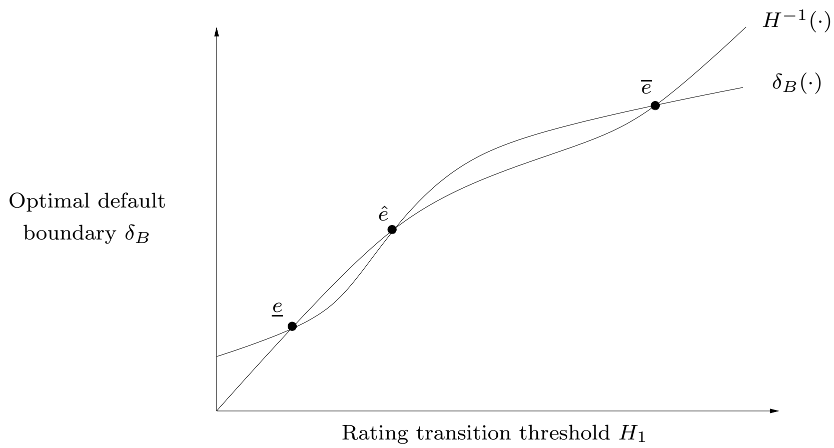

Fig. 1: *The figure plots best-response functions of the rating agency and the borrowing firm for the case in which there are two possible credit ratings ($I = 2$). Points $\underline{e}$, $\hat{e}$, and $\overline{e}$ are Markov equilibria of the game. The soft-rating-agency equilibrium is given by $\underline{e}$, while the tough-rating-agency equilibrium is given by $\overline{e}$. The point $\hat{e}$ corresponds to an intermediate equilibrium.*

the *tough-rating-agency equilibrium*. Fig. 1 plots the best response functions of the rating agency and the borrowing firm as well as the corresponding equilibria of the game for the case in which there are two possible credit ratings ($I = 2$). The tough-rating-agency equilibrium $\overline{e}$ has higher default and rating transition threshold than the soft-rating-agency equilibrium $\underline{e}$.

The following algorithm will be useful in computing equilibria of the game:

Algorithm 2. *Start from $n = 1$ and an arbitrary default boundary, x_0.*

1. *Calculate $H(x_{n-1})$ using Eq. (8), and then $x_n = \delta_B(H(x_{n-1}))$ using Algorithm 1.*

2. *If convergence has been achieved (i.e. $|x_n - x_{n-1}| \leq \epsilon$), output $(x_n, H(x_n))$. Otherwise, return to step 1 with $n = n + 1$.*

Proposition 3. *Algorithm 2 always converges to an equilibrium of the game. It converges to the soft-rating-agency equilibrium, if started from $x_0 = \delta_B(0, \ldots, 0)$, and to the tough-rating-agency equilibrium, if started from $x_0 = \delta_B(\infty, \ldots, \infty)$*

Algorithm 2 can thus be used to find out whether the game has a unique equilibrium.

Corollary 1. *The game has a unique Markov equilibrium if and only if Algorithm 2 yields the same equilibrium if started from* $x_0 = \delta_B(0, \ldots, 0)$ *or* $x_0 = \delta_B(\infty, \ldots, \infty)$.

Convergence of the algorithm to the same equilibrium point when started from $x_0 = \delta_B(0, \ldots, 0)$ or $x_0 = \delta_B(\infty, \ldots, \infty)$ is a necessary and sufficient condition for uniqueness.

If the capital structure of the firm can be represented by a fixed-coupon consol bond, there is no feedback effect of credit ratings on the firm. The following proposition shows that in this case equilibrium is unique.

Proposition 4. *If* C *is a fixed-coupon consol bond (i.e.* $C(i) = c$ *for all* i*), then the equilibrium is unique.*

The case of a fixed-coupon consol bond is a canonical model in the credit risk literature (Black and Cox (1976), Leland (1994)). In this model, there are no feedback effects of credit ratings. Rating agencies are merely observers trying to estimate the first-passage-time distribution through a constant threshold. The main departure of the current paper from this canonical model is that ratings affect credit quality, creating a circularity problem that makes the task of rating agencies more difficult. When credit ratings affect credit quality, multiple equilibria may exist, in which case there is more than one accurate rating policy that can be selected by the rating agency.

5. Equilibrium selection and welfare

The previous section shows that multiple equilibria may result from the interaction between the rating agency and the borrowing firm. An important question is which equilibrium is more likely to be selected in practice and what are the implications for social welfare.

Since in equilibrium ratings are always accurate, the only welfare losses in the model arise from bankruptcy costs. A higher equilibrium default boundary is thus associated with lower welfare due to higher bankruptcy costs. The following proposition summarizes this result.

Proposition 5. *Equilibria of the game are Pareto-ranked. The tough-rating-agency equilibrium is the worst equilibrium, while the soft-rating-agency equilibrium is the best equilibrium.*

To maximize total welfare, a rating agency should always select the soft-rating-agency equilibrium. In practice, though, rating agencies may fail in some instances to select the soft-rating-agency equilibrium. One reason could be simply because correctly understanding and incorporating the feedback effects of credit ratings is difficult. For example, in December 2001, a few days after the collapse of Enron, which had exposure to several rating triggers, Standard and Poor's issued a report explaining its policy on rating triggers:[12]

> How is the vulnerability relating to rating triggers reflected all along in a company's ratings? Ironically, it typically is not a rating determinant, given the circularity issues that would be posed. To lower a rating because we might lower it makes little sense – especially if that action would trip the trigger!

Ignoring rating triggers will clearly lead to inaccurate ratings. Otherwise equal firms with different exposure to rating triggers will default at different times, but will be given the same rating if their exposure to rating triggers is ignored. Almost three years after the earlier report, in October 2004, S&P republished the report, with a correction to reflect its more recent view that vulnerability relating to rating triggers can be reflected all along in a company's ratings, but that there remains questions over circularity.

[12] "Playing Out the Credit-Cliff Dynamic," Standard and Poor's, December 2001.

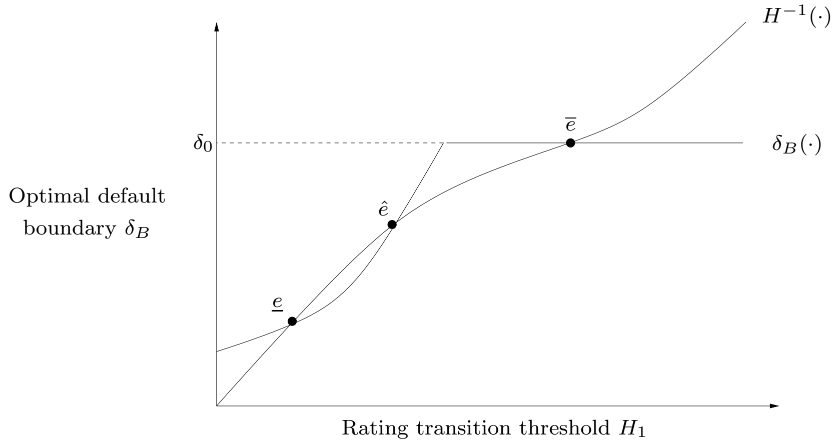

Fig. 2: *The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibria for the case in which there are two possible credit ratings ($I = 2$). In the situation illustrated in the figure, the borrowing firm would fail a stress test, since the tough-rating-agency equilibrium $\overline{e}$ involves immediate default. The firm would survive if the rating agency selected the soft-rating-equilibrium.*

Rating agencies may also fail to select the soft-rating agency equilibrium due to public pressure. As a response to the widespread criticism towards rating agencies in the aftermath of Enron's collapse, Moody's has clearly indicated that it would take rating triggers into account when assigning credit ratings. In a July 2002 report,[13] Moody's explained that from that point on it would require issuers to disclose any rating triggers and would incorporate the serious negative consequences of rating triggers in its ratings by conducting stress tests with firms that have exposure to such triggers. In these stress tests, to avoid downgrades, firms would need to be able to survive stress-case scenarios in which rating triggers are set off.

There is some ambiguity on how such stress tests are to be implemented in practice. However, in the presence of feedback effects, it is clear that assign-

[13] "Moody's Analysis of US Corporate Rating Triggers Heightens Need for Increased Disclosure," Moody's, July 2002.

ing credit ratings based on stress-case scenarios in which rating triggers are set off may produce unnecessary financial distress. Fig. 2 illustrates, for the case of two possible credit ratings ($I = 2$), a situation in which downgrades can be avoided without negative implications for accuracy even though under a stress-case scenario the firm would immediately default.[14] In this example, the rating agency will find that under the rating thresholds associated with the tough-rating-agency equilibrium the borrowing firm would default immediately, failing thus the type of stress test described above. However, under the soft-rating-agency equilibrium, ratings would be accurate and the firm would survive, implying higher welfare. The main point is that, while credit rating agencies should take account of rating triggers in assigning ratings, doing so makes it even more important that they choose the soft equilibrium.

To obtain the Pareto-preferred soft-rating-agency equilibrium, the objective function of the rating agency should incorporate, in addition to accuracy, some other concern. Among all equilibria, the soft-rating-agency equilibrium has the lowest default threshold, and consequently the lowest probability of default over a given horizon. Therefore, a concern about the survival of the borrowing firm may lead the rating agency to select the soft-rating-agency equilibrium.

One way this can be implemented in practice is by having the borrowing firm pay a small fee to the rating agency in exchange for its services. The rating agency would receive this fee continuously until the borrowing firm defaults. In the limit, as this fee gets close to zero, the rating agency's preference becomes lexicographic, so that it is concerned about rating accuracy in the first place and minimizing the probability of default of the borrowing

[14] In the figure, the flat part of the best response function corresponds to points in which the optimal default boundary δ_B given by Algorithm 1 is higher than the current cash flow level δ_0.

firm in the second place. Under this scheme, rating agencies would select the soft-rating equilibrium, since, among all accurate rating policies, it is the one that minimizes the probability of default, and thus maximizes the present value of fee payments.

The above scheme is in fact close to how the credit ratings industry is currently organized. For a rating agency, potential reputational losses from setting inaccurate ratings are likely to be much more important than the fees they receive from any individual issuer.[15] As noted by Thomas McGuire, former VP of Moody's, "what's driving us is primarily the issue of preserving our track record. That's our bread and butter."[16]

The fact that rating agencies are paid by the firms they rate has received intense criticism. The concern is that this practice may induce bias in favor of issuers. While this is a valid concern, the results of this paper suggest that small fees paid by issuers to the rating agencies may induce rating agencies to select the Pareto-preferred soft-rating-agency equilibrium, without introducing significant biases.

6. Stability and the credit-cliff dynamic

In this section, I study equilibrium stability and its implications for credit ratings. The following proposition analyzes the special case in which equilibrium is unique.

Proposition 6. *If the game has a unique Markov equilibrium, it is globally stable in terms of best-response dynamics.*

[15]Using corporate bond prices and ratings, Covitz and Harrison (2003) find evidence supporting the view that rating agencies are motivated primarily by reputation-related incentives. In contrast, He, Qian and Strahan (2010) and Cornaggia, Cornaggia and Hund (2012) find evidence that rating agencies reward large issuers of structured products by granting them unduly favorable ratings. In structured products markets, there are a small number of large issuers, weakening the reputational incentives.

[16]Institutional Investor, 10-1995, "Ratings Trouble."

Proposition 6 implies that if one starts from any Markov strategy, iterative best-response dynamics will lead to the unique equilibrium of the game. Milgrom and Roberts (1990) show that stability also holds with respect to several other types of learning dynamics. Therefore, when the equilibrium is unique, small perturbations to the parameters of the model or to the responses of players will only have a small impact on the equilibrium outcome, so that changes in credit ratings will be gradual.

As shown in the previous sections, however, the model does not always produce a unique equilibrium. Because this is a game of strategic complementarity there will typically exist multiple equilibria. When there are multiple equilibria, some of them may be unstable.[17] As such, small perturbations to the parameters of the model or to the responses of players may lead to large shifts in the equilibrium outcome. Multi-notch downgrades or even immediate default of highly rated corporations as response to small shocks are thus possible.

Fig. 3 illustrates one situation in which this happens for the case of two possible credit ratings ($I = 2$). In the figure, the soft-rating-agency equilibrium is locally unstable. Small perturbations to the best-response of either players may generate best-response dynamics that resemble what has been described as "credit-cliff dynamic." Starting from the soft-rating-agency equilibrium $\underline{e}$, if the rating agency becomes slightly tougher by increasing its ratings transition thresholds H, the firm's optimal response is to increase its default threshold δ_B. This in turn makes rating-agencies increase ratings thresholds even further. The credit-cliff dynamic only stops when the tough-rating-agency equilibrium is reached. In the situation depicted in Fig. 3, the tough-rating-agency equilibrium involves immediate default. Therefore, in

[17]For example, in Fig. 1 the soft-rating-agency equilibrium $\underline{e}$ and the tough-rating-agency equilibrium is given by $\overline{e}$ are locally stable, while equilibrium $\hat{e}$ is unstable in the sense of iterated best-response.

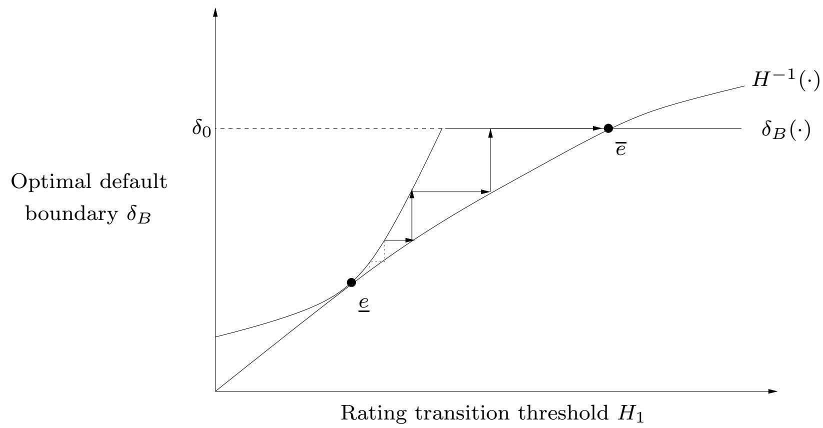

Fig. 3: *The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibria for the case in which there are two possible credit ratings ($I = 2$). The soft-rating-agency equilibrium $\underline{e}$ is unstable. Small shocks may produce a "credit-cliff dynamic" that leads to the tough-rating-agency equilibrium $\overline{e}$, which in this case involves immediate default.*

this case, the credit-cliff dynamic produces a "death spiral."

One may argue that situations such as the one illustrated by Fig. 3 are not generic because they require $H^{-1}(\cdot)$ to be exactly tangent to $\delta_B(\cdot)$ at the soft-rating-agency equilibrium point. Fig. 4 depicts, for the case of two possible credit ratings, a situation in which both the soft-rating-agency and the tough-rating-agency equilibrium are locally stable, but a small unanticipated shock to some parameter of the model (such as an increase in the discount rate r) makes the soft-rating-agency equilibrium $\underline{e}$ and the intermediate equilibrium $\hat{e}$ disappear. The only remaining equilibrium is the tough-rating-agency equilibrium. Small shocks to fundamentals may thus lead to multi-notch downgrades or even immediate default of a highly rated firm.

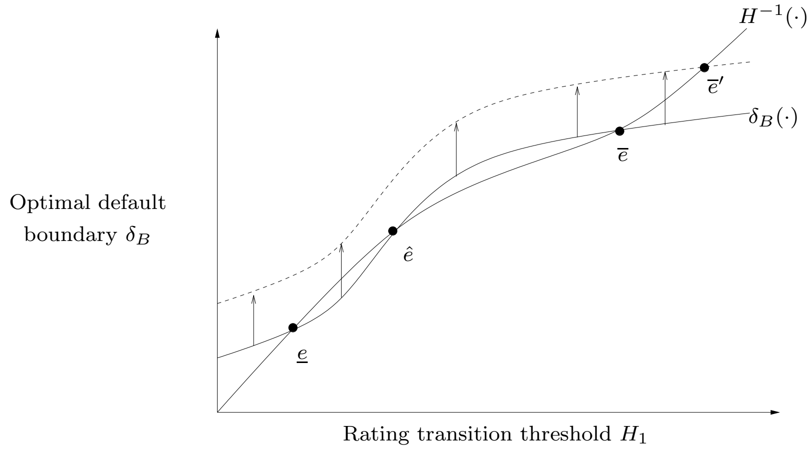

Fig. 4: *The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibria for the case in which there are two possible ratings ($I = 2$). A small shock to fundamentals may eliminate all equilibria except for the tough-rating-agency equilibrium $\overline{e}'$, leading to a multi-notch downgrade or even immediate default.*

7. Competition between rating agencies

In this section, I consider competition between rating agencies. The model is similar to the model considered in previous sections except that there are now two rating agencies $k \in \{1, 2\}$. Rating agency k assigns a rating R_t^k to the borrowing firm at each time t. The ratings-based PSD obligation C promises payments $C(R_t^1, R_t^2)$ from the borrowing firm to debtholders at each time t. The promised coupon payments are assumed to be decreasing in the credit ratings R_t^1 and R_t^2. Firms with higher ratings face lower coupon payments.

As in the base model, a rating agency prefers to assign accurate ratings to inaccurate ones. However, perhaps due to competition for market share, each rating agency is also concerned about assigning more accurate ratings than the other rating agency. As such, a rating agency preferred outcome is that it assigns accurate ratings while the other rating agency assigns inaccurate ratings.

As in the previous sections, I focus on Markov Perfect Equilibria of the game. The choice of rating transition thresholds $H = (H^1, H^2)$ by rating agencies 1 and 2 induces a step-up PSD obligation C^H promising payments $C^H(\delta_t) = C(i,j)$ whenever $\delta_t \in [H^1_i, H^1_{i-1}) \cap [H^2_j, H^2_{j-1})$. The optimal default threshold is of the form $\tau(\delta_B)$ and depends on the rating transition thresholds $H = (H^1, H^2)$ of both rating agencies.

Lemma 1. *With a ratings-based PSD obligation C whose coupon depends on R^1_t and R^2_t, any equilibrium involves rating agencies choosing symmetric rating transition thresholds ($H^1 = H^2$). The firm default boundary δ_B and the rating transition thresholds H^1 or H^2 are in the equilibrium set $\mathcal{E}$ of the game with a single rating agency.*

In equilibrium, the two rating agencies will choose the same rating transition thresholds ($H^1 = H^2$), which are in the equilibrium set $\mathcal{E}$ of the game with a single rating agency. However, not all equilibrium threholds H in $\mathcal{E}$ survive deviations by a single rating agency. To study this issue it becomes important to understand how coupon payments are determined when ratings are split (i.e. $R^1_t \neq R^2_t$).

If the ratings-based PSD obligation is induced by explicit contracts such as in the case of rating triggers, it is easy to find out the criterion to be applied when ratings are split. For a sample of bank loan contracts containing explicit rating triggers between 1993 and 2008, Wiemann (2010) manually checked 50 randomly selected contracts and found that 22 contracts used the highest rating, 20 contracts used the lowest rating, and the remaining 8 contracts used an average rating.[18]

Formally, the ratings-based PSD obligation C relies on the minimum rating if its promised payment depends only on $\min[R^1_t, R^2_t]$. It relies on the maximum rating if its promised payment depends only on $\max[R^1_t, R^2_t]$. The

[18] According to Wiemann (2010), the most common average is $(R^1_t + R^2_t)/2$ rounded to the higher rating.

next proposition studies equilibria of the model with rating agency competition when the ratings-based PSD contract relies on the minimum or maximum of the two ratings.[19]

Proposition 7. *If the ratings-based PSD obligation C relies on the minimum (maximum) of the ratings, then the unique Markov equilibrium of the game is the tough-rating-agency (soft-rating-agency) equilibrium.*

Therefore, the effects of competition depend on how the rating triggers are specified in the contract. In particular, the way in which rating splits are resolved has an important impact on the equilibrium outcome. Under contracts that rely on the minimum of the ratings, rating agencies cannot coordinate on any equilibrium other than the tough-rating-agency equilibrium. If they try to coordinate on any other equilibrium, one rating agency would have an incentive to deviate to a rating policy associated with a tougher equilibrium, affecting the default threshold of the borrowing firm and making the rating policy of the other agency inaccurate. Therefore, only the tough-rating-agency equilibrium survives under contracts that rely on the minimum of the two credit ratings. By a similar argument, under contracts that rely on the maximum of the two ratings, only the soft-rating-agency equilibrium survives.[20]

Even though, according to Wiemann (2010), the vast majority of the contracts rely on either the maximum or the minimum credit rating, it would be interesting to understand what happens when interest payments depend on some average of the available ratings. In an earlier version of the paper, I show that if a single rating agency can drive the firm to immediate default by

[19] The restriction to Markov Perfect Equilibrium is important here. If one allows for strategies that depend on the whole history of the game, sufficiently patient rating agencies would be able to sustain coordination of any equilibrium in $\mathcal{E}$.

[20] Propositions 7 together with Proposition 5 imply that rating triggers should be specified using the maximum rating if more than one rating is assigned. In Section 10, I discuss other frictions that could justify the use of alternative contractual forms.

adopting the rating transition thresholds associated with the tough-rating-agency equilibrium, then the only equilibrium that survives is the tough-rating-agency equilibrium. The intuition and proof of this result are similar to the ones in Proposition 7.

8. Comparative statics

In this section, I study how the tough-rating-agency equilibrium and the soft-rating-agency equilibrium respond to changes in some of the parameters of the model.

Proposition 8. *The equilibrium default boundary δ_B and rating transition thresholds H associated with the tough-rating-agency equilibrium and the soft-rating-agency equilibrium are*

1. *increasing in the coupon payments C.*
2. *increasing in the interest rate r.*
3. *decreasing in the drift $\mu(\cdot)$ of the cash flow process.*
4. *decreasing in the target rating transition thresholds G.*

These comparative statics are easy to see graphically taking for example Fig. 1 as a starting point. Increases (decreases) in coupon payments C or interest rate r do not affect the rating agency's best response curve $H^{-1}(\cdot)$ but shift the equityholders' best response curve $\delta_B(\cdot)$ up (down) leading to higher (lower) soft- and tough-rating-agency equilibria. Increases (decreases) in target transition thresholds do not affect the equityholders' best response curve $\delta_B(\cdot)$ but shift the rating agency's best response curve $H^{-1}(\cdot)$ left (right) leading to higher (lower) soft- and tough-rating-agency equilibria. Increases (decreases) in the drift $\mu(\cdot)$ shift the equityholders' best response curve $\delta_B(\cdot)$ down (up) and the rating agency's best response curve $H^{-1}(\cdot)$ left (right) leading to higher (lower) soft- and tough-rating-agency equilibria.

It is important to notice that these are comparative statics on the set of equilibria. They admit situations such as the one described in Fig. 4, in which a shock to parameters of the model (for example, an increase in interest rate r) reduces the number of equilibria from three to one. In this particular example, the tough- and soft-rating-agency equilibrium increase with the shock, as the only remaining equilibrium is both the tough- and soft-rating-agency equilibrium and is higher than any of the three former equilibria.

9. Equilibrium computation

In this section, I compute the best-response functions δ_B and H and equilibria when the cash flow process δ is a geometric Brownian motion or a mean-reverting process. The computation of the default threshold δ_B involves solving an ordinary differential equation, while the computation of the rating transition thresholds H involves computing the first-passage time distribution through a constant threshold. Equilibria of the game can then be computed by best-response iteration as explained in Algorithm 2.

Geometric Brownian Motion. When the cash flow process δ of the firm follows a geometric Brownian motion,

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dB_t, \tag{10}$$

equilibrium of the game is unique and can be solved in closed-form. This example is discussed in Manso, Strulovici and Tchistyi (2010). In Appendix Appendix C, I show how to compute numerically the best response functions δ_B and H and then how to find the equilibria of the game when the cash flow process follows a geometric Brownian motion.

Fig. 5 plots the best-response and the corresponding unique equilibrium of the game when the cash flow process is a geometric Brownian motion. As shown in Appendix Appendix C, there is always a unique equilibrium in this case.

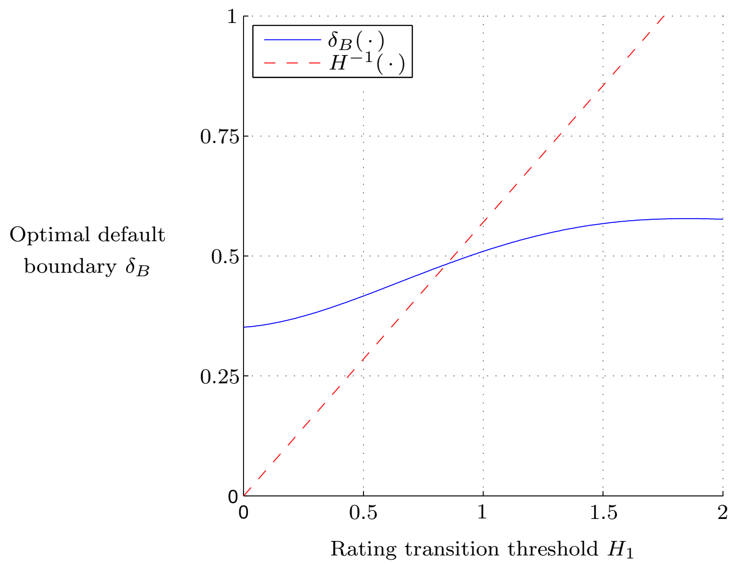

Fig. 5: *The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibrium for the case in which the cash flow process follows a geometric Brownian motion. The parameters used to plot the figure are* $r = 0.06$, $\mu = 0.02$, $\sigma = 0.25$, $I = 2$, $c_1 = 1$, $c_2 = 1.5$, *and* $G_1 = 2\%$.

Mean-reverting process. I now assume that the cash-flow process δ follows a mean-reverting process with proportional volatility:

$$d\delta_t = \lambda(\mu - \delta_t)dt + \sigma\delta_t dB_t \tag{11}$$

where λ is the speed of mean reversion, μ is the long-term mean earnings level to which δ reverts, and σ is the volatility.

In contrast to the case of a geometric Brownian motion, a mean-reverting cash flow process allows for transitory and permanent shocks. As Bhattacharya (1978) notes, "…mean-reverting cash flows are likely to be more relevant than the extrapolative random walk process in Myers and Turnbull (1977) and Treynor and Black (1976) for sound economic reasons. In a competitive economy, we should expect some long-run tendency for project cash flows to revert to levels that make firms indifferent about new investments

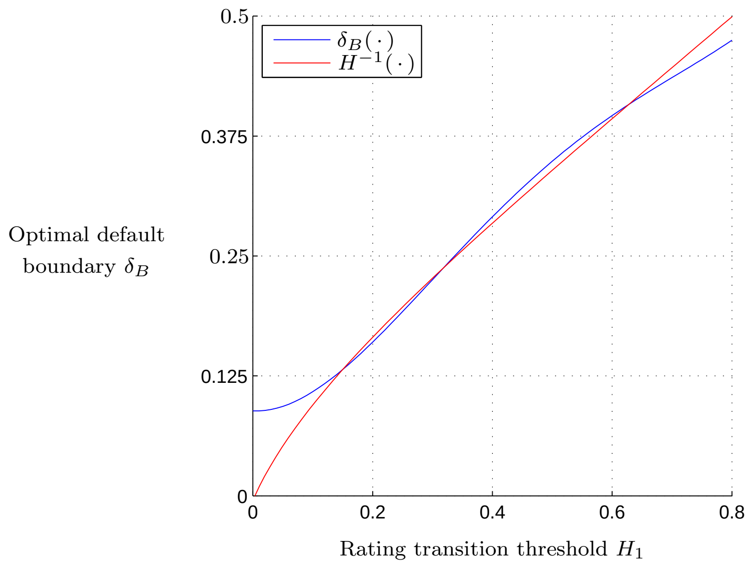

Fig. 6: *The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibrium for the case in which the cash flow process follows the mean-reverting process* (11)*. The parameters used to plot the figure are* $r = 0.06$*,* $\lambda = 0.15$*,* $\mu = 1$*,* $\sigma = 0.4$*,* $I = 2$*,* $c_1 = 1.3$*,* $c_2 = 0.75$*, and* $G_1 = 20\%$*.*

in the particular type of investment opportunities that a given project represents, rather than 'wandering' forever." Several empirical studies indeed find that earnings are mean-reverting (Freeman, Ohlson and Penman (1982), Kormendi and Lipe (1987), Easton and Zmijewski (1989), Fama and French (2000)).

Sarkar and Zapatero (2003) study the optimal default decision of equity-holders when cash flows follow a mean-reverting process and the firm issues a consol bond with fixed coupon payments c. Here I consider the situation in which the firm issues a ratings-based PSD obligation C. In Appendix Appendix C, I show how to compute numerically the best response functions δ_B and H and then how to find the equilibria of the game when the cash flow process follow a mean-reverting process.

Fig. 6 plots the best response functions in case the cash flows follow the mean-reverting process (11). For this particular example there are three possible equilibria. Under the soft-rating-agency equilibrium, the present value of bankruptcy costs are close to zero. In contrast, under the tough-rating-agency equilibrium, the present value of bankruptcy costs corresponds to 10% of the firm asset value when upon bankruptcy 20% of the firm asset value is lost ($\rho(x) = 0.2x$). This shows that the selection of equilibria by the rating agency can have a big impact on welfare.

10. Additional discussion

In the model, bankruptcy costs are the only source of inefficiency. For this reason, the soft-rating-agency equilibrium is always preferred to the tough-rating-agency equilibrium. In practice, however, there may be other frictions, such as agency costs, that could justify the use of other forms of contracts. For example, default may be welfare-enhancing if it fosters the replacement of negligent managers (Jensen (1993)). In such setting, the tough-rating-agency equilibrium would induce higher welfare than the soft-rating-agency equilibrium.

The paper studies the rating agency's problem of assigning credit ratings taking as given that the borrower has issued performance-sensitive debt (PSD). To justify why borrowers issue performance-sensitive debt, I rely on previous work, which has argued that borrowers issue performance-sensitive debt in response to adverse selection (Manso, Strulovici and Tchistyi (2010)) or moral hazard (Tchistyi (2011)) problems.[21] Therefore, assuming these benefits associated with performance-sensitive debt, firms will choose to issue at least some performance-sensitive debt even when they anticipate a positive probability of the tough-rating-agency equilibrium being selected.

[21] Flannery (1986) and Diamond (1991) obtain similar results for the case of short-term debt, whose rollover makes it implicitly performance-sensitive.

This could happen if the equilibrium selection by the rating agency depended on some random variable, such as public pressure for rating agencies to be tougher or even sunspots.[22] The random variable need not convey anything fundamental about the borrower. Once the rating agency assigns tougher ratings, default of the borrower becomes more likely and being tougher is accurate for the rating agency.

11. Conclusion

After the recent crisis, governments have recognized the significant market impact of rating agencies. To mitigate this impact, they have proposed changes that reduce the reliance of regulation and supervisory practices on credit ratings.[23] To the extent that credit ratings are informative, market participants will rely on credit ratings, introducing the feedback effects studied in this paper.

Rather than proposing ways to eliminate the feedback effects of credit ratings, I analyze the consequences of different regulations and practices of the credit rating industry in the presence of feedback effects. I show that rating agencies that have a small bias towards the survival of the borrower, which can be achieved via the issuer-pay model, are likely to select the Pareto-preferred soft-rating-equilibrium. Stress tests, on the other hand, may lead to the selection of the Pareto-dominated tough-rating-agency equilibrium. Even if rating agencies pursue an accurate rating policy, multi-notch downgrades or immediate default may occur as responses to small shocks to fundamentals. Increased competition between rating agencies may lead to rating downgrades, increasing default frequency and reducing welfare.

[22] Azariadis (1981) and Cass and Shell (1983) provide a formal analysis of this point in a general setting. Diamond and Dybvig (1983) apply this idea to the context of bank runs.

[23] "Principles for Reducing Reliance on CRA Ratings," Financial Stability Board, 27 October 2010; "Dodd-Frank Wall Street Reform and Consumer Protection Act," (Pub.L. 111–203, H.R. 4173, Section 939), 21 July 2010.

The model specification is flexible to capture realistic cash-flow processes, and thus potentially allows rating agencies and other market participants to incorporate the feedback effects of credit ratings into debt valuation and rating policies.[24] Numerical examples suggest significant welfare implications. In the base-case example with mean-reverting cash flows, I find that the present value of bankruptcy losses in the tough-rating-agency equilibrium is substantially higher than in the soft-rating-agency equilibrium.

There are several unanswered questions. One question involves the effects of rating agencies on systemic risk. Rating downgrades of one firm could create pressure for the downgrades of other firms, in a form of feedback effect not studied in the current paper. It would also be interesting to study the interactions of investment decisions of the firm with the rating policy of the credit rating agency. Finally, it would be interesting to understand how global games may affect the selection of equilibrium in the setting studied in this paper. I leave these questions for future research.

[24] The model follows the tradition of the credit risk literature (Merton (1974), Black and Cox (1976), Leland (1994)) and is similar to models used by investors and rating agencies, such as the Moody's KMV model, but it incorporates the feedback effects of credit ratings.

Appendices

Appendix A. Technical conditions

The following technical conditions guarantee the existence of a unique strong solution to Eq. (1).[25]

Condition 1. *The drift μ and diffusion σ of the cash flow process* (1) *satisfy:*

$$|\mu(x) - \mu(y)| \leq K|x - y|, \tag{A.1}$$

$$|\sigma(x) - \sigma(y)| \leq h(|x - y|) \tag{A.2}$$

for every $x, y \in [0, \infty)$, where K is a positive constant and $h : [0, \infty) \mapsto [0, \infty)$ is a strictly increasing function with $h(0) = 0$ and

$$\int_{(0,\epsilon)} h^{-2}(u)du = \infty, \forall \epsilon > 0 \tag{A.3}$$

Appendix B. Proofs

Proof of Proposition 1: It is enough to show that the firm's equity value $\tilde{W}(\delta_B, H)$ has increasing differences in δ_B and H. If $H' \geq H$,

$$\widetilde{W}(\delta, \delta_B, H') - \widetilde{W}(\delta, \delta_B, H) = E_x \left[\int_0^{\tau(\delta_B)} e^{-rt} \left[C^H(\delta_t) - C^{H'}(\delta_t) \right] dt \right] \tag{B.1}$$

is increasing in δ_B, since $C^H(\delta_t) - C^{H'}(\delta_t) \leq 0$. ■

Proof of Proposition 2: It follows from the fact that $P(\tau(\delta_B) \leq T \,|\, \mathcal{F}_t)$ is increasing in δ_B. ■

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[25] See e.g. Karatzas and Shreve (1991, pp. 291-292).

Proof of Theorem 1: Let the function $F : \mathbb{R}^{I+1} \times \mathbb{R} \mapsto \mathbb{R} \times \mathbb{R}^{I+1}$ be such that $F(x, y) = (\delta_B(y), H(x))$. From Propositions 1 and 2, F is monotone. The set $\mathcal{E}$ correspond to fixed points $(x, y) = F(x, y)$. Let Y be such that

$$Y = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{I+1}; 0 \leq x \leq \delta_B(\infty, \ldots \infty) \\ \text{and } (0, \ldots, 0) \leq y \leq H(\delta_B(\infty, \ldots \infty))\}.$$

The set Y is a complete lattice with the usual partial order on Euclidean spaces. The function $G = F|_Y$ maps Y into Y and is monotone. By the Tarski fixed point theorem, the set $\mathcal{E}$ of Markov equilibria is a complete lattice. ■

Proof of Proposition 3: Because δ_B and H are increasing, the sequence $\{x_n\}$ produced by Algorithm 2 is either increasing or increasing. Since the sequence is bounded above by $\delta_B(\infty, \ldots, \infty)$ and bounded below by 0, it must converge to some point e. The claim is that $(e, H(e))$ is an equilibrium of the game. Let $y \in \mathbb{R}$ be any other default strategy for the borrowing firm and take any sequence $\{y_n\}$ converging to y. By construction,

$$W(y, H(e)) = \lim_{n \to \infty} W(y_n, H(x_{n-1}) \leq \lim_{n \to \infty} W(x_n, H(x_{n-1})) = W(e, H(e))$$

where the first and last equality follow from the continuity of H and W. Therefore $(e, H(e))$ is an equilibrium of the game.

It remains to show that if $x_0 = \delta_B(0, \ldots, 0)$, then the algorithm converges to the lowest equilibrium $(\underline{e}, H(\underline{e}))$ of the game. If $(e, H(e))$ is any other element of $\mathcal{E}$, $x_0 \leq e$, and $x_n \leq e$ implies $x_{n+1} = \delta_B(H(x_n)) \leq \delta_B(H(e)) = e$. By induction, $(\underline{e}, H(\underline{e}))$ is the smallest element in $\mathcal{E}$.

The proof of convergence of the algorithm to the largest equilibrium when $x_0 = \delta_B(\infty, \ldots, \infty)$ is symmetric. ■

Proof of Proposition 4: If C is a fixed-coupon consol bond paying coupon c, then

$$\widetilde{W}(\delta, \delta_B, H) \equiv E_x \left[\int_0^{\tau(\delta_B)} e^{-rt} \left[\delta_t - c\right] dt \right],$$

does not depend on H. Therefore, the default policy $\delta_B(H)$ that maximizes $\widetilde{W}(\delta, \delta_B, H)$ does not depend on H, and Algorithm 2 must converge to the same point in one iteration when started from either $x_0 = \delta_B(0, \ldots, 0)$ or $x_0 = \delta_B(\infty, \ldots, \infty)$. ■

Proof of Proposition 6: From Proposition 3, the sequence produced by an algorithm that iterates best-response functions converges to an equilibrium if started from any default threshold x_0. Therefore, if the equilibrium of the game is unique, it is globally stable. ■

Proof of Lemma 1: The proof is by contradiction. Suppose there was an equilibrium in which $H^1 \neq H^2$. Then it must be the case that $H^1 \neq H(\delta_B(H^1, H^2))$ or $H^2 \neq H(\delta_B(H^1, H^2))$. Suppose, without loss of generality, that rating agency 1 is inaccurate (i.e. $H^1 \neq H(\delta_B(H^1, H^2))$). One needs to show that it can improve its ratings.

For a fixed H^2, $\delta_B(H^1, H^2)$ is increasing in H^1 since $C(i, j)$ is decreasing in i, and the problem becomes similar to the one studied in Section 4. For a fixed H^2, let $\widehat{\mathcal{E}}$ be the set of equilibria δ_B and H^1. It follows from Theorem 1 that $\widehat{\mathcal{E}}$ is non-empty. Therefore, given H^2, there exists an accurate policy for rating agency 1, making this a profitable deviation. ■

Proof of Proposition 7: Suppose that ratings-based PSD obligation C relies on the minimum of the ratings. From Lemma 1, the only possible equlibria are in the set $\mathcal{E}$ and involve rating agencies playing symmetric strategies. Let $\overline{e} = (\overline{\delta_B}, \overline{H})$ correspond to the tough-rating-agency equilibrium. Suppose that there exists an equilibrium of the game with $(\widetilde{\delta_B}, \widetilde{H}) \neq (\overline{\delta_B}, \overline{H})$. Rating agency 1 could then deviate and choose $H^1 = \overline{H}$. Because C relies on the minimum of the ratings, and $\overline{H} \geq \widetilde{H}$, under this deviation, rating agency 1 would have accurate ratings while rating agency 2 would have inaccurate ratings.

It remains to show that the tough-rating-agency equilibrium is indeed an equilibrium. If agency 2 selects ratings thresholds $H^2 = \overline{H}$, then agency

1 cannot do better than selecting $H^1 = \overline{H}$. Any deviation $H^1 \leq \overline{H}$ would make its ratings inaccurate, since the default boundary would stay at $\overline{\delta_B}$. Any deviation $H^1 \geq \overline{H}$ would also make its ratings inaccurate, since even though it could move the default boundary to a level higher than $\overline{\delta_B}$, H^1 would not be accurate by the definition of the tough-rating-agency equilibrium. Finally, deviations in which $H_i^1 < \overline{H_i}$ for some i and $H_i^1 \geq \overline{H_i}$ for some i cannot lead to accurate ratings either since they would move the default boundary to a higher level than $\overline{\delta_B}$, but for some i the rating transition threshold H_i^1 would be lower than $\overline{H_i}$, the accurate rating transition threshold under $\overline{\delta_B}$.

The proof for when the ratings-based PSD obligation C relies on the maximum of the ratings is similar. ■

Proof of Proposition 8: It is enough to show that the best-response functions δ_B and H increase when there is an increase in the parameter of interest. If this is the case, the sequence produced by Algorithm 2 under the higher parameter will be greater than or equal to the sequence produced by Algorithm 2 under the lower parameter. Since the soft-rating-agency and the tough-rating-agency equilibrium are the limits of such sequences, they will also be higher under the higher parameter.

I first study comparative statics with respect to C. To show that the best response function δ_B is increasing in C it is enough to show that the firm's equity value $\tilde{W}(\delta_B, H; C)$ has increasing differences in δ_B and C. If $\hat{C} \geq C$,

$$\widetilde{W}(\delta_B, H; \hat{C}) - \widetilde{W}(\delta_B, H; C) = E\left[\int_0^{\tau(\delta_B)} e^{-rt}\left[C^H(\delta_t) - \hat{C}^H(\delta_t)\right] dt\right] \quad \text{(B.2)}$$

is increasing in δ_B, since $C^H(\delta_t) - \hat{C}^H(\delta_t) \leq 0$. On the other hand, the best-response function H is unaffected by changes in C.

Next, I study comparative statics with respect to r. Theorem 2 of Quah and Strulovici (2010) guarantees that δ_B is increasing in r. On the other hand, the best-response function H is not affected by changes in r.

Next, I study comparative statics with respect to $\mu(\cdot)$. To show that δ_B is decreasing in $\mu(\cdot)$ it is enough to show that the firm's equity value $\tilde{W}(\delta_B, H; \mu)$ has increasing differences in δ_B and $-\mu$. Let $\hat{\mu} \geq \mu$ and δ_t $(\hat{\delta}_t)$ be the cash-flow process under μ $(\hat{\mu})$. We then have that

$$\widetilde{W}(\delta_B, H; \hat{\mu}) - \widetilde{W}(\delta_B, H; \mu) = \\ E\left[\int_0^{\tau(\delta_B)} e^{-rt}\left\{\left[\hat{\delta}_t - \delta_t\right] + \left[C^H(\delta_t) - C^H(\hat{\delta}_t)\right]\right\} dt\right],$$

is decreasing in δ_B, since C^H is decreasing and $\hat{\delta}_t \geq \delta_t$ in every path of B_t. The rating transition thresholds H are decreasing in $\mu(\cdot)$ since $\hat{\delta}_t \geq \delta_t$ for every path of B_t.

Finally, I study comparative statics with respect to G. The best-response function δ_B is unaffected by changes in G. The rating transition thresholds H are decreasing in G, since $P(\tau(\delta_B) \leq T \mid \mathcal{F}_t)$ is decreasing in δ_t. ■

Appendix C. Particular cash-flow processes

Geometric Brownian Motion. Based on Algorithm 1, the equity value W and default threshold δ_B under a step-up PSD obligation C^H with transition thresholds H solve:

$$W(x) = \begin{cases} 0, & x \leq \delta_B\ , \\ L_1^i x^{-\gamma_1} + L_2^i x^{-\gamma_2} + \frac{x}{r-\mu} - \frac{C(i)}{r}, & H_i \leq x \leq H_{i+1}\ , \end{cases} \tag{C.1}$$

for $i = 1, \ldots, I$, where $\gamma_1 = \dfrac{m + \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}$, $\gamma_2 = \dfrac{m - \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}$, $m = \mu - \dfrac{\sigma^2}{2}$, and where δ_b, L_1^i and L_2^i solve the following system of equations:

$$W(\delta_B) = 0, \quad W'(\delta_B) = 0\ , \tag{C.2}$$

and for $i = 1, \ldots, I-1$,

$$W(H_i-) = W(H_i+)\,, \quad W'(H_i-) = W'(H_i+)\,. \tag{C.3}$$

Because the market value of equity is non-negative and cannot exceed the asset value,[26]

$$L_2^I = 0. \tag{C.4}$$

The system (C.2)–(C.4) has $2I+1$ equations with $2I+1$ unknowns (L_j^i, $j \in \{1,2\}$, $i \in \{1,\ldots,I\}$, and δ_B). Substituting (C.1) into (C.2)–(C.4) and solving gives

$$\begin{aligned}
L_1^1 &= \frac{(\gamma_2+1)\frac{\delta_B}{r-\mu} - \gamma_2 \frac{c_1}{r}}{(\gamma_1-\gamma_2)\,\delta_B^{-\gamma_1}}\,, \\
L_2^1 &= \frac{-(\gamma_1+1)\frac{\delta_B}{r-\mu} + \gamma_1 \frac{c_1}{r}}{(\gamma_1-\gamma_2)\,\delta_B^{-\gamma_2}}\,, \\
L_1^j &= L_1^1 + \frac{\gamma_2}{(\gamma_1-\gamma_2)r}\sum_{i=1}^{j-1}\frac{c_i-c_{i+1}}{H_{i+1}^{-\gamma_1}},\ j=2,\ldots,I\,, \\
L_2^j &= L_2^1 - \frac{\gamma_1}{(\gamma_1-\gamma_2)r}\sum_{i=1}^{j-1}\frac{c_i-c_{i+1}}{H_{i+1}^{-\gamma_2}},\ j=2,\ldots,I\,,
\end{aligned}$$

$$0 = -(\gamma_1+1)\frac{\delta_B}{r-\mu} + \frac{\gamma_1}{r}\left(c_1 - \sum_{i=1}^{I-1}(c_i-c_{i+1})\left(\frac{\delta_B}{H_{i+1}}\right)^{-\gamma_2}\right)\,, \tag{C.5}$$

where, for convenience, I let $c_i \equiv C(i)$. Therefore, the best response $\delta_B(H)$ is given by the solution of (C.5).

To derive the best-response $H(\delta_B)$ one needs to study the first-passage time distribution of the process δ. Since δ is a geometric Brownian motion, its first-passage time distribution is an inverse Gaussian:

$$P(\tau(\delta_B) - t \le T \,|\, \mathcal{F}_t) = 1 - \Phi\left(\frac{m(T-t)-x}{\sigma\sqrt{T-t}}\right) + e^{\frac{2mx}{\sigma^2}}\,\Phi\left(\frac{x+m(T-t)}{\sigma\sqrt{T-t}}\right),$$

where, $x = \log\left(\frac{\delta_B}{\delta_t}\right)$, $m = \mu - \frac{1}{2}\sigma^2$, δ_t is the current level of assets, and Φ is the normal cumulative distribution function. Since $P(\tau(\delta_B) \le T \,|\, \mathcal{F}_t)$

[26]Since $\gamma_1 > 0$ and $\gamma_2 < 0$, the term $L_2^I x^{-\gamma_2}$ would necessarily dominate the other terms in the Eq. (C.1) violating the inequality $0 \le W(x) \le x/(r-\mu)$, unless $L_2^I = 0$.

depends on δ_t only through $\frac{\delta_B}{\delta_t}$, we have the linearity of $H(\cdot)$.

$$H(\delta_B) = \delta_B\, h, \tag{C.6}$$

where $h \in \mathbb{R}^{I+1}$ is such that $h_0 = 0$, $h_I = \infty$, and $h_{i+1} \geq h_i$.

Equilibrium needs to satisfy $(x, y) = (\delta_B(y), H(x))$, or alternatively, $x = \delta_B(H(x))$. Plugging (C.6) into (C.5) and solving for δ_B one obtains the unique equilibrium default threshold δ_B^*, which is given by:

$$\delta_B^* = \frac{\gamma_1(r-\mu)}{(\gamma_1+1)r}\widehat{C}, \tag{C.7}$$

where

$$\widehat{C} = \sum_{i=1}^{I}\left[\left(\frac{1}{h_{i+1}}\right)^{-\gamma_2} - \left(\frac{1}{h_i}\right)^{-\gamma_2}\right] c_i.$$

The equilibrium rating transition thresholds H^* are thus given by:

$$H^* = \frac{\gamma_1(r-\mu)}{(\gamma_1+1)r}\widehat{C}h$$

Mean-Reverting Process. The equity value W that solves (7) for the mean-reverting process (11) can be written as:

$$W\,(x) = \begin{cases} 0, \quad x \leq \delta_B\ , \\ L_1^i x^{-\eta_1} M_1(x) + L_2^i x^{-\eta_2} M_2(x) \\ \qquad + \frac{x}{\lambda+r} + \frac{\lambda\mu}{(\lambda+r)r} - \frac{C(i)}{r}, \quad H_i \leq x \leq H_{i+1}\ , \end{cases} \tag{C.8}$$

for $i = 1, \ldots, I$, where η_1 and η_2 are roots of the quadratic equation

$$\frac{1}{2}\sigma^2\eta(\eta-1) - \lambda\eta - r = 0,$$

$M_1(x) = M(-\eta_1, 2-2\eta_1+2\lambda/\sigma^2; 2\lambda\mu/\sigma^2 x)$, $M_2(x) = M(-\eta_2, 2-2\eta_2+2\lambda/\sigma^2; 2\lambda\mu/\sigma^2 x)$, and where M is the confluent hypergeometric function given by the infinite series $M(a,b;z) = 1 + az/b + \{[a(a+1)]/[b(b+1)]\}(z^2/2!) + \{[a(a+1)(a+2)]/(b(b+1)(b+2)]\}(z^3/3!) + \ldots$

The default threshold δ_b, and constants L_1^i and L_2^i thus solve the following system of equations:

$$W\left(\delta_B\right)=0, \quad W'\left(\delta_B\right)=0\ , \tag{C.9}$$

and for $i=1,\dots,I-1$,

$$W\left(H_i-\right)=W\left(H_i+\right), \quad W'\left(H_i-\right)=W'\left(H_i+\right). \tag{C.10}$$

Because the market value of equity is non-negative and cannot exceed the asset value,

$$L_2^I=0. \tag{C.11}$$

The system (C.9)–(C.11) has $2I+1$ equations with $2I+1$ unknowns (L_j^i, $j\in\{1,2\}$, $i\in\{1,\dots,I\}$, and δ_B). Substituting (C.8) into (C.9)–(C.11) and solving numerically gives the best-response δ_B to any rating transition thresholds H.

The solution to this system of equations is:

$$\begin{aligned}
L_1^1 &= \frac{\frac{1}{\lambda+r}g_2(\delta_B)-(\frac{1}{\lambda+r}\delta_B+\frac{\lambda\mu}{(\lambda+r)r}-\frac{c_1}{r})g_2^{'}(\delta_B)}{g_1(\delta_B)g_2^{'}(\delta_B)-g_1^{'}(\delta_B)g_2(\delta_B)}\\
L_2^1 &= \frac{\frac{1}{\lambda+r}g_1(\delta_B)-(\frac{1}{\lambda+r}\delta_B+\frac{\lambda\mu}{(\lambda+r)r}-\frac{c_1}{r})g_1^{'}(\delta_B)}{g_2(\delta_B)g_1^{'}(\delta_B)-g_2^{'}(\delta_B)g_1(\delta_B)}\\
L_1^j &= L_1^1+\frac{1}{r}\sum_{i=1}^{j-1}\frac{g_2^{'}(H_{i+1})(c_{i+1}-c_i)}{g_1(H_{i+1})g_2^{'}(H_{i+1})-g_1^{'}(H_{i+1})g_2(H_{i+1})},\ j=2,...I\\
L_2^j &= L_2^1+\frac{1}{r}\sum_{i=1}^{j-1}\frac{g_1^{'}(H_{i+1})(c_{i+1}-c_i)}{g_2(H_{i+1})g_1^{'}(H_{i+1})-g_2^{'}(H_{i+1})g_1(H_{i+1})},\ j=2,...I
\end{aligned}$$

$$\begin{aligned}
0=&\frac{\frac{1}{\lambda+r}g_1(\delta_B)-(\frac{1}{\lambda+r}\delta_B+\frac{\lambda\mu}{(\lambda+r)r}-\frac{c_1}{r})g_1^{'}(\delta_B)}{g_2(\delta_B)g_1^{'}(\delta_B)-g_2^{'}(\delta_B)g_1(\delta_B)}\\
&+\frac{1}{r}\sum_{i=1}^{I-1}\frac{g_1^{'}(H_{i+1})(c_{i+1}-c_i)}{g_2(H_{i+1})g_1^{'}(H_{i+1})-g_2^{'}(H_{i+1})g_1(H_{i+1})}
\end{aligned} \tag{C.12}$$

where

$$g_i(x) = x^{\eta_i} M_i(x).$$

and $c_i \equiv C(i)$. Therefore, the best response $\delta_B(H)$ is given by the solution of $(C.12)$.

In the case of mean-reverting cash flows, there is no closed-form solution for the first-passage-time distribution. Therefore, I compute the best-response rating transition thresholds H using Monte Carlo simulation.

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