Quantum Noise as an Entanglement Meter

Leonid Levitov

MIT and KITP UCSB

Landau memorial conference
Chernogolovka, 06/22/2008

Part I:
Quantum Noise as an Entanglement Meter

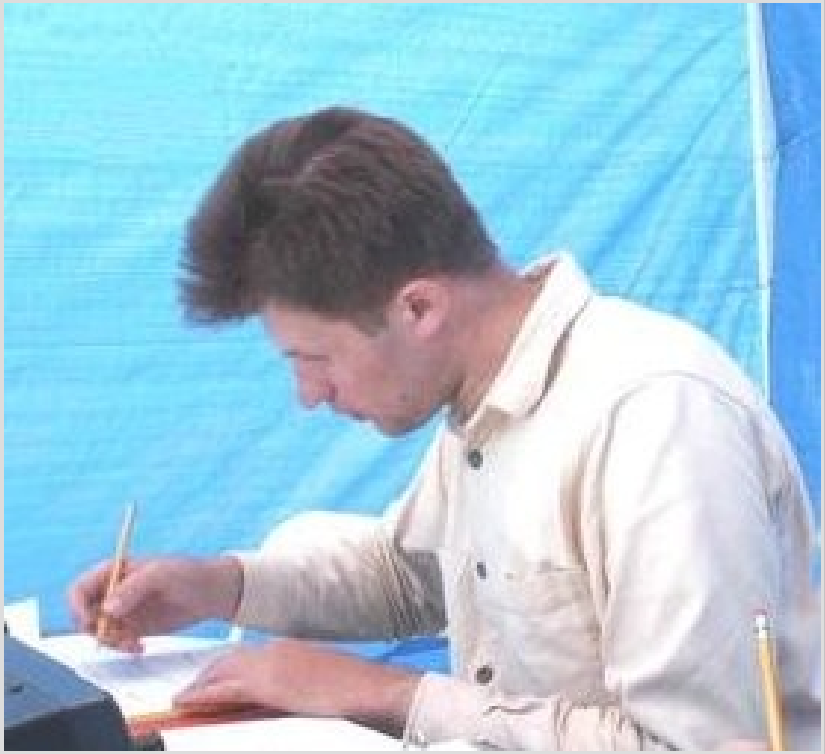
with Israel Klich (2008); arXiv: 0804.1377

Part II:
Coherent Particle Transfer in an On-Demand Single-Electron Source

with Jonathan Keeling and Andrei Shytov (2008)
arXiv: 0804.4281

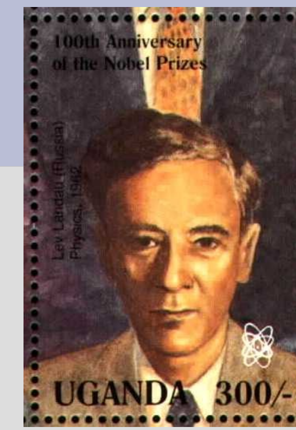
Israel Klich
UCSB

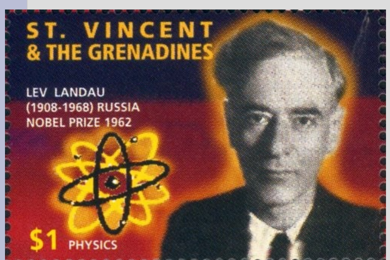
Jonathan Keeling
Cambridge Univ.



Andrei Shytov
Utah

Density matrix





Pure state vs. mixed state

$$|\Psi\rangle = \sum_i a_i |\psi_i\rangle .$$

$$E(B) = \sum_{i,j} a_i^* a_j \langle i|B|j\rangle .$$

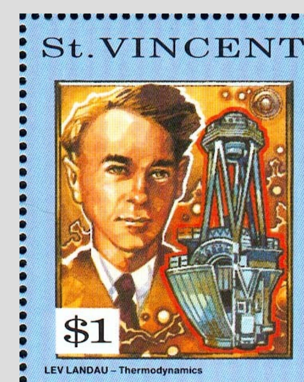
Density matrix
Landau 1927

$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j| ,$$

$$E(B) = Tr(\rho B).$$

Quantum-statistical entropy
von Neumann 1927

$$S(\rho) = -\mathrm{Tr}(\rho \ln \rho),$$

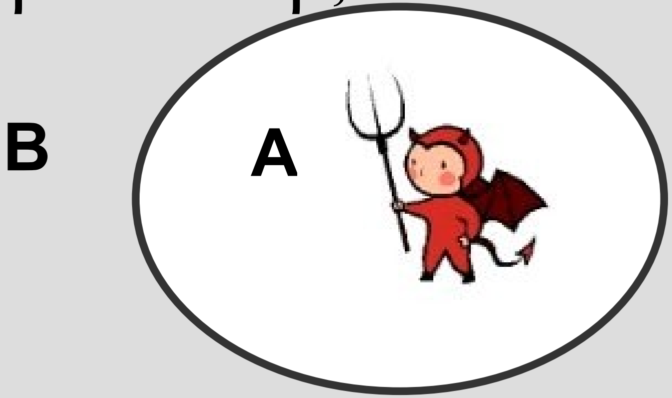


Entanglement Entropy

- Expresses complexity of a quantum state
- Describes correlations between two parts of a many-body system
- Useful in: field theory, black holes, quantum quenches, phase transitions, quantum information, numerical studies of strongly correlated systems

$S = -\mathrm{Tr}\, \rho_A \log \rho_A$

$\rho_A = \mathrm{Tr}_B\, \rho, \quad V=A+B$

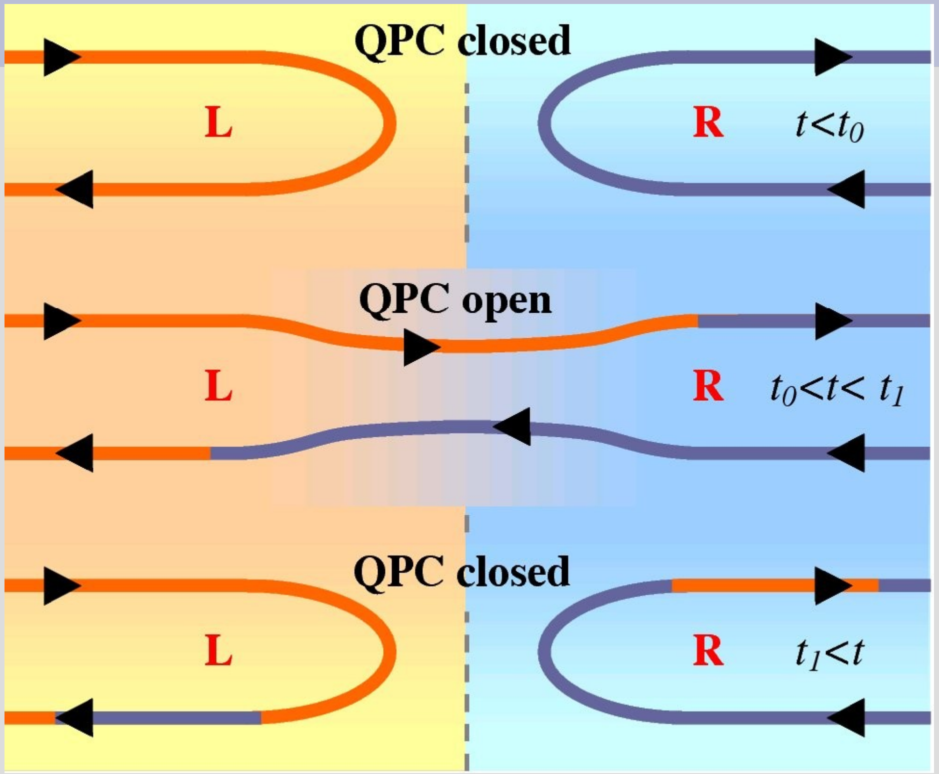


Wilczek, Bekenstein, Vidal, Kitaev, Preskill, Cardy, Bravyi, Hastings, Verstraete, Klich, Fazio, Levin, Wen, Fradkin...

Can it be measured?

arXiv: 0804.1377

- Relate to the electron transport
- Quantum point contact (QPC) with transmission tunable in time
- Open and close “door” between reservoirs R, L, let particles from R & L mix
- Statistics of current fluctuations encode S!



$$\mathcal{S}_L = -\mathbf{Tr}_L\left(\rho_L(t)\log\rho_L(t)\right)$$

$$\rho_L(t) = \mathbf{Tr}_R\left(\mathbf{U}(t)\rho(t=0)\mathbf{U}^\dagger(t)\right)$$

Current fluctuations, counting statistics

- Probability distribution of transmitted charge
- Recently measured up to 5th moment in tunnel junctions, quantum dots and QPC (Reulet, Prober, Reznikov, Fujisawa, Ensslin)
- Well understood theoretically

Generating function

Probabilities

Cumulants

$$\chi(\lambda) = \sum_n P_n e^{i\lambda n} = \exp\left(\sum_m \frac{(i\lambda)^m C_m}{m!}\right)$$

A universal relation between noise and entanglement entropy

Electron noise cumulants

True for arbitrary protocol of QPC driving

$$\mathcal{S} = \sum_{m>0} \frac{\alpha_m}{m!} C_m, \quad \alpha_m = \begin{cases} (2\pi)^m |B_m|, & \text{m even} \\ 0, & \text{m odd} \end{cases},$$

B_m are Bernoulli numbers ($B_2 = \frac{1}{6}$, $B_4 = -\frac{1}{30}$, $B_6 = \frac{1}{42} \ldots$),

$$\mathcal{S} = \frac{\pi^2}{3} C_2 + \frac{\pi^4}{15} C_4 + \frac{2\pi^6}{945} C_6 + \ldots$$

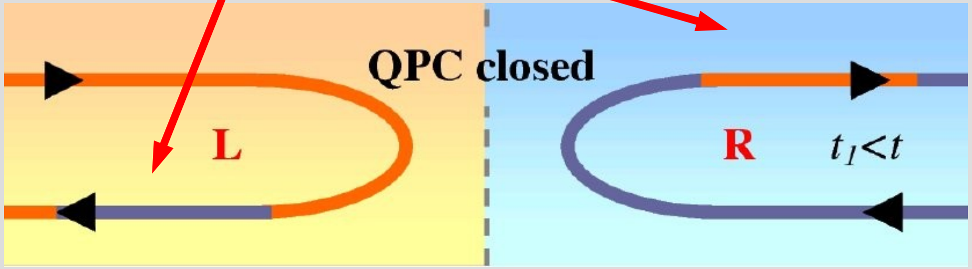
For free fermions Full Counting Statistics accounts for ALL correlations relevant for the entanglement entropy

Example: abrupt on/off switching

- Counting statistics computed explicitly
- Only C_2 is nonzero
- Logarithmic charge fluctuations, logarithmic entropy
- Agrees with field-theoretic calculations
- Can use electric noise to measure central charge

$$\mathcal{S}_L = \frac{1}{3}\log\frac{t}{\tau} \qquad \mathcal{S} = \frac{c+\bar{c}}{6}\log\frac{\ell}{a} \qquad \ell = v_F t.$$

Heuristically, number fluctuations in a time-dependent interval:



Space-time duality: use time window (door open/close) instead of space interval at a fixed time

Possible Experimental Realization

- Periodic switching: particle fluctuations and entropy proportional to total time;
- Fixed increment ΔS per driving period;
- DC shot noise reproduces ΔS:

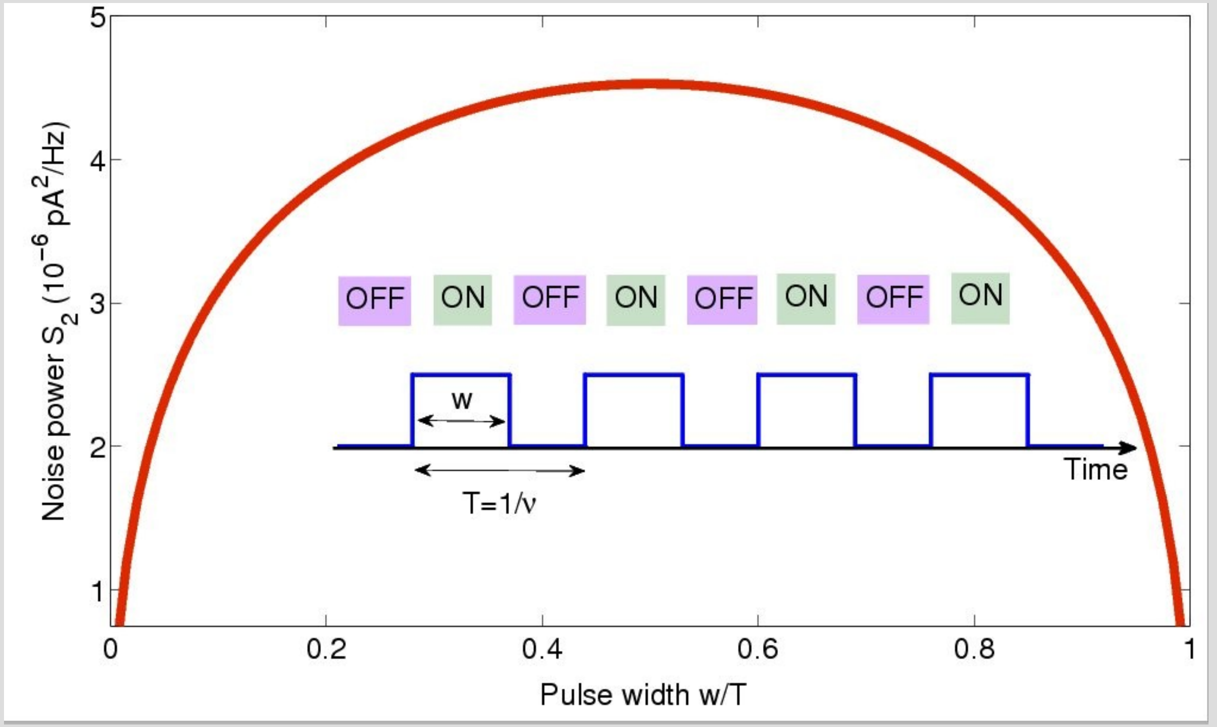
Total # of periods

$$C_2(N) \approx \frac{N}{\pi^2} \log \frac{\sin \pi \nu w}{\pi \nu \tau}, \qquad \nu = 1/T$$

$$C_2 \propto N \qquad \mathrm{d}\mathcal{S}/\mathrm{d}t = \frac{1}{3}\nu \log \frac{\sin \pi \nu w}{\pi \nu \tau}$$

$$S_2 = \frac{e^2 \nu}{\pi^2} \log \frac{\sin \pi \nu w}{\pi \nu \tau}$$

For ν=500 Mhz, T_{noise}=25 mK



Step 1: Relate many-body and one-particle quantities

Projected density matrix (gaussian for thermal state):

T=0
or
T>0

$$\rho_L \propto e^{-\tilde{H}_{ij} a_i^\dagger a_j}$$

$$M_{ij} = \mathbf{Tr}_L \rho_L a_i^\dagger a_j \qquad \tilde{H} = \log((I-M)M^{-1})$$

$$i, j \in L,$$

Find the entropy of an evolved state:

$$\mathcal{S}_L = -\mathrm{Tr}\left(M \log M + (1-M)\log(1-M)\right)$$

$$\mathcal{S}_L = -\int_0^1 dz\, \mu(z)\left(z \log z + (1-z)\log(1-z)\right)$$

$$\mu(z) = -\frac{1}{\pi}\,\mathrm{Im}\,\mathrm{Tr}\frac{1}{z-M+i0} = -\frac{1}{\pi}\partial_z\,\mathrm{Im}\,\log\det(z-M+i0)$$

Step 2: Counting statistics yields same quantity M

Functional determinant in an original form (LL, Lesovik '92)

$$\chi(\lambda) = \det\left(1 - n_\epsilon + n_\epsilon U^\dagger e^{i\lambda P_L} U e^{-i\lambda P_L}\right)$$

Scattering operator

Recently: Klich, Ivanov, Abanov, Nazarov, Vanevic, Belzig

$$\chi(\lambda) = \det\left((1 - M + M e^{i\lambda P_L}) e^{-i\lambda n P_L}\right)$$

$$g(z) = \log\det(1 - M + M e^{i\lambda P_L})$$

$$z^{-1} = 1 - e^{i\lambda}$$

$$g(z) = \log\det(z - M) - \operatorname{rank}(M)\log z$$

The quantity M

- Matrix in the single-particle Hilbert space;
- Describes partition of the modes between A and B: either statistical or dynamical;
- Intrinsic to the Full Counting Statistics
- Provides spectral representation for the entropy

Step 3: Combine results 1 and 2

$$\mu(z) = \frac{1}{\pi}\mathrm{Im}\,\partial_z\, g(z) + \mathrm{rank}(M)\delta(z)$$

$$\log\chi = \sum \frac{(i\lambda)^m}{m!} C_m$$

$$\mu(z) = \frac{1}{\pi}\sum_m \frac{C_m}{m!}\,\mathrm{Im}\,\partial_z \left(i\pi + \log\frac{1-z}{z}\right)^m + \mathrm{rank}(M)\delta(z)$$

Entanglement entropy

$$\mathcal{S} = \sum_{m=1}^{\infty} \frac{1}{m!} C_m \alpha_m$$

Noise cumulants

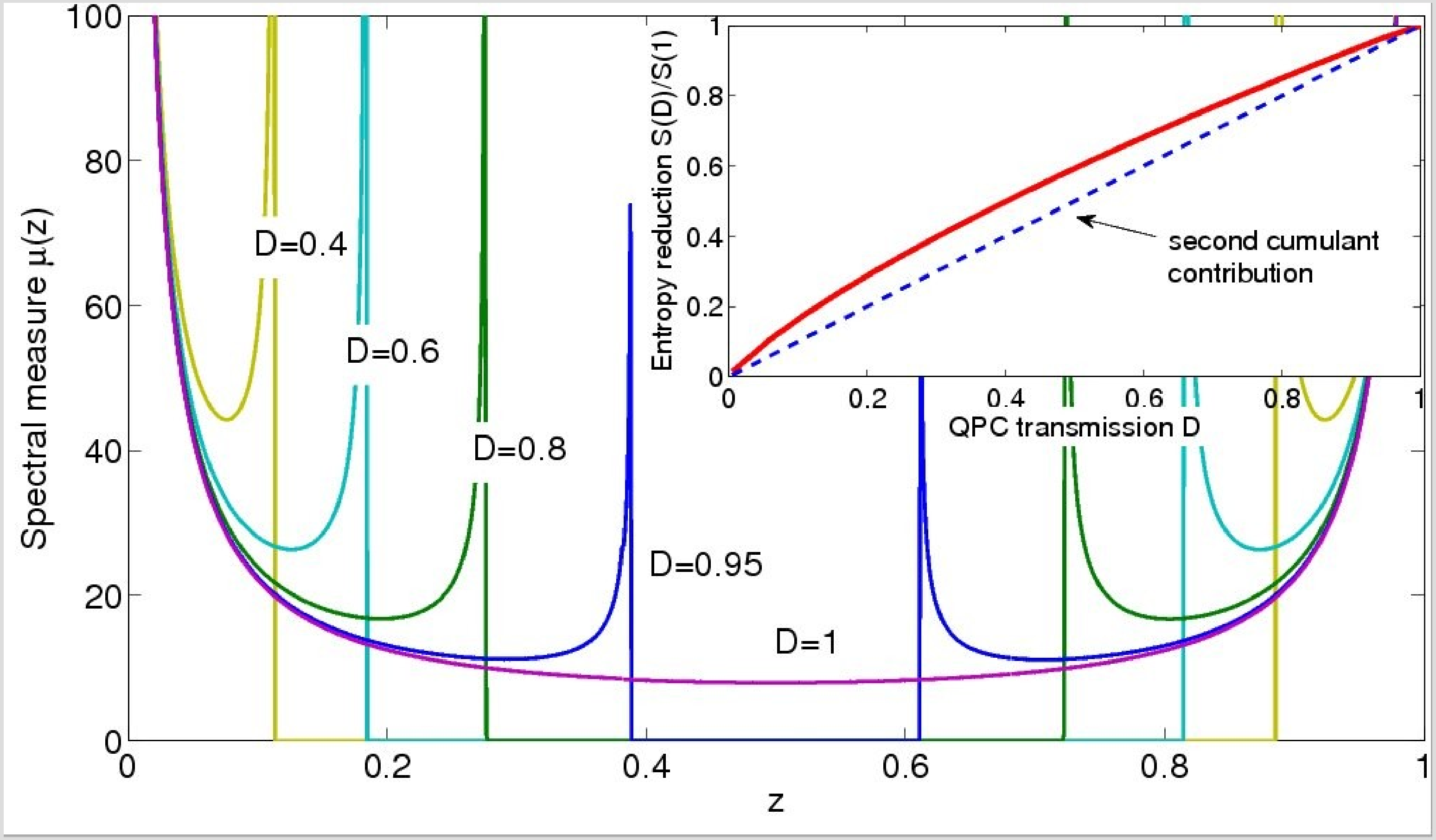
Relation of α_m to Bernoulli numbers:

$$\int_0^\infty \frac{u^{2m}\,\mathrm{d}u}{\sinh^2 u} = \pi^{2m}|B_{2m}|$$

The spectrum of M for a non-unit QPC transmission

Dependence on the parameters of driving unchanged
(up to a rescaling factor)

$$\mathcal{S} \sim \log \sin \pi \nu w$$
$$F = \mathcal{S}(D)/\mathcal{S}(1)$$



Summary & Outlook

- Universal relation between entanglement entropy and noise
- A new interpretation of Full Counting Statistics
- Generalization to other entropies (Renyi, etc);
- Opens way to measure S by electric transport (by pulsing QPC through on/off cycle)

- *Realize in cold atoms: particle number statistics*
- *Restricted vs. unrestricted entanglement*
- *Interacting systems? Neutral modes?*
- *A similar relation of entropy and noise (FCS) for Luttinger liquid is found*

?

Part II
Coherent Particle Transfer in an On-Demand Single-Electron Source

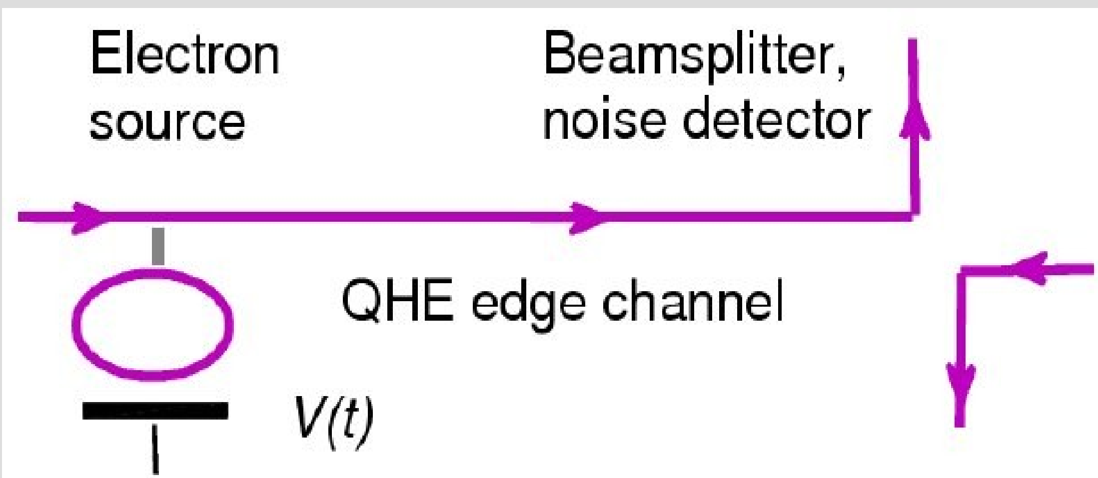
with Jonathan Keeling and Andrei Shytov (2008)
arXiv: 0804.4281

Noiseless particle source

- Transfer a particle from a localized state to a continuum without creating other excitations
- Populate a one-particle state in a Fermi gas without perturbing the rest of the Fermi sea
- Minimally entangled states in electron systems: coherent, noiseless current pulses
- Extend notion of quantized electron states (quantum dots, turnstiles) to states that can travel at a high Fermi velocity
- Bosons? Luttinger liquids?

Eject a localized electron into a Fermi continuum in a noiseless fashion

Electron system:



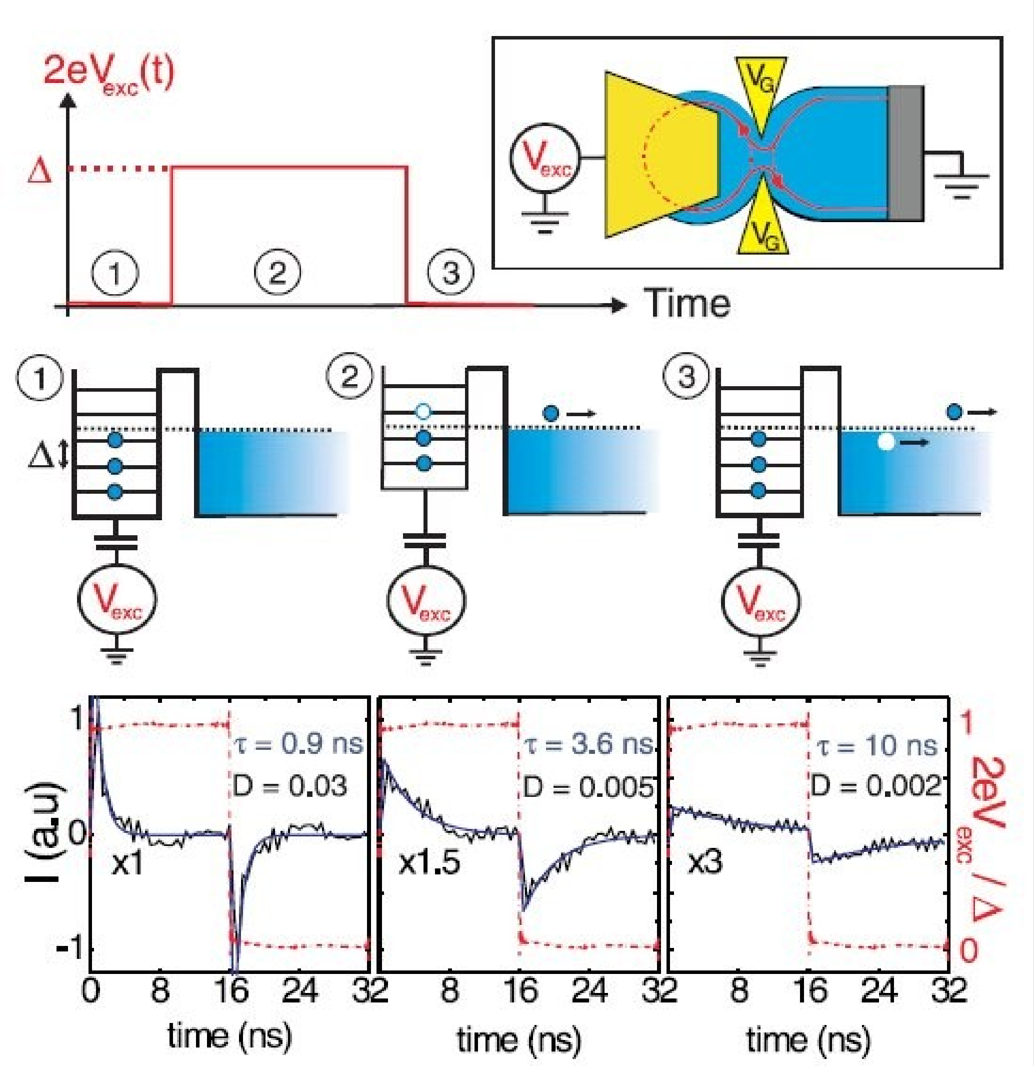
Cold atoms:

Quantum Tweezers (one-atom optical trap in a quantum gas)

Too noisy?

Experimental realization in a 1d QHE-edge electron system

Quantized current pulses in an On-Demand Coherent Single-Electron Source

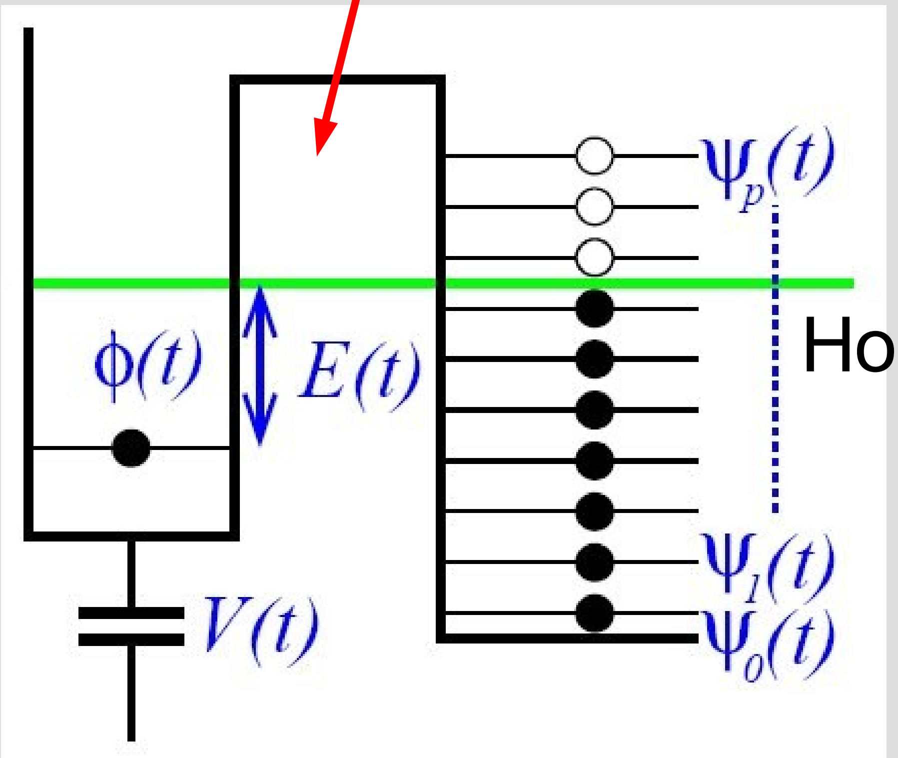


G. Feve et al. Science 316, 1169 (2007)

Excitation content: particles and holes

Tunnel coupling

Particle states



Hole states

No splash, Captain?

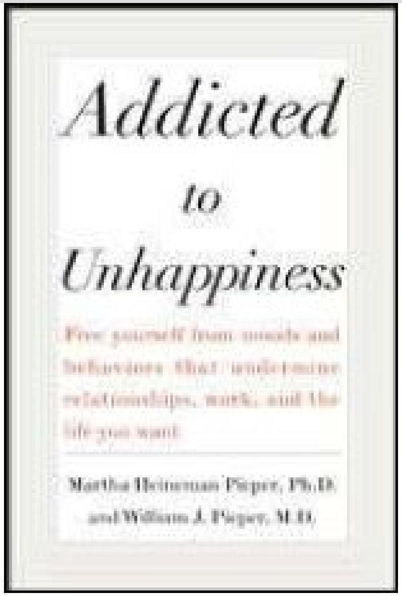
The number of excitations: **unhappiness** = $N_p + N_h$

Minimize unhappiness?

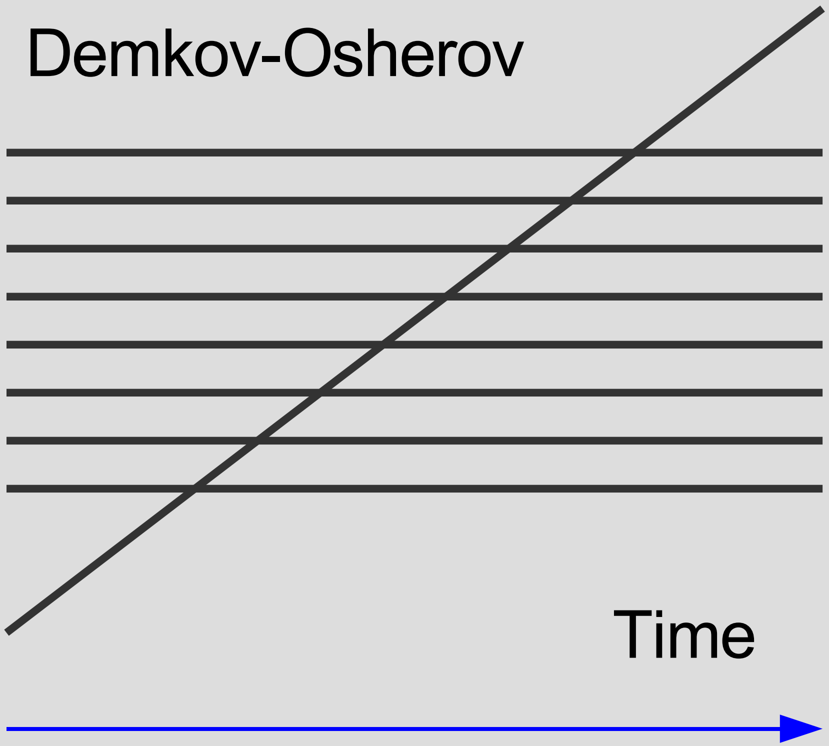
Optimize driving so that

$N_{ex} = N_e + N_h =$ **min**, $\Delta N = N_e - N_h = 1$

Localized and delocalized particles indistinguishable: Excitation unavoidable? No.



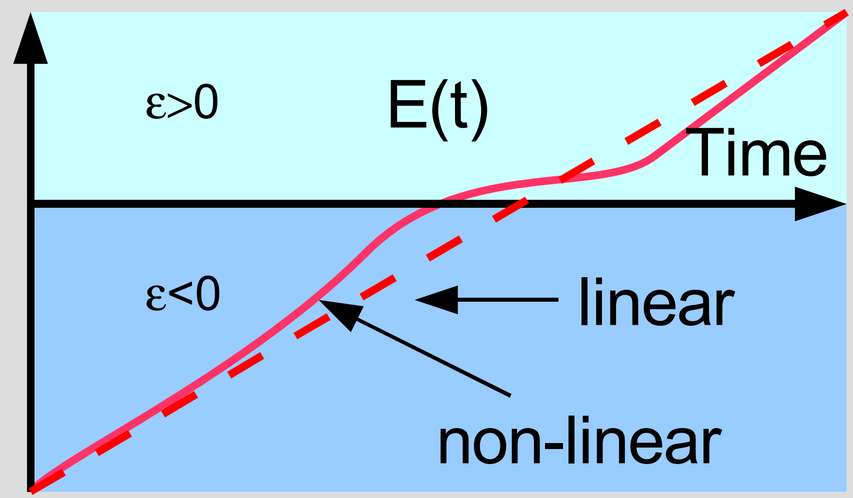
Multilevel Landau-Zener problems, exact S-matrix



Discrete states,
linear driving

Our problem:

Continuous spectrum,
arbitrary driving



Time-dependent S-matrix

Gate voltage, tunnel coupling

$$[i\partial_t - E(t)]\,\phi(t) = \sum_p \lambda(t)\psi_p(t),$$

$$[i\partial_t - \epsilon_p]\,\psi_p(t) = \lambda^*(t)\phi(t),$$

Quasi 1D scattering channel representation:

$$\psi(t,x) = \sum\nolimits_p e^{ipx}\psi_p(t) \quad \epsilon_p \to -iv_F\partial_x$$

$$[i\partial_t - E(t)]\,\phi(t) = \lambda(t)\int dx\delta(x)\psi(t,x)$$

$$[i\partial_t + iv_F\partial_x]\,\psi(t,x) = \lambda^*(t)\delta(x)\phi(t)$$

In-state:

$$\psi(t,x<0) = \psi_0(t,x) = \frac{1}{\sqrt{2\pi}}e^{-i\epsilon'(t-x/v_F)}$$

The S-matrix:

Out-state:

$$U(\epsilon,\epsilon') = \int \frac{dt}{\sqrt{2\pi}}\psi(t,x>0)\exp\left[i\epsilon\left(t-\frac{x}{v_F}\right)\right]$$

Find the S-matrix:

$$\psi(t,x) = \psi_0\left[t - \frac{x}{v_F}\right] - \frac{i}{v_F}\lambda^*\left[t - \frac{x}{v_F}\right]\phi\left[t - \frac{x}{v_F}\right]\theta(x).$$

$$\left[i\partial_t - E(t) + i\frac{|\lambda(t)|^2}{2v_F}\right]\phi(t) = \lambda(t)\psi_0(t)$$

$$\phi(t) = -i\int_{-\infty}^{t} dt'\lambda(t')\psi_0(t')e^{X(t,t')}$$

Resonance width:

$$|\lambda(t)|^2/v = \Gamma(t)$$

$$i\partial_t X(t,t') = [E(t) - i\Gamma(t)/2]\,X(t,t')$$

ANSWER:

$$U(\epsilon,\epsilon') = \delta(\epsilon-\epsilon') - \int_{-\infty}^{\infty}dt\int_{-\infty}^{t}dt'\,\frac{\lambda(t)\lambda(t')}{2\pi v_F}e^{A(t,t')}$$

$$A(t,t') = i(\epsilon t - \epsilon' t') - \int_{t'}^{t}d\tau\left[\frac{\Gamma(\tau)}{2} + iE(\tau)\right]$$

Number of excitations

Energy representation:

$$N^+ = \langle\Omega| U^\dagger \sum_{\epsilon>\epsilon_F} a_\epsilon^\dagger a_\epsilon U |\Omega\rangle = \int_{\epsilon_F}^{\infty} d\epsilon \int_{-\infty}^{\epsilon_F} d\epsilon' \, |U(\epsilon,\epsilon')|^2$$

Time representation:

$$N^+ = -\left(\frac{\Gamma}{2\pi}\right)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} ds \int_{-\infty}^{s} ds'$$

$$\frac{\exp\left[-\frac{\Gamma}{2}(t-t'+s-s') - i\int_{t'}^{t} E(\tau)d\tau + i\int_{s'}^{s} E(\tau)d\tau\right]}{(t-s+i0)(t'-s'+i0)}$$

Excitation number depends on the protocol, E(t)

Optimal driving?

Linear driving minimizes unhappiness

rapidity

$$E(t) = ct,$$

Slow or fast rapidity, degeneracy in c

Resulting state depends on c value

Relevant energy window: $|\varepsilon-\varepsilon_F|$ of order Γ

S-matrix for linear driving

$$A(T,\tau) = i(\epsilon + \epsilon')\frac{\tau}{2} - i(\epsilon' - \epsilon)T - \frac{\Gamma}{2}\tau - icT\tau.$$

$$t = T + \tau/2,\ t' = T - \tau/2,\ \text{with } \tau > 0$$

S-matrix: rank-one particle/hole block

$$U(\epsilon \neq \epsilon') = \theta(\epsilon - \epsilon')\frac{\Gamma}{c}e^{-\frac{\Gamma}{2c}(\epsilon-\epsilon')+\frac{i}{2c}(\epsilon^2-\epsilon'^2)}.$$

$$N^+ = 1 \qquad N^- = 0 \qquad N^+ - N^- = 1$$

No e/h pairs: $U_{ab}\,U_{a'b'} - U_{ab'}\,U_{a'b} = 0$

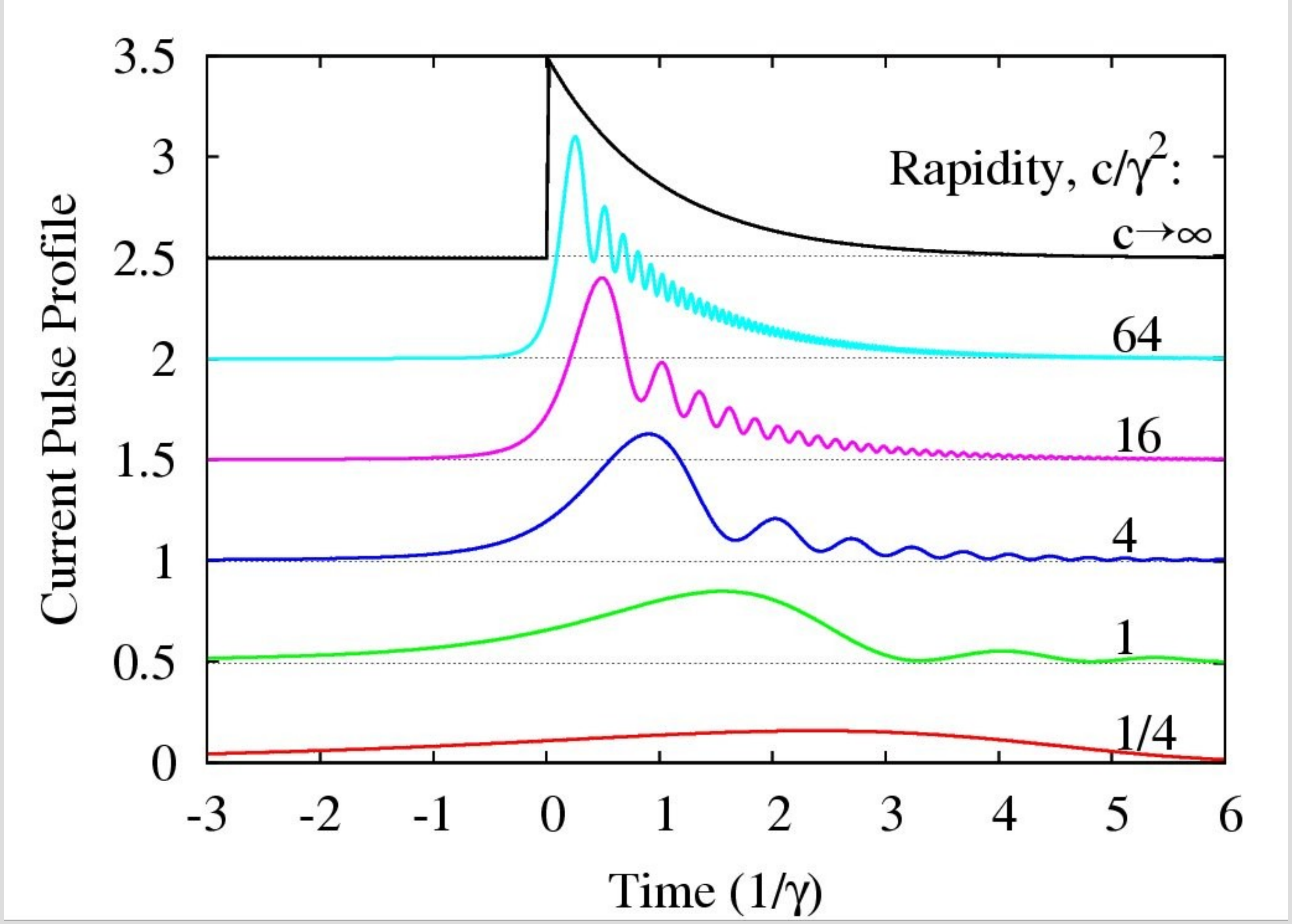
Current pulse profile at different rapidities

$$\psi(t,x) = \sqrt{\frac{\Gamma}{c}} \int_0^\infty \frac{d\epsilon}{\sqrt{2\pi}} \exp\left[-i\epsilon\left(t - \frac{x}{v_F}\right) - \frac{\Gamma\epsilon}{2c} + i\frac{\epsilon^2}{2c}\right]$$

High c: exponential profile

One-electron pulse with fringes on the trailing side

Low c: Lorentzian profile



Energy excitation and e/h pair production suppressed by Fermi statistics

Pauli principle helps to eliminate entanglement

Use noise to measure unhappiness

- Send current pulses on a QPC (beamsplitter): The partition noise generated at QPC is a direct measure of the excitation number
- Use a periodic train of pulses, vary frequency, protocol, duty cycle, etc, to demonstrate noise minimum
- At finite temperature must have $h\nu > kT$:

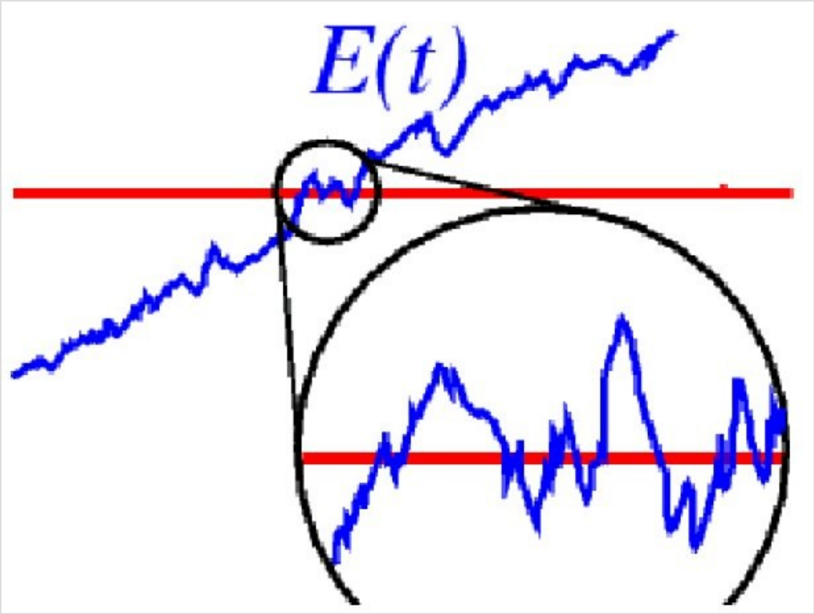
  e.g. $T = 10$ mK, $\nu > 200$ MHz

More examples

- Harmonic driving, $E(t)=E_0+\cos\Omega t$, simulates repeated linear driving;
- Linear driving + classical noise:

$$E(t) = ct + \delta V(t),$$

$$\langle \delta V(t)\delta V(t')\rangle = \gamma_2\, \delta(t-t')$$



Total number of excitations:

$N_{ex} = 1$ for fast driving;

$$N^{\mathrm{ex}} \approx \frac{2\gamma\gamma_2}{\pi c} \ln \frac{\omega_0}{\gamma_*}$$

for slow driving (multiple crossings of the Fermi level);

Crossover at $c \sim \gamma\gamma_2$

Slow driving

A more intuitive picture at slow driving:
quasistationary time-dependent scattering phase

$$\theta(t) = \arctan((\varepsilon - E(t))/\Gamma)$$

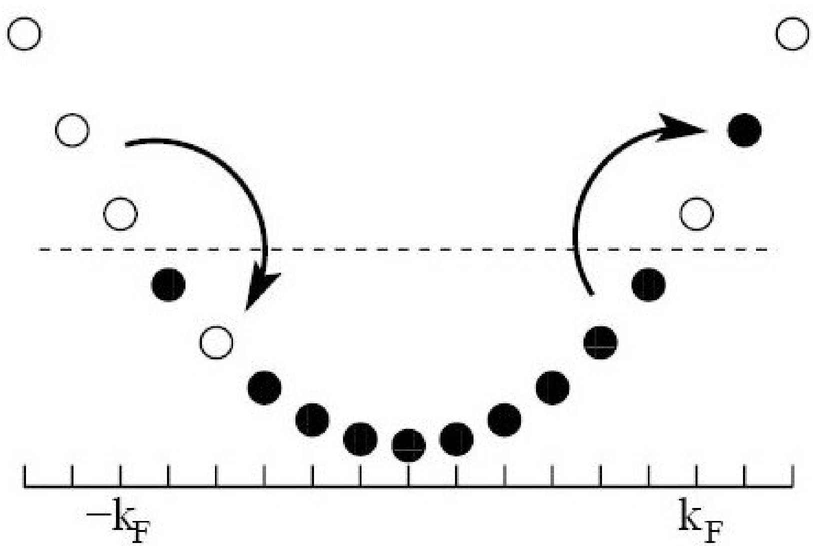
Translates into an effective time-dependent ac voltage:

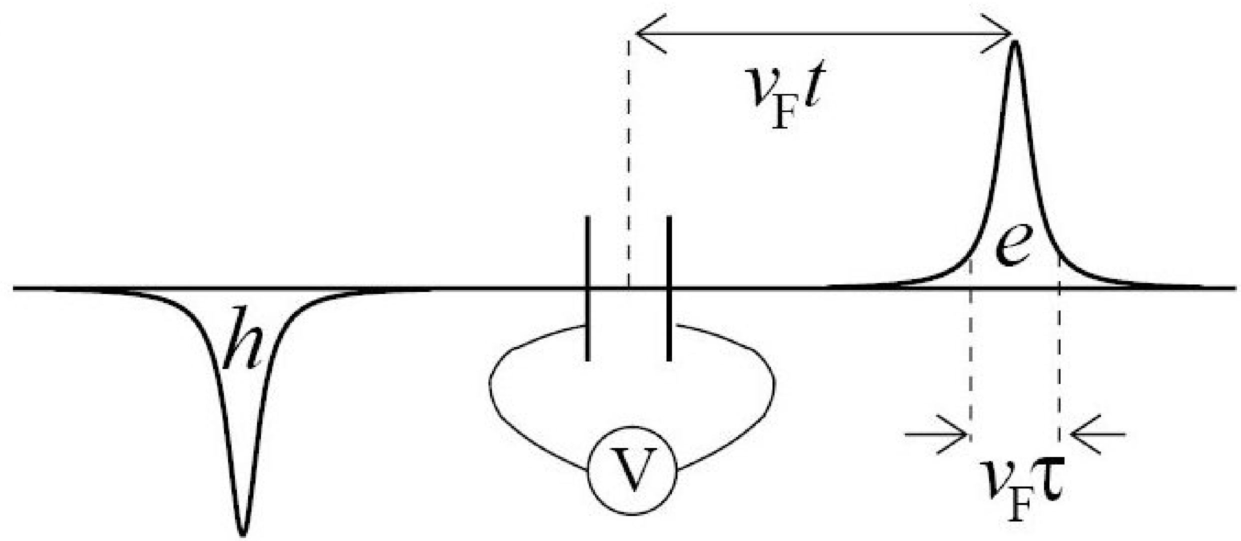
$$V(t) = (h/e)\, d\theta/dt$$

Noiseless excitation realized for Lorentzian pulses
of quantized area (PRL 97, 116403 (2006))

Clean excitation by a voltage pulse

- Particle excited above E_F;
- Other particles filling the void at $E < E_F$ (near $+k_F$);
- Undisturbed Fermi sea (no mess left behind);
- Counter-propagating hole (similar, near $-k_F$).





Minimal noise requirement

$$N_{ex} = N_e + N_h \;\rightarrow\; \min, \quad \Delta q/e = N_e - N_h = n = const$$

- An interesting variational problem, solved by pulses of integer area $2\pi n$:

$$V(t) = \frac{\hbar}{e} \sum_{i=1\ldots n} \frac{2\tau_i}{(t-t_i)^2 + \tau_i^2} \quad (\tau_i > 0)$$

Lorentzian pulses (overlapping or non-overlapping): $N_h = 0$ or $N_e = 0$

Degeneracy: $N_{ex} = n$, the same for all t_i, τ_i

When does a unitary evolution excites at most one particle?

Evolve a Fermi sea, $n = \sum_{E_k < E_F} |k\rangle\langle k|$:

$$n \to U n U^{-1}, \qquad U_{+-} = (1-n)Un, \qquad U_{-+} = nU(1-n)$$

Criterion: IF and ONLY IF $U_{-+} = 0$, $U_{+-} = c|\phi_+\rangle\langle\phi_-|$ a rank one matrix.

Proof: $\langle k'|U|k\rangle = \langle k'|\phi_+\rangle\langle\phi_-|k\rangle$ for $E_k < E_F$, $E_{k'} > E_F$;

$U_{a\to a'}U_{b\to b'} - U_{a\to b'}U_{b\to a'} = 0$, — at most one particle excited.

Transition amplitude for Lorentzian pulses $\psi(t,x) = \psi(0, x+vt)e^{i\phi(t)}$:

$$e^{i\phi(t)} = \frac{t + i\xi_k^*}{t - i\xi_k}, \quad \xi_k = \tau_k - it_k$$

Fourier transform: $\int e^{i\phi(t)+i\omega t}dt = \delta(\omega) + \sqrt{2\tau}e^{-\xi\omega}\theta(\omega)$, $\omega = E_{k'} - E_k$;
Criterion fulfilled due to multiplicativity of exp!

Features:

- A many body excitation which conspires to behave like a single particle;
- Direct product of e and h;
- Energy distribution width $\hbar/\tau$ — inverse pulse width;
- Generalized to many pulses of equal sign. "Laughlin" algebra:

$$\prod_{k=1}^{n} e^{i\phi_k(t)}|0\rangle = \prod_{k<k'} \frac{\xi_k + \xi_{k'}^*}{\xi_k - \xi_{k'}} A_n^\dagger A_{n-1}^\dagger \ldots A_1^\dagger|0\rangle, \qquad A_k^\dagger = \sum_{\epsilon > E_F} e^{-\xi_k \epsilon} a_k^\dagger$$

- Pulses of opposite sign: entangled e-h pairs and an undisturbed Fermi sea;

$$e^{-i\phi_1(t)} e^{i\phi_2(t)}|0\rangle = \frac{\xi_k - \xi_{k'}}{\xi_k + \xi_{k'}^*} A_1^\dagger B_2^\dagger|0\rangle + \frac{2\sqrt{\tau_k \tau_{k'}}}{\xi_k + \xi_{k'}^*}|0\rangle$$

- Gereralized for chiral Luttinger liquid (QHE edge state):

$e \to e_* = e/m$, $\int V dt = h/e_*$ — fractional charge pulses.

Summary

- Many-body states that conspire to behave like one-particle states
- Release/trap a particle in/from a Fermi sea in a clean, noiseless way
- Single-particle source can be realized using quantum dots: a train of quantized pulses of high frequency
- Can employ particle dynamics with high Fermi velocity 10^8 cm/s to transmit quantized states in solids