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# TWO-PASS 3D MIGRATION AND LINEARIZED INVERSION IN THE (x, *t*)-DOMAIN[1]

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## Abstract

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3D Kirchhoff migration and acoustic Born inversion of zero-offset seismic data in a constant-velocity medium can be uniformly factored as a cascade of two 2D diffraction integrals. The formal argument is based on a straightforward implementation of the original time-domain approach of Gibson, Larner and Levin. The factorization differs from the factorization described by Jakubowicz and Levin in omitting all time-dependent filters from the 2D operators in favour of 1D filtrations performed as a preprocess and a postprocess.


## Introduction

Gibson, Larner and Levin (1983) introduced an efficient approach to 3D migration by means of a simple heuristic argument showing that the Kirchhoff-summation method could be accomplished by successive applications of the standard 2D technique. In a companion paper, which has been the subject of some further discussion (Stolt 1984; Fokkema, Lörtzer and Ziolkowski 1986; Jakubowicz and Levin 1986), Jakubowicz and Levin (1983) argued that 3D wave-equation migration could be precisely factored as a cascade of two $2\frac{1}{2}$D wave-equation migrations by considering migration in both the (**x**, $\omega$) and (**k**, $\omega$) domains. They declined to treat the (**x**, *t*) domain on the grounds that the half-derivative in $2\frac{1}{2}$D Kirchhoff migration rendered the analysis intractable.

In this note we sidestep the issue of the half-derivative and observe that the 3D wave-equation migration can be handled precisely by the original time-domain algorithm provided both the input data and output image are modified by the application of suitable 1D filters. This is both computationally efficient and natural with respect to the original heuristic. Furthermore, since Kirchhoff migration based on exploding reflectors differs from multidimensional inversion based on the Born

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approximation only in the data preparation step, the factorization applies equally to these algorithms.

## THE CASCADED TWO-PASS 3D DIFFRACTION INTEGRAL

Recall the time-domain analysis of Gibson, Larner and Levin (1983): given a homogeneous medium with two-way slowness $a = 2/c$, in computing

$$\text{image}\,(x_0, y_0, t_0) = \iint \mathrm{d}x\,\mathrm{d}y\ \text{data}\,(x, y, t = \sqrt{t_0^2 + a^2(\Delta x^2 + \Delta y^2)}),$$

we can first form temporary ('in-line') sums:

$$\text{temp}\,(x, y_0, t_1) = \int \mathrm{d}y\ \text{data}\,(x, y, t = \sqrt{t_1^2 + a^2\,\Delta y^2})$$

and then combine these temporary sums by stacking in the orthogonal ('cross-line') direction:

$$\text{image}\,(x_0, y_0, t_0) = \int \mathrm{d}x\ \text{temp}\,(x, y_0,\ t_1 = \sqrt{t_0^2 + a^2\,\Delta x^2}),$$

$$= \int \mathrm{d}x \int \mathrm{d}y\ \text{data}\,(x, y, t = \sqrt{(\sqrt{t_0^2 + a^2\,\Delta x^2})^2 + a^2\,\Delta y^2}). \quad (1)$$

Algebraically, the factorization rests on the trivial identity

$$\sqrt{t_0^2 + a^2(\Delta x^2 + \Delta y^2)} = \sqrt{(\sqrt{t_0^2 + a^2\,\Delta x^2})^2 + a^2\,\Delta y^2}. \quad (2)$$

Since the temporary sum is independent of the final image point, the total number of terms to be computed per output trace is reduced from $(ni)(nx)(ny)$ to $(ni)(nx + ny)$ where $nx$ and $ny$ are the number of $x$ and $y$ offsets contributing to the stack, and $ni$ is the number of points in each image trace (and each temporary trace).

When we replace the diffraction stack by a wave-equation migration, (1) becomes a Kirchhoff integral (Schneider 1978):

$$I_{\text{Kirch}}(x_0, y_0, t_0) = -\frac{1}{2\pi}\frac{\partial}{\partial z}\int \mathrm{d}x\,\mathrm{d}y\,\frac{1}{r}\ \text{data}\,(x, y, t = ar), \quad (3)$$

where $z = t_0/a$ and $r = \sqrt{\Delta x^2 + \Delta y^2 + z^2}$.
Equation (3) refines (1) (and complicates the problem of cascading) by the addition of a weighting term $1/r$ that depends on both $x$ and $y$, plus a differentiation filter that is applied after integration. Jakubowicz and Levin (1983) suggested that the 3D migration operator defined by (3) could be factored as a cascade of two $2\frac{1}{2}$D migrations. In space-time the standard $2\frac{1}{2}$D operator has the form (Schneider 1978):

$$I_{\text{Kirch2D}}(x_0,\ t_0) = -\frac{1}{\pi}\frac{\partial}{\partial z}\int \mathrm{d}x\left[\int_{t=t_1}^{\infty} \mathrm{d}t\ \frac{\text{data}\,(x, t)}{\sqrt{t^2 - t_1^2}}\right]_{t_1 = \sqrt{t_0{}^2 + a^2\,\Delta x^2}}. \quad (4)$$

This operator involves a 2D diffraction stack and the application of 1D filters before and after stack. The integral can be written in other forms involving a single 1D filter (e.g. Stolt and Benson 1986, p. 105). These extra filters add to the computational complexity of the cascaded 3D integral and make it difficult to analyse directly. Jakubowicz and Levin (1983) attacked that difficulty by working with equivalent forms in the ($\mathbf{x}$, $\omega$) and ($\mathbf{k}$, $\omega$) domains. This indirect treatment is tricky and the original paper contained some errors (Stolt 1984; Fokkema, Lörtzer and Ziolkowski 1986). The ($\mathbf{k}$, $\omega$) argument can be corrected (Jakubowicz and Levin 1986; Stolt and Benson 1986, pp. 97–106), but requires careful handling of imaginary wavenumbers and a stationary-phase argument to convert the $2\frac{1}{2}$D operator to space-time.

In fact, the situation is much simpler than this extended discussion suggests. By directly re-examining the 3D space-time operator, we can describe a factorization which is different from that described in the previous work and is simpler both conceptually and computationally. The key is to separate the diffraction integral from 1D filters and to factor only the former. In the following section we show that three standard migration/inversion operators can uniformly be implemented as a three step process:

1. Preprocess by replacing the original input traces data $(x, y, t)$ by appropriately filtered traces $\widehat{\text{data}}\,(x, y, t)$. This first step is accomplished by an operator that depends only on $t$ and can be applied tracewise.
2. Form the simple diffraction stack

$$\text{image}\,(x_0, y_0, t_0) = \iint \mathrm{d}x\,\mathrm{d}y\,\widehat{\text{data}}\,\left(x, y, t = \sqrt{t_0^2 + a^2(\Delta x^2 + \Delta y^2)}\right).$$

3. Postprocess by replacing image $(x, y, t)$ by the final output

$$I(x_0, y_0, t_0) = t_0\,\text{image}\,(x_0, y_0, t_0).$$

The second step has the trivial factorization described in (1). The 1D operations described in steps one and three replace four 1D operations required to cascade (4). The various migration/inversion operators differ only in the preprocessing step.

## Two-pass 3D Migration and Inversion in the ($\mathbf{x}$, $t$) Domain

In view of the above discussion, it suffices to show that each of our migration/inversion operators can be written in the normal form

$$I(x_0, y_0, t_0) = t_0 \iint \mathrm{d}x\,dy\,\widehat{\text{data}}\,\left(x, y, t = \sqrt{t_0^2 + a^2(\Delta x^2 + \Delta y^2)}\right) \tag{5}$$

by making an appropriate choice of $\widehat{\text{data}}$. We consider them in order of complexity.

### *Approximate Born Inversion*

The simplest 3D zero-offset migration operator to put into normal form is derived from the approximate inversion of the acoustic Generalized Radon Transform (e.g. Miller, Oristaglio and Beylkin 1987, equation (29)):

$$I_{\text{GRT}}(x_0, y_0, t_0) = 2a^3 \iint \mathrm{d}x\,\mathrm{d}y\,\frac{t_0}{t}\,\widehat{\text{data}}\,\left(x, y, t = \sqrt{t_0^2 + a^2(\Delta x^2 + \Delta y^2)}\right). \tag{6}$$

This is clearly reduced to (5) by substituting the trivial preprocessing

$$\widehat{\text{data}}_{\text{GRT}}(x, y, t) = \frac{2a^3}{t}\,\text{data}(x, y, t). \tag{7}$$

### *Kirchhoff Migration*

Consider (3). It has the equivalent form

$$I_{\text{Kirch}}(x_0, y_0, t_0) = \int \mathrm{d}x\,\mathrm{d}y \left[ -\frac{t_0}{2\pi t}\frac{\partial}{\partial t}\frac{\text{data}(x, y, t)}{t} \right]_{t=\sqrt{t_0{}^2 + a^2(\Delta x^2 + \Delta y^2)}}. \tag{8}$$

Equation (8) is reduced to (5) by the preprocessing

$$\widehat{\text{data}}_{\text{Kirch}}(x, y, t) = -\frac{1}{2\pi t}\frac{\partial}{\partial t}\frac{\text{data}(x, y, t)}{t}. \tag{9}$$

### *Exact Born Inversion*

A more complicated 3D inversion operator for zero-offset data which is exact within the Born approximation is derived in Cheng and Coen (1984). In their equation (41) they provide a prefilter which should be applied to the data before applying the Kirchhoff operator

$$\widetilde{\text{data}}(x, y, t) = -8\pi c \left[ 2\int_0^t \mathrm{d}\tau \int_0^\tau \text{data}(x, y, t')\,\mathrm{d}t' + \int_0^t \tau\,\text{data}(x, y, \tau)\,\mathrm{d}\tau \right]. \tag{10}$$

The exact Born inversion is then obtained by substituting $\widetilde{\text{data}}$ for data in (3) or (8). The same substitution in (9) will yield a rather complicated preprocessing for this case. In fact, the operator can be substantially simplified. Making use of integration by parts with $A = t$, and $\mathrm{d}B = \text{data}(x, y, t)$, we can rewrite (10) as

$$\widetilde{\text{data}}(x, y, t) = -8\pi c \left[ 2t\int_0^t \text{data}(x, y, \tau)\,\mathrm{d}\tau - \int_0^t \tau\,\text{data}(x, y, \tau)\,\mathrm{d}\tau \right]. \tag{11}$$

Substituting this expression for data $(x, y, t)$ in (9), we obtain a simpler preprocess for exact 3D Born inversion

$$\widehat{\text{data}}_{\text{exact Born}}(x, y, t) = \frac{8a}{t}\left[ \text{data}(x, y, t) + t^{-2}\int_0^t \tau\,\text{data}(x, y, \tau)\,\mathrm{d}\tau \right]. \tag{12}$$

The authors are grateful to C. Esmersoy and M. Oristaglio for this last reduction. The factor of $c^2$ by which (7) differs from the leading term in (12) is explained by the difference between two definitions of the scattering potential. Linearized inversion in the ($\mathbf{k}$, $\omega$) domain is discussed in Stolt and Benson (1986, pp. 150–156).

## REMARKS

The postprocessing multiplication by $t_0$ is easily incorporated into the cascaded integral. Writing $\overline{\text{data}} = t\,\widehat{\text{data}}$, we can put each migration operator into the alternative normal form

$$I(x_0, y_0, t_0) = -\iint \mathrm{d}x\,\mathrm{d}y\,\frac{t_0}{t}\,\overline{\text{data}}\,(x, y, t = \sqrt{t_0^2 + a^2(\Delta x^2 - \Delta y^2)}).$$

The cosine weighting term $t_0/t$ can be factored as a product of cross-line cosine term $t_0/t_1$ with an inline cosine term $t_1/t$ to give the variant of the cascaded 3D operator:

$$I(x_0, y_0, t_0) = \int \mathrm{d}x\,\frac{t_0}{t_1}\,\text{temp}\,(x, y_0, t_1 = \sqrt{t_0^2 + a^2\,\Delta x^2}),$$

$$= \int \mathrm{d}x\,\frac{t_0}{t_1}\int \mathrm{d}y\,\frac{t_1}{t}\,\overline{\text{data}}\,(x, y, t = \sqrt{(\sqrt{t_0^2 + a^2\,\Delta x^2})^2 + a^2\,\Delta y^2}). \quad (13)$$

This approach also yields a simple uniform approach to $2\frac{1}{2}$D migration and inversion. Assume that the earth is invariant under changes in $y$. Then so is the data and we can write

$$\text{temp}\,(x, y_0, t_1) = \text{temp}\,(x, t_1),$$

$$= \int \mathrm{d}y\,\frac{t_1}{t}\,\overline{\text{data}}\,(x, t = \sqrt{t_1^2 + a^2 y^2}). \quad (14)$$

Make the change of variables $y = (\pm\sqrt{t^2 - t_1^2})/a$ in (13) and recollect terms to obtain the $2\frac{1}{2}$D operator:

$$\text{image}\,(x_0, t_0) = \frac{2t_0}{a}\int \mathrm{d}x\left[\int_{t=t_1}^{\infty} \mathrm{d}t\,\frac{\overline{\text{data}}\,(x, t)}{\sqrt{t^2 - t_1^2}}\right]_{t_1 = \sqrt{t_0^2 + a^2\,\Delta x^2}}. \quad (15)$$

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