

CISM Summer School, Udine Italy, June 22-26 2009.  
Electrokinetics and Electrohydrodynamics of Microsystems

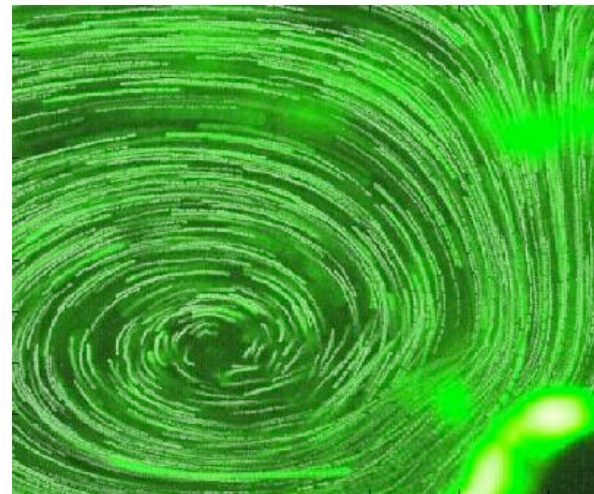
# Induced-Charge Electrokinetic Phenomena

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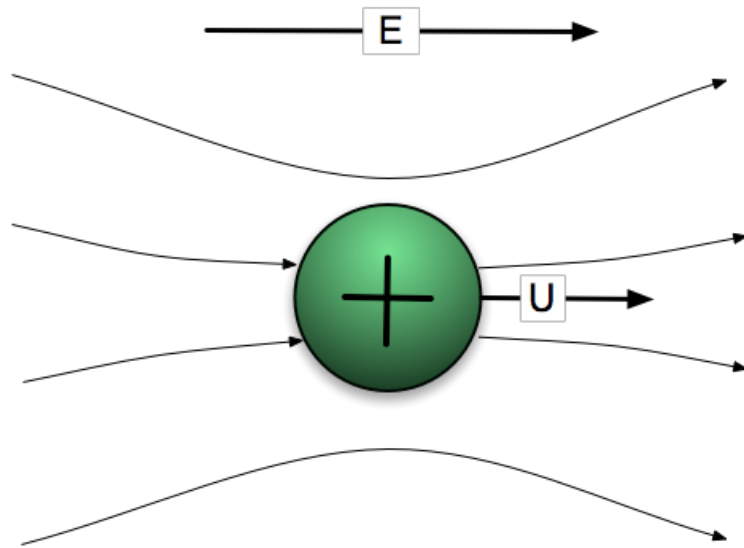
## Lectures

1. Introduction
2. Low-voltage theory
3. Particle motion
4. Fluid motion
5. Large-voltage theory



# Background: Linear Electrophoresis

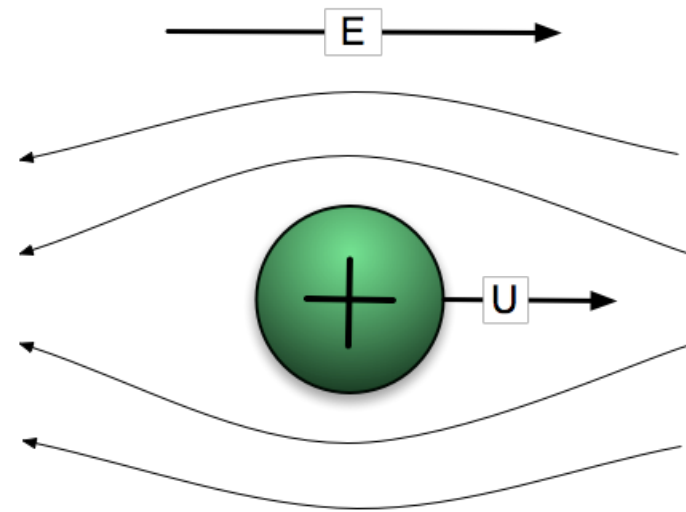
# Electrophoresis in a dielectric liquid



Lab frame

$$U = \frac{qE}{6\pi\eta R}$$

Size-dependent velocity



Particle frame

$$u \sim \frac{R}{r}$$

Long-range flow perturbation

# Electrophoresis in an electrolyte

Smoluchowski (1907)

Electro-osmotic slip

$$u_s = -\mu_e E_t$$

Electrophoretic mobility

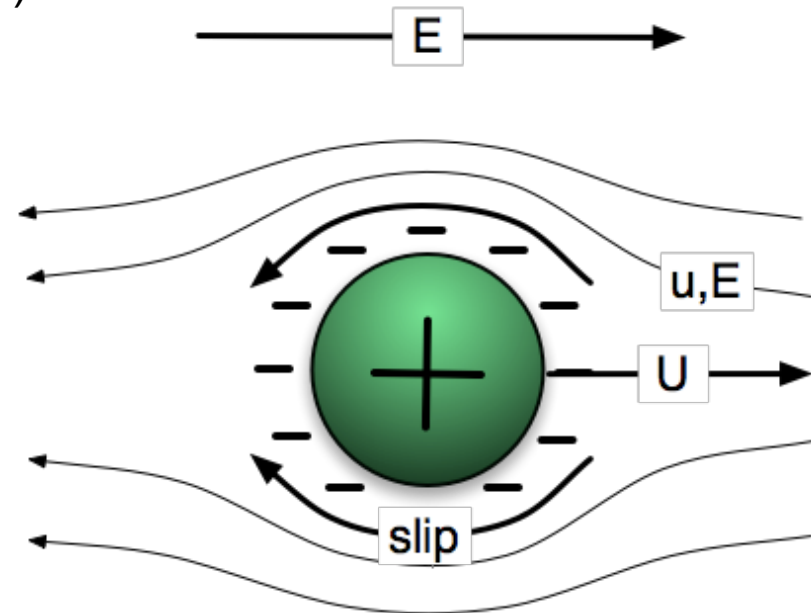
$$\mu_e = \frac{\epsilon \zeta}{\eta}$$

Zeta potential

$$\zeta = \Delta\phi_D = q/C_D$$

Size-independent velocity

$$U = \mu_e E_\infty$$



force-free motion

$$u \sim \left(\frac{R}{r}\right)^3$$

short-range flow perturbation

# Electrophoresis of colloids

Morrison (1970) – thin double layers; uniform, small surface charge

$$\nabla^2 \phi = 0, \quad \vec{E} = -\nabla \phi, \quad \hat{n} \cdot \vec{E} = 0$$

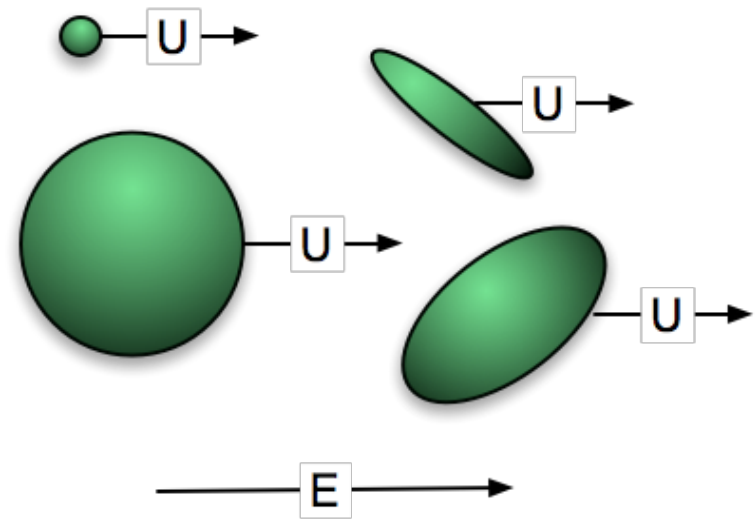
$$\nabla^2 \vec{u} = \nabla p, \quad \nabla \cdot \vec{u} = 0, \quad \hat{n} \cdot \vec{u} = 0$$

$$\vec{u}_s = -\mu_e \vec{E}_t$$

Solution for uniform mobility: potential flow

$$\vec{u} = -\mu_e \vec{E}, \quad p = 0$$

$$\vec{U} = \mu_e \vec{E}_\infty$$

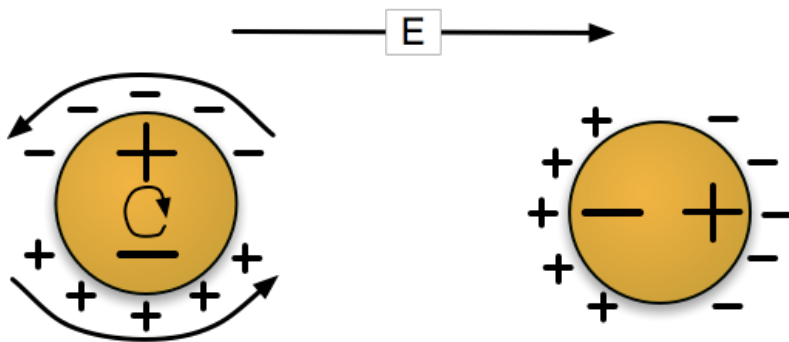


Zero relative motion!

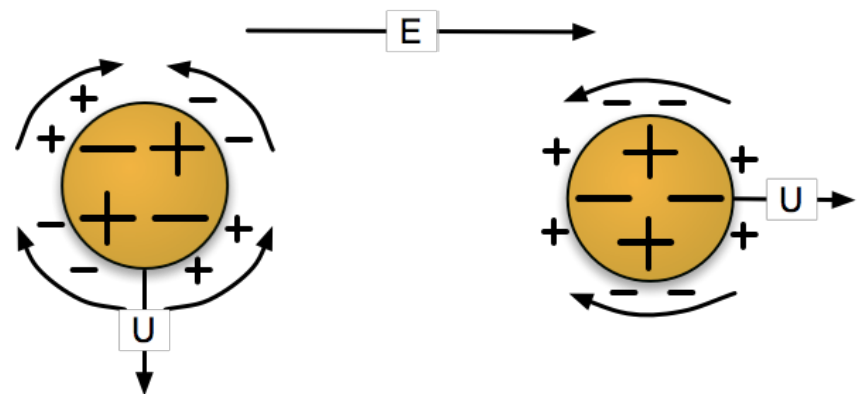
# Heterogeneous Particles

Anderson 1984; Ajdari 1995, Long & Ajdari 1998

An inhomogeneous sphere rotates to align dipole with  $E$ . Electrophoresis enhances forced electro-rotation.



Transverse EP is possible, for certain orientations



EP mobility is not related to total or average charge!

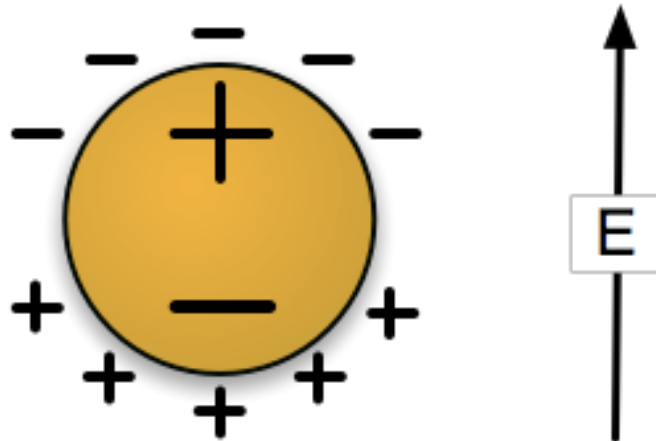
Non-spherical AND inhomogeneous particles can rotate along one axis or exhibit transverse EP in all orientations.

Nonlinear “induced-charge”  
electrophoresis of polarizable  
homogeneous particles

# Electrophoresis of **Polarizable** Particles

Classical result, e.g. Levich (1956):

Electrophoretic mobility depends on the total charge, but not on the induced dipole moment



Induced dipole

...BUT this only holds for linear response to small  $E$



# Stotz-Wien-Dukhin Effect

For nonlinear double-layer capacitance, the mobility depends on  $E$ , since induced charge must redistribute to maintain the same the total charge (AS Dukhin 1992)

$$\mu_e(E) \sim \frac{\varepsilon}{\eta} \left( \zeta - \frac{3}{8} \frac{C'(\zeta)}{C(\zeta)} (ER)^2 + \dots \right)$$

Can exploit for separation in “unbalanced” AC fields

$$\langle E \rangle = 0, \quad \text{but} \quad \langle E^3 \rangle \neq 0$$

(SS Dukhin et al 1986, R Chimenti, patent 1986)

# Ion-specific mobility

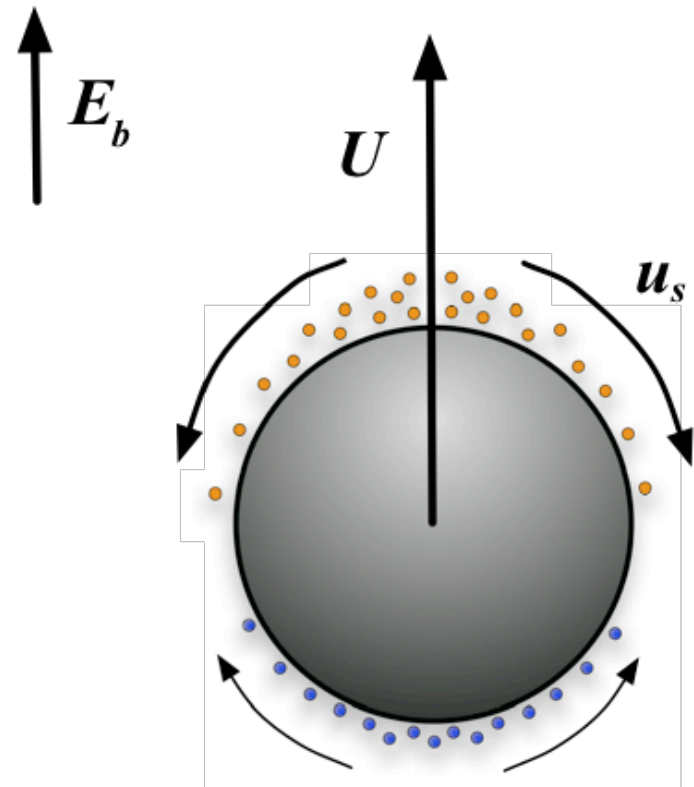
Bazant, Kilic, Storey, Ajdari, Adv Coll Interface Sci (2009)

## Dukhin effect in an *asymmetric electrolyte*

Mobility must also depend on ion charges, sizes, etc.

Even an *uncharged sphere* can move, if it is polarizable

Its apparent charge is that of ions which screen at higher density



# Dielectrophoresis (DEP)

Maxwell-Wagner induced dipole moment

$$p = 4\pi\epsilon \left( \frac{\sigma_p - \sigma + i\omega(\epsilon_p - \epsilon)}{\sigma_p + \sigma + i\omega(\epsilon_p + \epsilon)} \right) R^3 E$$

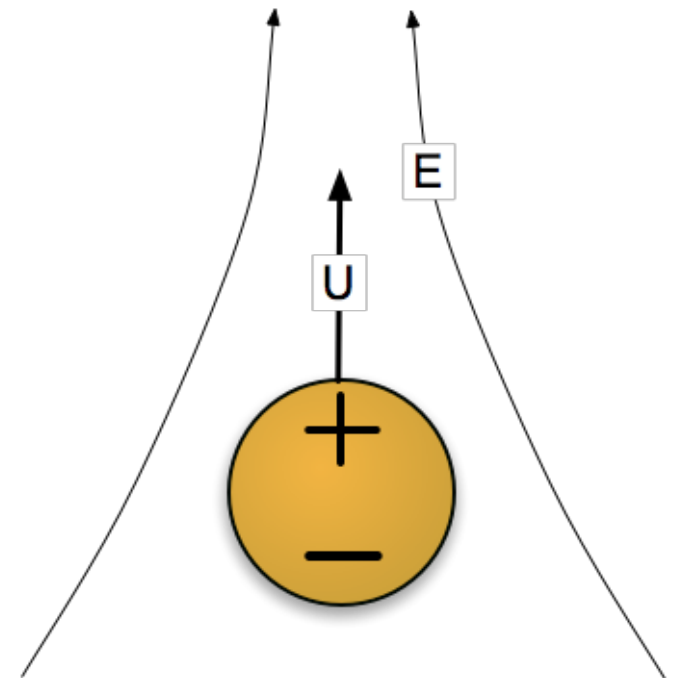
Time-averaged force and torque

$$\langle \vec{F} \rangle = (\text{Re } \vec{p}) \cdot \nabla \vec{E}$$

$$\langle \vec{T} \rangle = (\text{Re } \vec{p}) \times \vec{E}$$

Velocity in Stokes flow

$$\vec{U} = \frac{\langle F \rangle}{6\pi\eta R} \sim \frac{\epsilon R^2 \nabla E^2}{\eta}$$



# “Dipolophoresis” = DEP + ICEP

Shilov & Simonova, Colloid J. USSR (1981, 2001).

Squires & Bazant, J. Fluid Mech. (2006).

T. Miloh, Phys. Fluids (2008)

Metal sphere dipolophoresis

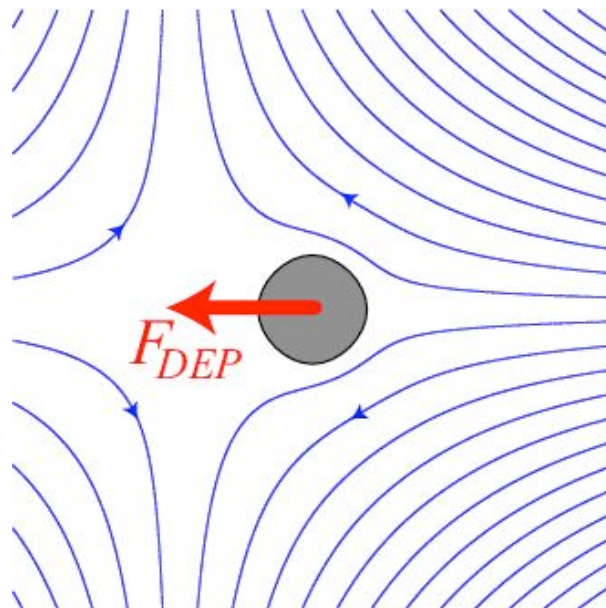
General problem of DEP + ICEP

Thick DL dipolophoresis theory

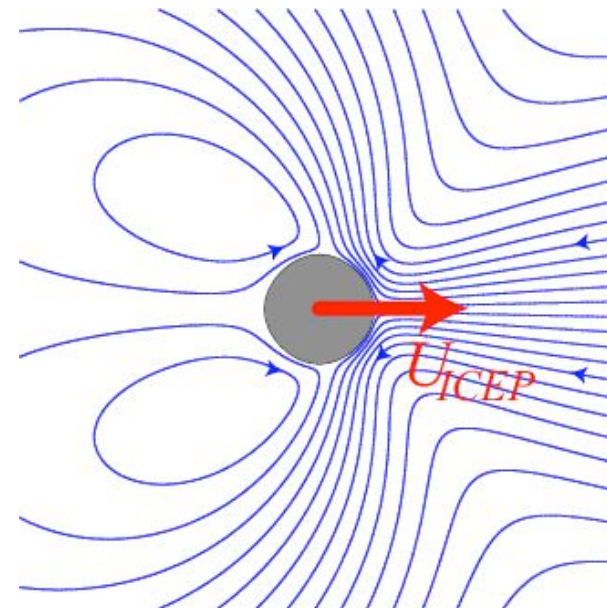
- In an electrolyte, “induced-charge electrophoresis” opposes DEP
- Both effects have the same scaling (and cancel for a metal sphere).

$$U_{ICEP} \sim -U_{DEP} \sim \frac{\varepsilon R^2}{\eta} \nabla |E|^2$$

$\nabla |E| \parallel E$



Electric Field



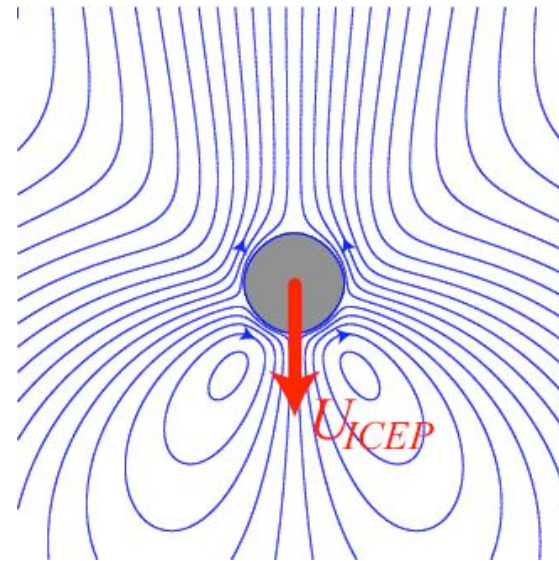
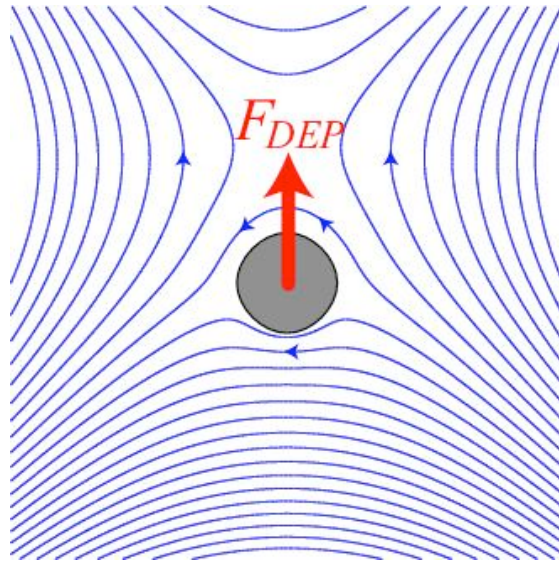
Fluid Streamlines

General solution for any 2d shape in any non-uniform E field by *complex analysis*...

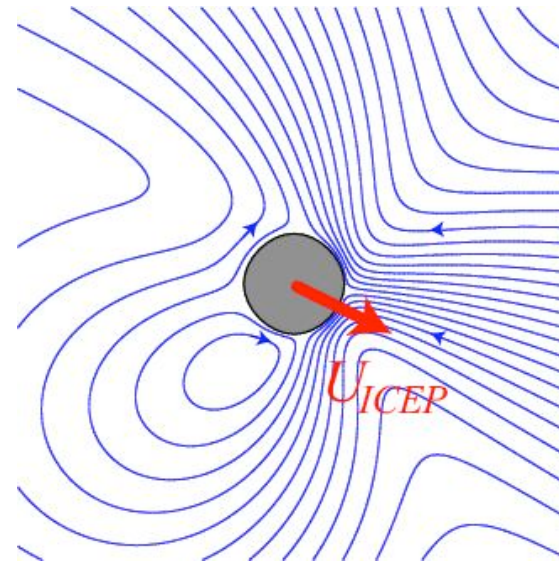
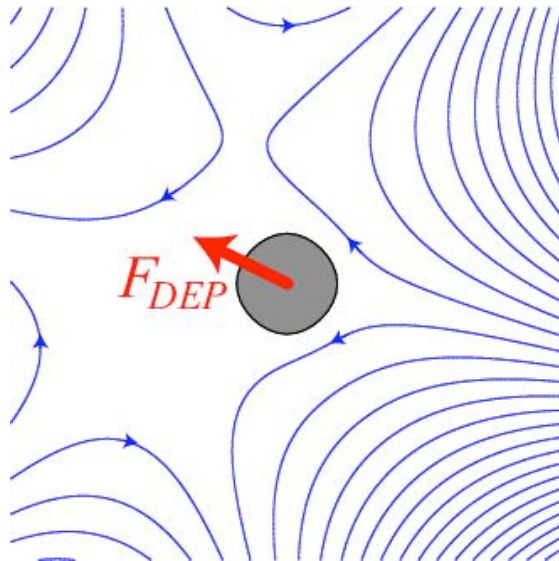
Electric Field

Fluid Streamlines

$$\nabla|E| \perp E$$

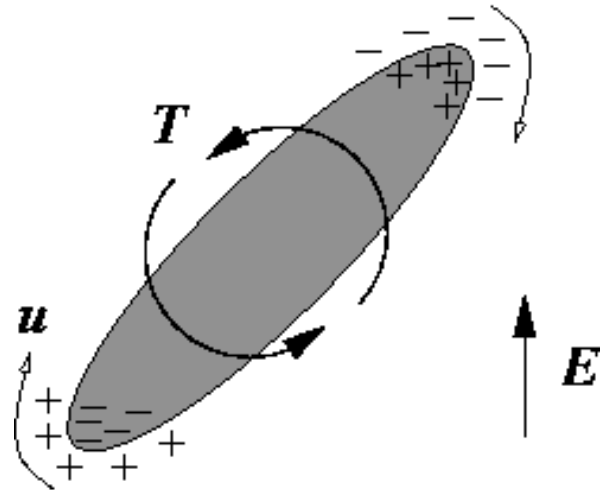


Non-uniform  
 $\nabla E$



Induced-charge electrophoresis  
of *heterogeneous* particles

# Electrokinetic motion of rod-like metal particles



Perturbation theory:  
Squires & Bazant, JFM (2006)

$$\Omega_{ICEP} \sim \frac{\epsilon R E^2}{\eta}$$

$$\sim p \times E = T_{DEP}$$

Rose & Santiago,  
Phys Rev E (2006):  
Experiments on alignment  
of “nano-barcode” particles



Field off

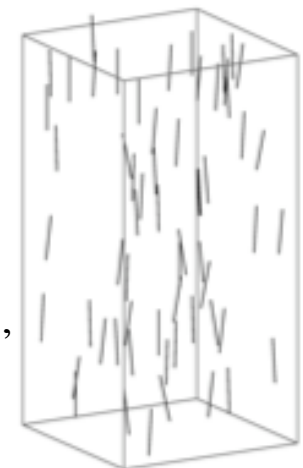


Field on

Saintillan, Darve & Shaqfeh,  
J Fluid Mech (2006): theory  
for spheroids + simulations

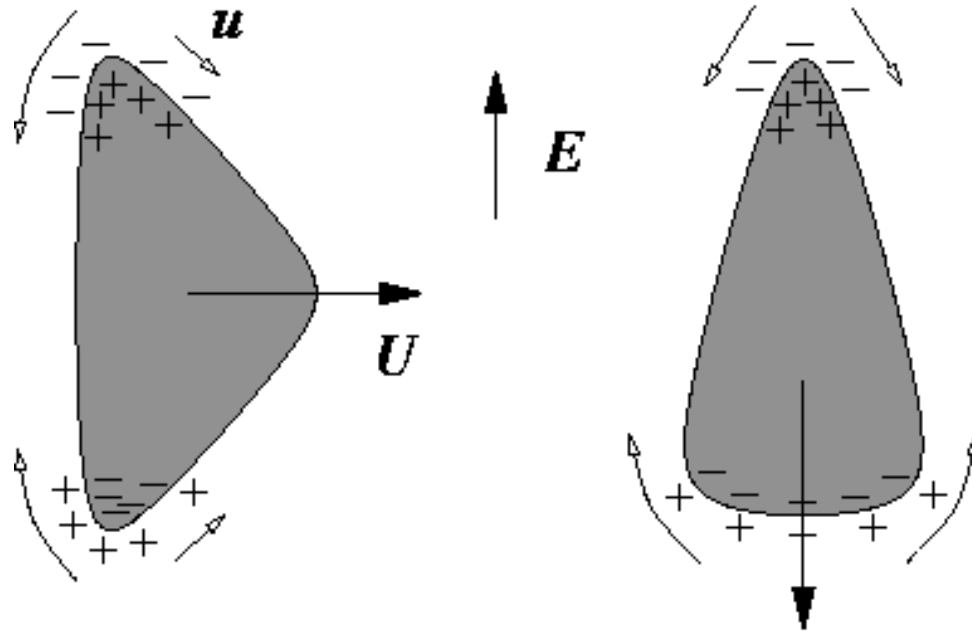


(a)  $Pe = 0$



(c)  $Pe = 1000$

# ICEP of Irregular Shapes



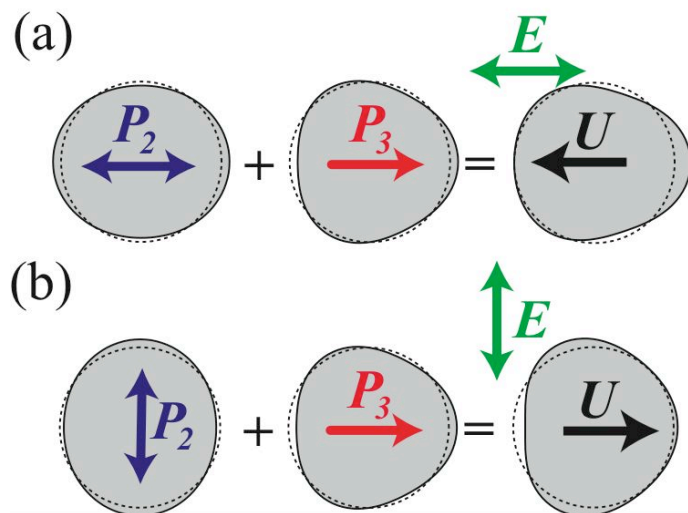
ICEP can separate particles of the same material and same size by shape alone. Any direction is possible.

Expts on quartz particles: Gamayunov & Murtsovkin (1992). Theory: Bazant & Squires (2004)  
Tensor relations: Yariv (2005); Perturbation analysis: Squires & Bazant (2006).



# ICEP of Asymmetric Shapes

Squires & Bazant, J. Fluid Mech. (2006).

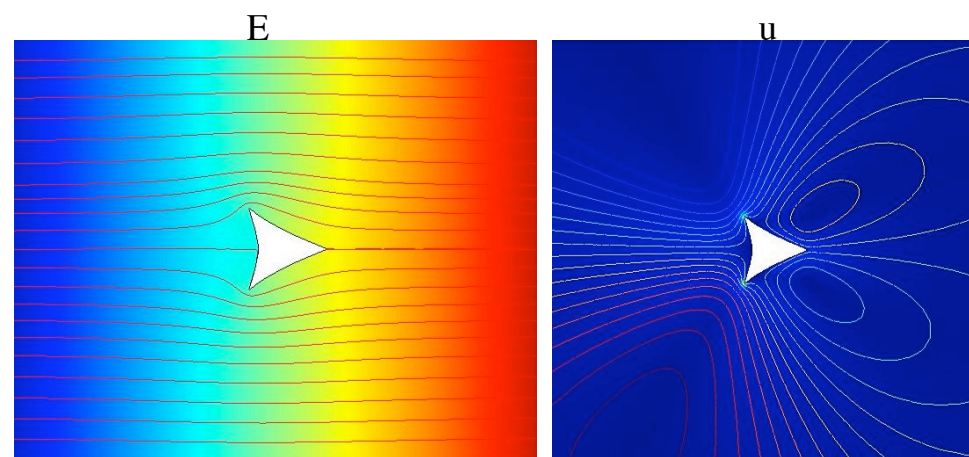


Perturbation analysis

*ICEP can separate polarizable colloids by shape and size in a uniform DC or AC electric field, while normal (linear) electrophoresis cannot.*

- long axis rotates to align with  $E$
- a “thin arrow” swims parallel to  $E$ , towards its “blunt” end
- a “fat arrow” swims transverse to  $E$  towards its “pointed” end

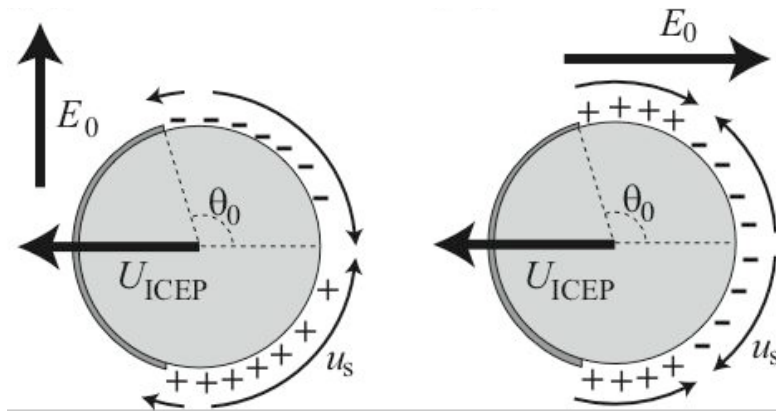
*An asymmetric metal post can pump fluid in any direction* in a uniform DC or AC field, but ICEO flow has quadrupolar rolls, very different from normal EOF.



FEMLAB finite-element simulation (Yuxing Ben)

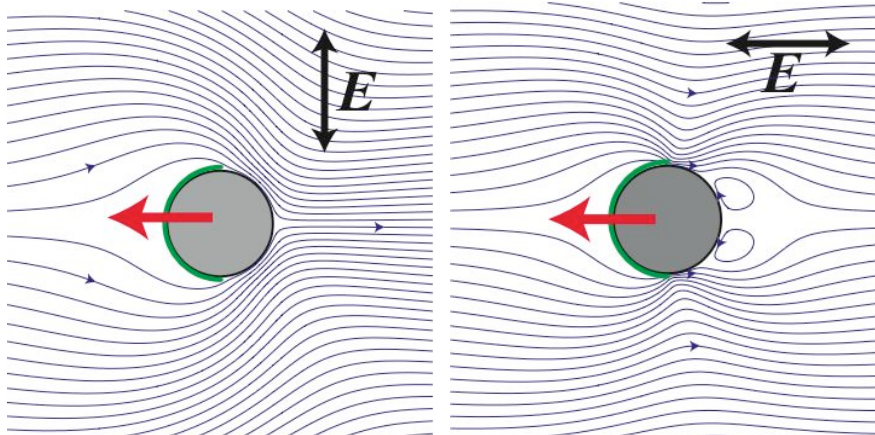
# ICEP of Inhomogeneous Particles

Bazant & Squires, Phys. Rev. Lett. (2004); Squires & Bazant, J. Fluid Mech (2006)



Stable

Unstable



## Example: Janus particle

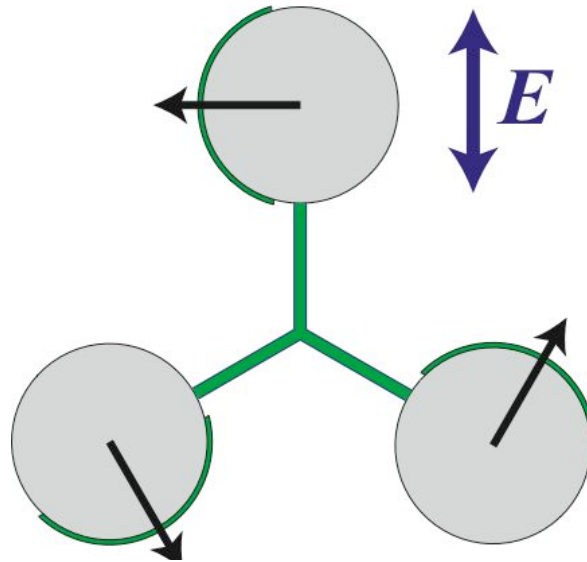
A metal/dielectric sphere in a uniform E field always moves toward its dielectric face, which rotates to perpendicular to E.

The particle swims sideways.

$$U = \frac{9}{64} \frac{\varepsilon a E^2}{\eta}$$

An even more surprising example (Squires & Bazant J Fluid Mech 2006):

## An Electrophoretic Pinwheel

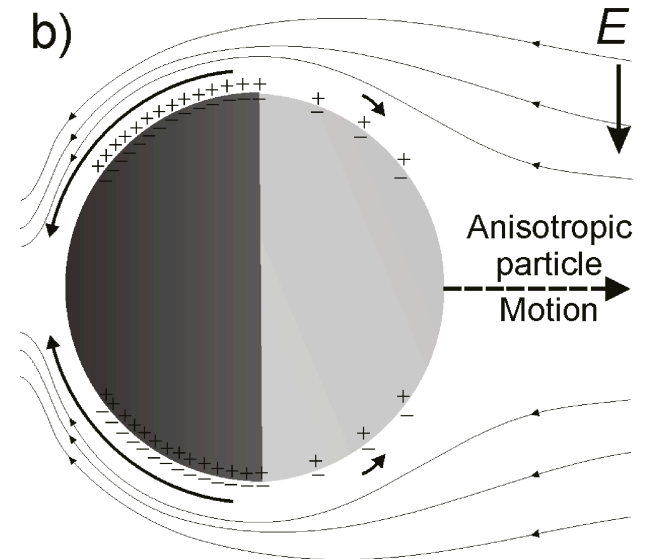
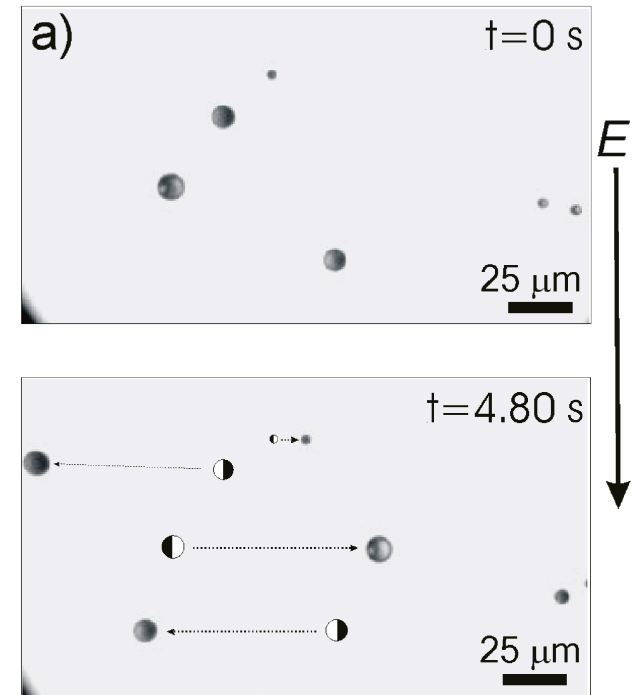
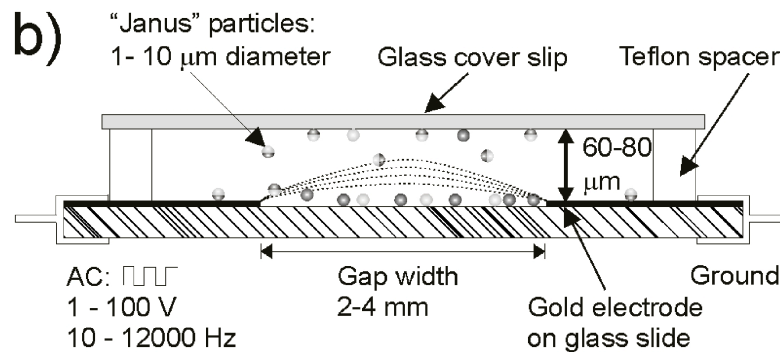
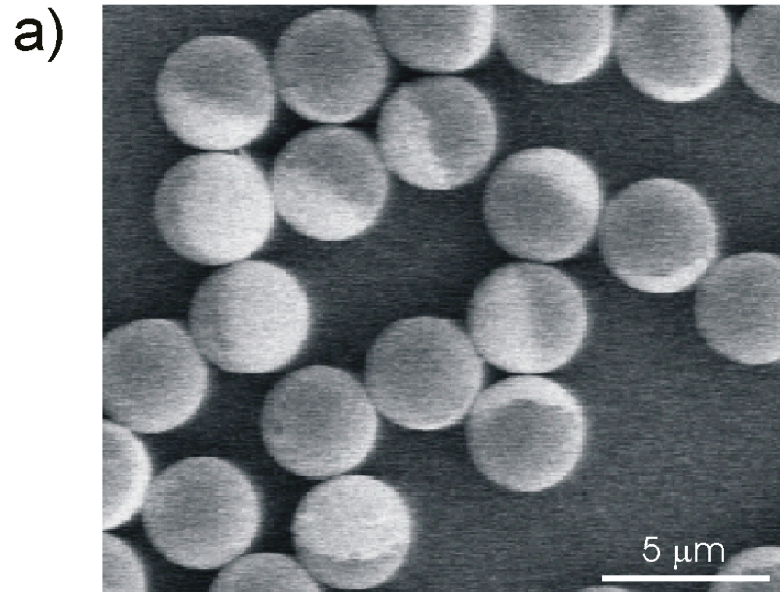


- Responds to any electric field by rotating (  $\vec{\Omega} \perp \vec{E}$  )
- Could be used to apply torques to molecules or cells?

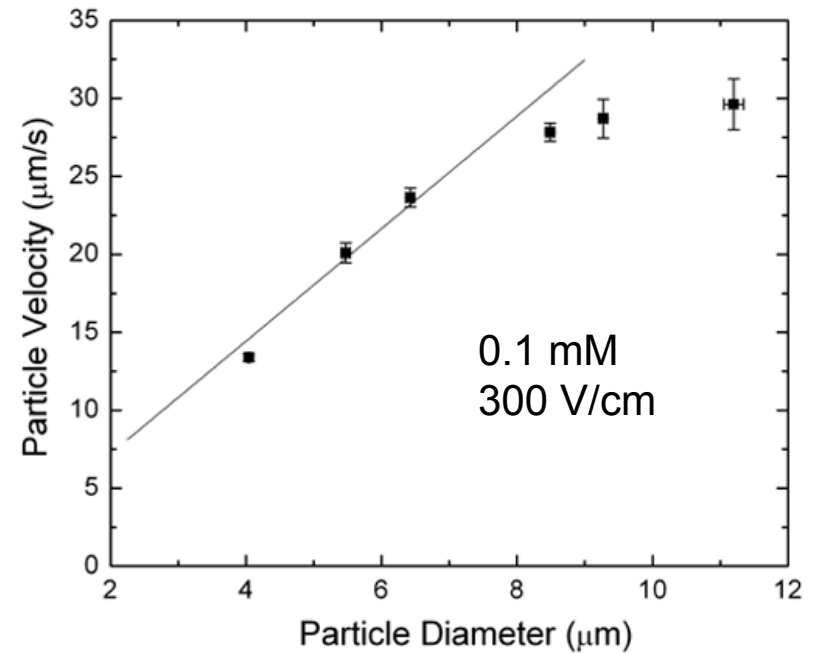
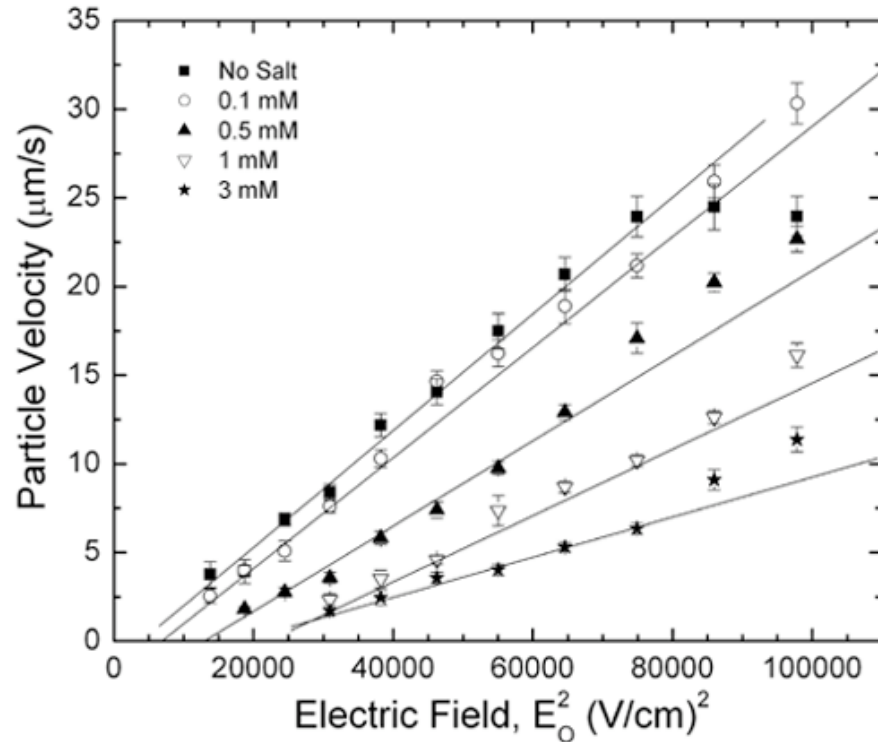
# ICEP Experiments

S. Gangwal, O. Cayre, MZB, O.Velev, Phys Rev. Lett (2008).

## Gold/latex Janus spheres in NaCl



# Experimental data

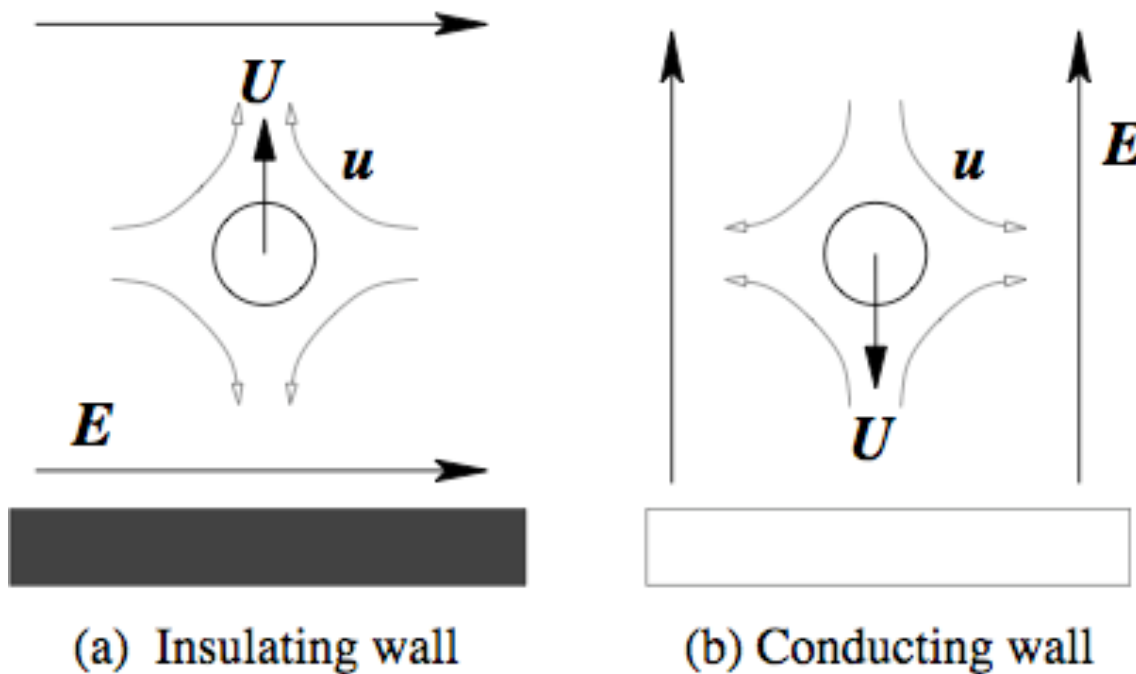


Good fit to theory for

$$c_0 < 1\text{mM}, ER < 10kT/e, \delta = \frac{C_s}{C_D} \approx 10$$

but particles move near the wall... why?

# ICEO wall interactions

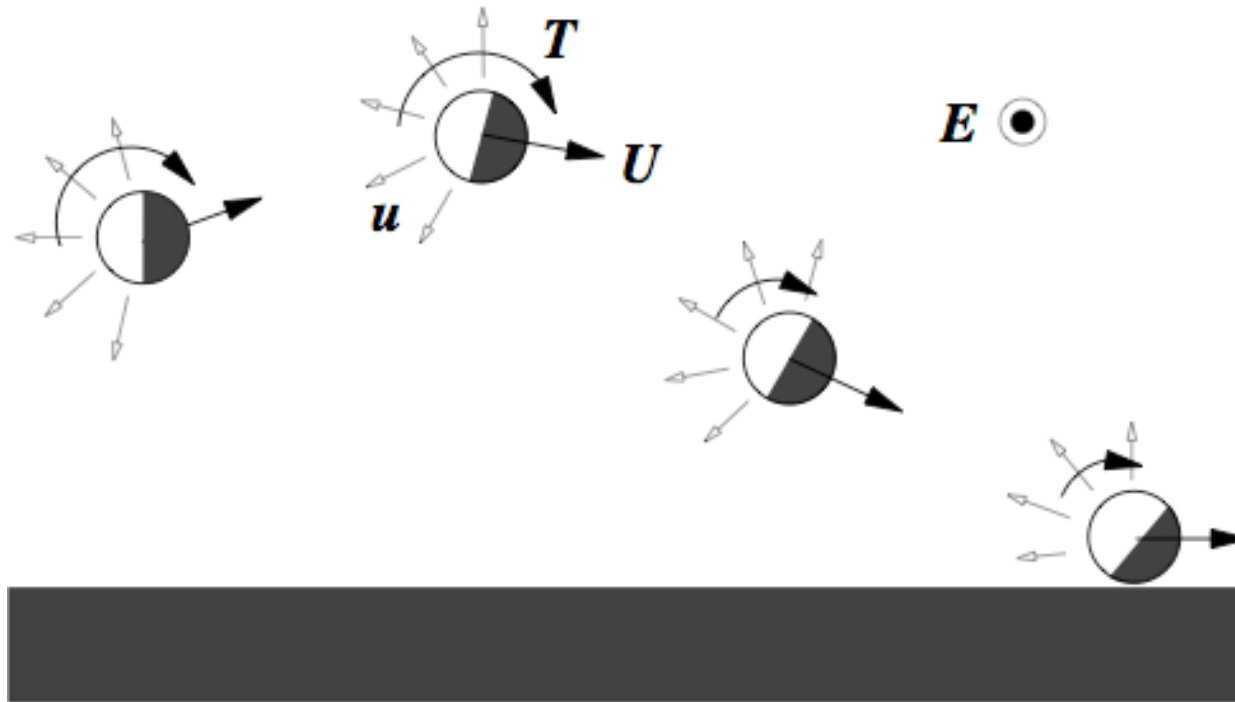


Zhao & Bau, Langmuir (2007)

We expect repulsion from non-polarizable walls, but the Janus particles are attracted to the wall. Why?

# ICEP of Janus particles near a wall

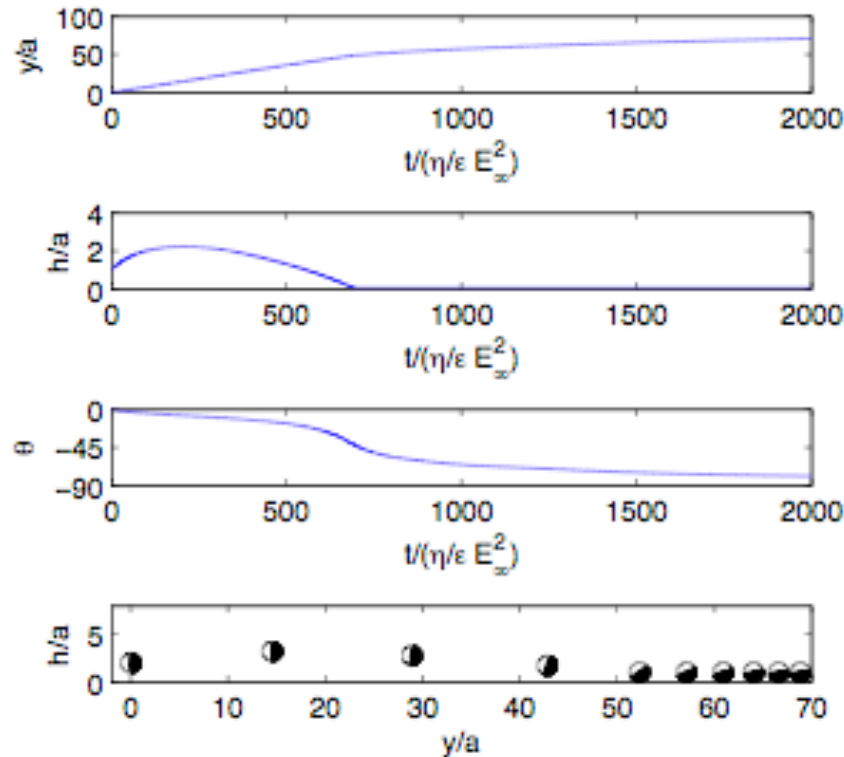
Kilic & Bazant, preprint, arXiv:0712.0453



Without **tilting**, the particle is indeed repelled, but it always rotates to meet the wall and can translate with stable angle.

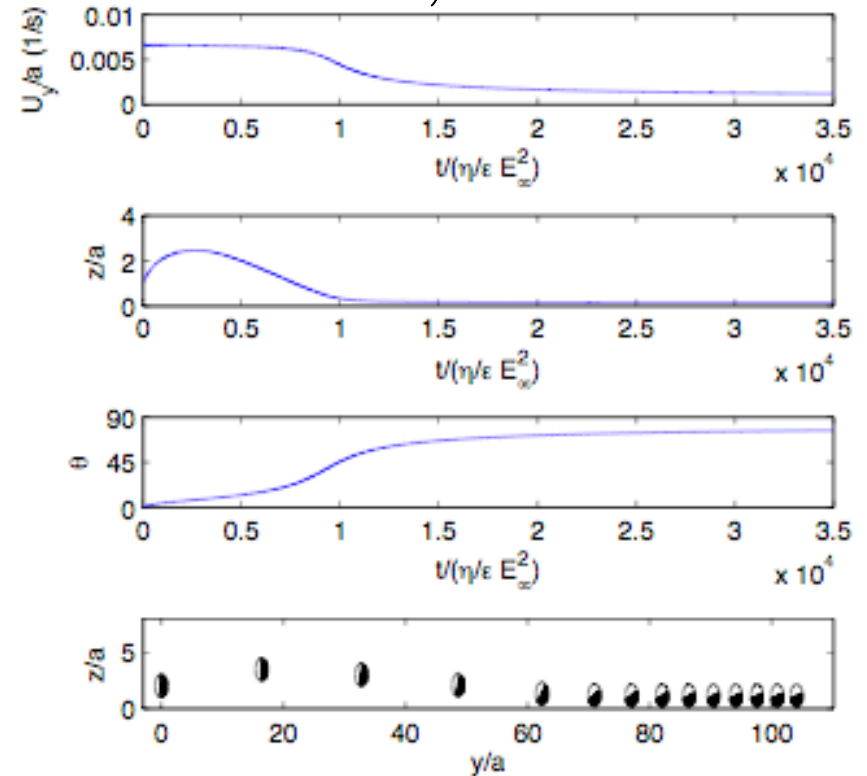
# Simulations of Janus-Wall interaction

$$\delta = 0, \omega = 0$$



Strong ICEO flows make particle face the wall and get stuck.

$$\delta > 5, \omega > 5$$

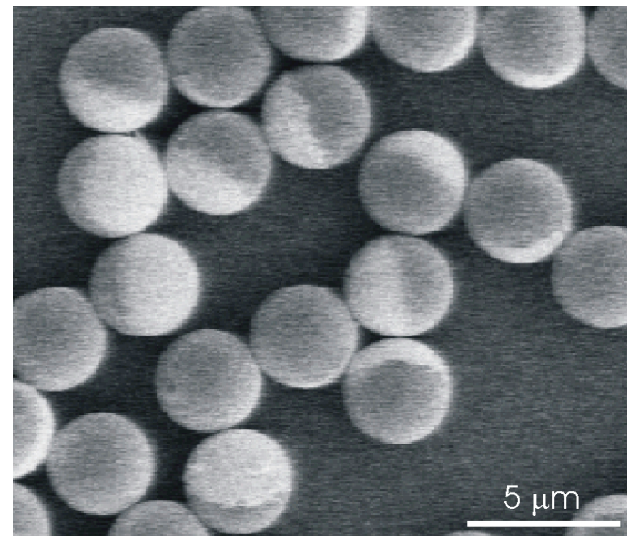
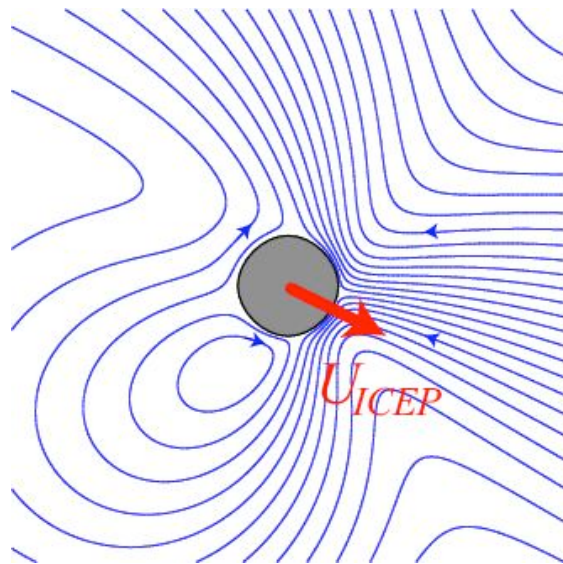


Weaker ICEO flows produce tilted translation as observed (due to DEP).



# Lecture 3: Conclusion

Nonlinear “induced-charge” electrophoresis leads to many new phenomena in colloids with applications to separation, manipulation, self-assembly. Open questions remain.



Papers, slides... <http://web.mit.edu/bazant/www/ICEO>