

Conjugate Gradient for Domain Wall Fermions with 4-d EO preconditioning

Version 1.1.5

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Abstract

This document presents an implementation of a conjugate gradient solver for the Domain Wall Fermion Dirac operator using Pentium 4 streaming SIMD extension (SSE). The code targets SciDAC's cluster machines implementing the QMP protocol.

This version is suboptimal in overlapping communication and computation because of deficiencies of the QMP implementation.

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1 INTRODUCTION

The code below interfaces with a Chroma-like upper level environment to provide file access and machine initialization and configuration. In fact, this file is an implementation of a level 3 routine for solving the Dirac equation. There are some restrictions on input parameters imposed by the algorithm and a particular way the SSE is used by the implementation. There are the following restrictions on the lattice geometry:

- All four-dimensional extends of the lattice should be even. This is required for even-odd decomposition used in the preconditioner.
- The fifth-dimension extend should be a multiple of 4. It is needed for efficient use of SSE registers and simplification of vector code.
- The implementation supports up to four dimensional tori as a network topology.

Because of many issues involved in optimizing the code, it is advantageous to put together some definitions and outline here the optimization strategy used.

1.1 Definitions

Lattice extend is the total size of the lattice in a given dimension.

Network is the logical topology of the network presented by QMP to the application.

Node is a computing element in the network which runs an execution thread. For this implementation we assume that there is one compute node per network location. If an SMP is used, it is the responsibility of QMP to provide a proper abstraction to the application.

Sublattice is the part of the lattice that resides on a compute node.

Site is a point on the lattice.

1.2 Optimization Strategy

For this code we assume that scarcity of resources makes us run the inverter on a small number of nodes compared to the number of sites. This is based on the observation that physics needs grow faster than SciDAC budget and computer deployment plans. We also assume, that the current trend in computer industry persists, namely, that the processors grow faster while memory speed and latency continues to lag in relative terms. We also want a solver whose performance would degrade gracefully when one moves out of the optimization domain. In particular, we impose no limitation on the size of sublattice. There is even no requirement that all sublattices should be of the same size.

For the optimization sweetspot, we assume that the typical problem is too large to fit into the cache hierarchy and mostly resides in main memory. This is true now for existing and proposed clusters and is like to remain true for the future, since large scale computations tend to use larger lattices most of the time.

2 PHYSICS

Here we give the fermion action and γ -matrix and other conventions.

2.1 Dirac Operator

The Domain Wall Fermion Dirac operator is

$$\begin{aligned}\chi_{s,x} = D\psi &= M_0\psi_{s,x} + \sum_{\mu} \left((1 + \gamma_{\mu})U_{x,\mu}\psi_{s,x+\hat{\mu}} + (1 - \gamma_{\mu})U_{x-\hat{\mu},\mu}^{\dagger}\psi_{s,x-\hat{\mu}} \right) \\ &+ (1 + \gamma_5)M_s^{(+)}\psi_{s+1,x} + (1 - \gamma_5)M_s^{(-)}\psi_{s-1,x}\end{aligned}$$

where

$$M_s^{(+)} = \begin{cases} 1, & \text{if } s < N_s - 1 \\ -m_f, & \text{if } s = N_s - 1 \end{cases}$$

and

$$M_s^{(-)} = \begin{cases} 1, & \text{if } s > 0 \\ -m_f, & \text{if } s = 0 \end{cases}$$

We also assume that $\psi_{N_s,x} = \psi_{0,x}$ and $\psi_{-1,x} = \psi_{N_s-1,x}$.

2.2 Gamma matrices

We use the same γ -matrix basis as Chroma to simplify conversion between two codes. The choice below could be changed with a few modifications to the rest of the code, if γ_5 is kept diagonal, and one of other γ -matrices has all nonzero entries equal to +1.

$$\gamma_0 = -\sigma_2 \otimes \sigma_1 = \begin{pmatrix} 0 & i\sigma_1 \\ -i\sigma_1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_0)$:

- 5a $\langle \text{Build } (1 + \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 5a} \rangle \equiv$
 $g \rightarrow f[0][c].re = f \rightarrow f[0][c].re - f \rightarrow f[3][c].im;$
 $g \rightarrow f[0][c].im = f \rightarrow f[0][c].im + f \rightarrow f[3][c].re;$
 $g \rightarrow f[1][c].re = f \rightarrow f[1][c].re - f \rightarrow f[2][c].im;$
 $g \rightarrow f[1][c].im = f \rightarrow f[1][c].im + f \rightarrow f[2][c].re;$
- 5b $\langle \text{Unproject and accumulate } (1 + \gamma_0) \text{ link 5b} \rangle \equiv$
 $qs \rightarrow f[0][c].re += hh[k].f[0][c].re; qs \rightarrow f[3][c].im -= hh[k].f[0][c].re;$
 $qs \rightarrow f[0][c].im += hh[k].f[0][c].im; qs \rightarrow f[3][c].re += hh[k].f[0][c].im;$
 $qs \rightarrow f[1][c].re += hh[k].f[1][c].re; qs \rightarrow f[2][c].im -= hh[k].f[1][c].re;$
 $qs \rightarrow f[1][c].im += hh[k].f[1][c].im; qs \rightarrow f[2][c].re += hh[k].f[1][c].im;$

Now, same for $(1 - \gamma_0)$:

- 5c $\langle \text{Build } (1 - \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 5c} \rangle \equiv$
 $g \rightarrow f[0][c].re = f \rightarrow f[0][c].re + f \rightarrow f[3][c].im;$
 $g \rightarrow f[0][c].im = f \rightarrow f[0][c].im - f \rightarrow f[3][c].re;$
 $g \rightarrow f[1][c].re = f \rightarrow f[1][c].re + f \rightarrow f[2][c].im;$
 $g \rightarrow f[1][c].im = f \rightarrow f[1][c].im - f \rightarrow f[2][c].re;$
- 5d $\langle \text{Unproject and accumulate } (1 - \gamma_0) \text{ link 5d} \rangle \equiv$
 $qs \rightarrow f[0][c].re += hh[k].f[0][c].re; qs \rightarrow f[3][c].im += hh[k].f[0][c].re;$
 $qs \rightarrow f[0][c].im += hh[k].f[0][c].im; qs \rightarrow f[3][c].re -= hh[k].f[0][c].im;$
 $qs \rightarrow f[1][c].re += hh[k].f[1][c].re; qs \rightarrow f[2][c].im += hh[k].f[1][c].re;$
 $qs \rightarrow f[1][c].im += hh[k].f[1][c].im; qs \rightarrow f[2][c].re -= hh[k].f[1][c].im;$

$$\gamma_1 = \sigma_2 \otimes \sigma_2 = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_1)$:

- 5e $\langle \text{Build } (1 + \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 5e} \rangle \equiv$
 $g \rightarrow f[0][c].re = f \rightarrow f[0][c].re - f \rightarrow f[3][c].re;$
 $g \rightarrow f[0][c].im = f \rightarrow f[0][c].im - f \rightarrow f[3][c].im;$
 $g \rightarrow f[1][c].re = f \rightarrow f[1][c].re + f \rightarrow f[2][c].re;$
 $g \rightarrow f[1][c].im = f \rightarrow f[1][c].im + f \rightarrow f[2][c].im;$
- 5f $\langle \text{Unproject and accumulate } (1 + \gamma_1) \text{ link 5f} \rangle \equiv$
 $qs \rightarrow f[0][c].re += hh[k].f[0][c].re; qs \rightarrow f[3][c].re -= hh[k].f[0][c].re;$
 $qs \rightarrow f[0][c].im += hh[k].f[0][c].im; qs \rightarrow f[3][c].im -= hh[k].f[0][c].im;$
 $qs \rightarrow f[1][c].re += hh[k].f[1][c].re; qs \rightarrow f[2][c].re += hh[k].f[1][c].re;$
 $qs \rightarrow f[1][c].im += hh[k].f[1][c].im; qs \rightarrow f[2][c].im += hh[k].f[1][c].im;$

Now, same for $(1 - \gamma_1)$:

- 5g $\langle \text{Build } (1 - \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 5g} \rangle \equiv$
 $g \rightarrow f[0][c].re = f \rightarrow f[0][c].re + f \rightarrow f[3][c].re;$
 $g \rightarrow f[0][c].im = f \rightarrow f[0][c].im + f \rightarrow f[3][c].im;$
 $g \rightarrow f[1][c].re = f \rightarrow f[1][c].re - f \rightarrow f[2][c].re;$
 $g \rightarrow f[1][c].im = f \rightarrow f[1][c].im - f \rightarrow f[2][c].im;$
- 5h $\langle \text{Unproject and accumulate } (1 - \gamma_1) \text{ link 5h} \rangle \equiv$
 $qs \rightarrow f[0][c].re += hh[k].f[0][c].re; qs \rightarrow f[3][c].re += hh[k].f[0][c].re;$
 $qs \rightarrow f[0][c].im += hh[k].f[0][c].im; qs \rightarrow f[3][c].im += hh[k].f[0][c].im;$
 $qs \rightarrow f[1][c].re += hh[k].f[1][c].re; qs \rightarrow f[2][c].re -= hh[k].f[1][c].re;$
 $qs \rightarrow f[1][c].im += hh[k].f[1][c].im; qs \rightarrow f[2][c].im -= hh[k].f[1][c].im;$

$$\gamma_2 = -\sigma_2 \otimes \sigma_3 = \begin{pmatrix} 0 & i\sigma_3 \\ -i\sigma_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_2)$:

6a $\langle \textit{Build } (1 + \gamma_2) \textit{ projection of } \mathbf{f} \textit{ in } \mathbf{*g} \textit{ 6a} \rangle \equiv$
 $\text{g} \rightarrow \text{f}[0][c].\text{re} = \text{f} \rightarrow \text{f}[0][c].\text{re} - \text{f} \rightarrow \text{f}[2][c].\text{im};$
 $\text{g} \rightarrow \text{f}[0][c].\text{im} = \text{f} \rightarrow \text{f}[0][c].\text{im} + \text{f} \rightarrow \text{f}[2][c].\text{re};$
 $\text{g} \rightarrow \text{f}[1][c].\text{re} = \text{f} \rightarrow \text{f}[1][c].\text{re} + \text{f} \rightarrow \text{f}[3][c].\text{im};$
 $\text{g} \rightarrow \text{f}[1][c].\text{im} = \text{f} \rightarrow \text{f}[1][c].\text{im} - \text{f} \rightarrow \text{f}[3][c].\text{re};$

6b $\langle \textit{Unproject and accumulate } (1 + \gamma_2) \textit{ link 6b} \rangle \equiv$
 $\text{qs} \rightarrow \text{f}[0][c].\text{re} += \text{hh}[k].\text{f}[0][c].\text{re}; \text{qs} \rightarrow \text{f}[2][c].\text{im} -= \text{hh}[k].\text{f}[0][c].\text{re};$
 $\text{qs} \rightarrow \text{f}[0][c].\text{im} += \text{hh}[k].\text{f}[0][c].\text{im}; \text{qs} \rightarrow \text{f}[2][c].\text{re} += \text{hh}[k].\text{f}[0][c].\text{im};$
 $\text{qs} \rightarrow \text{f}[1][c].\text{re} += \text{hh}[k].\text{f}[1][c].\text{re}; \text{qs} \rightarrow \text{f}[3][c].\text{im} += \text{hh}[k].\text{f}[1][c].\text{re};$
 $\text{qs} \rightarrow \text{f}[1][c].\text{im} += \text{hh}[k].\text{f}[1][c].\text{im}; \text{qs} \rightarrow \text{f}[3][c].\text{re} -= \text{hh}[k].\text{f}[1][c].\text{im};$

Now, same for $(1 - \gamma_2)$:

6c $\langle \textit{Build } (1 - \gamma_2) \textit{ projection of } \mathbf{*f} \textit{ in } \mathbf{*g} \textit{ 6c} \rangle \equiv$
 $\text{g} \rightarrow \text{f}[0][\text{c}].\text{re} = \text{f} \rightarrow \text{f}[0][\text{c}].\text{re} + \text{f} \rightarrow \text{f}[2][\text{c}].\text{im};$
 $\text{g} \rightarrow \text{f}[0][\text{c}].\text{im} = \text{f} \rightarrow \text{f}[0][\text{c}].\text{im} - \text{f} \rightarrow \text{f}[2][\text{c}].\text{re};$
 $\text{g} \rightarrow \text{f}[1][\text{c}].\text{re} = \text{f} \rightarrow \text{f}[1][\text{c}].\text{re} - \text{f} \rightarrow \text{f}[3][\text{c}].\text{im};$
 $\text{g} \rightarrow \text{f}[1][\text{c}].\text{im} = \text{f} \rightarrow \text{f}[1][\text{c}].\text{im} + \text{f} \rightarrow \text{f}[3][\text{c}].\text{re};$

6d $\langle \textit{Unproject and accumulate } (1 - \gamma_2) \textit{ link 6d} \rangle \equiv$
 $\text{qs} \rightarrow \text{f}[0][\text{c}].\text{re} += \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{re}; \text{qs} \rightarrow \text{f}[2][\text{c}].\text{im} += \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{re};$
 $\text{qs} \rightarrow \text{f}[0][\text{c}].\text{im} += \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{im}; \text{qs} \rightarrow \text{f}[2][\text{c}].\text{re} -= \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{im};$
 $\text{qs} \rightarrow \text{f}[1][\text{c}].\text{re} += \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{re}; \text{qs} \rightarrow \text{f}[3][\text{c}].\text{im} -= \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{re};$
 $\text{qs} \rightarrow \text{f}[1][\text{c}].\text{im} += \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{im}; \text{qs} \rightarrow \text{f}[3][\text{c}].\text{re} += \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{im};$

$$\gamma_3 = \sigma_1 \otimes 1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_3)$:

6e $\langle \text{Build } (1 + \gamma_3) \text{ projection of } \mathbf{*f} \text{ in } \mathbf{*g} \text{ 6e} \rangle \equiv$
 $\text{g} \rightarrow \text{f}[0][\text{c}].\text{re} = \text{f} \rightarrow \text{f}[0][\text{c}].\text{re} + \text{f} \rightarrow \text{f}[2][\text{c}].\text{re};$
 $\text{g} \rightarrow \text{f}[0][\text{c}].\text{im} = \text{f} \rightarrow \text{f}[0][\text{c}].\text{im} + \text{f} \rightarrow \text{f}[2][\text{c}].\text{im};$
 $\text{g} \rightarrow \text{f}[1][\text{c}].\text{re} = \text{f} \rightarrow \text{f}[1][\text{c}].\text{re} + \text{f} \rightarrow \text{f}[3][\text{c}].\text{re};$
 $\text{g} \rightarrow \text{f}[1][\text{c}].\text{im} = \text{f} \rightarrow \text{f}[1][\text{c}].\text{im} + \text{f} \rightarrow \text{f}[3][\text{c}].\text{im};$

6f $\langle \text{Unproject } (1 + \gamma_3) \text{ link 6f} \rangle \equiv$
 $\text{qs} \rightarrow \text{f}[0][\text{c}].\text{re} = \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{re}; \text{qs} \rightarrow \text{f}[2][\text{c}].\text{re} = \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{re};$
 $\text{qs} \rightarrow \text{f}[0][\text{c}].\text{im} = \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{im}; \text{qs} \rightarrow \text{f}[2][\text{c}].\text{im} = \text{hh}[\text{k}].\text{f}[0][\text{c}].\text{im};$
 $\text{qs} \rightarrow \text{f}[1][\text{c}].\text{re} = \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{re}; \text{qs} \rightarrow \text{f}[3][\text{c}].\text{re} = \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{re};$
 $\text{qs} \rightarrow \text{f}[1][\text{c}].\text{im} = \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{im}; \text{qs} \rightarrow \text{f}[3][\text{c}].\text{im} = \text{hh}[\text{k}].\text{f}[1][\text{c}].\text{im};$

Now, same for $(1 - \gamma_3)$:

6g $\langle \text{Build } (1 - \gamma_3) \text{ projection of } *f \text{ in } *g \text{ 6g} \rangle \equiv$
 $g \rightarrow f[0][c].re = f \rightarrow f[0][c].re - f \rightarrow f[2][c].re;$
 $g \rightarrow f[0][c].im = f \rightarrow f[0][c].im - f \rightarrow f[2][c].im;$
 $g \rightarrow f[1][c].re = f \rightarrow f[1][c].re - f \rightarrow f[3][c].re;$
 $g \rightarrow f[1][c].im = f \rightarrow f[1][c].im - f \rightarrow f[3][c].im;$

6h $\langle \text{Unproject and accumulate } (1 - \gamma_3) \text{ link 6h} \rangle \equiv$
 $qs \rightarrow f[0][c].re += hh[k].f[0][c].re; qs \rightarrow f[2][c].re -= hh[k].f[0][c].re;$
 $qs \rightarrow f[0][c].im += hh[k].f[0][c].im; qs \rightarrow f[2][c].im -= hh[k].f[0][c].im;$
 $qs \rightarrow f[1][c].re += hh[k].f[1][c].re; qs \rightarrow f[3][c].re -= hh[k].f[1][c].re;$
 $qs \rightarrow f[1][c].im += hh[k].f[1][c].im; qs \rightarrow f[3][c].im -= hh[k].f[1][c].im;$

3 CONJUGATE GRADIENT

Here we develop the algorithm used in the solver.

3.1 Standard Algorithm

The basic conjugate gradient algorithm 1 is simple. Its only requirement is that matrix A is hermitian. Otherwise, it appears suited for DWF better than other iterative solvers.

```
Input:  $A$ , the matrix  
Input:  $b$ , the right hand side of the linear equation  
Input:  $x_0$ , an initial guess  
Input:  $n$ , the maximum number of iterations  
Input:  $\epsilon$ , required precision  
Output:  $x$ , approximate solution  
Output:  $\rho$ , final residue  
Output:  $k$ , number of iterations used  
begin  
   $x \leftarrow x_0$   
   $p \leftarrow r \leftarrow b - Ax$   
   $\rho \leftarrow \langle r, r \rangle$   
   $k \leftarrow 0$   
  while  $\rho > \epsilon$  or  $k < n$  do  
     $q \leftarrow Ap$   
     $\alpha \leftarrow \rho / \langle p, q \rangle$   
     $r \leftarrow r - \alpha q$   
     $x \leftarrow x + \alpha p$   
     $\gamma \leftarrow \langle r, r \rangle$   
    if  $\gamma < \epsilon$  then  
       $\rho \leftarrow \gamma$   
      break  
    end  
     $\beta \leftarrow \gamma / \rho$   
     $\rho \leftarrow \gamma$   
     $p \leftarrow r + \beta p$   
     $k \leftarrow k + 1$   
  end  
  return  $x, \rho, k$ .  
end
```

Algorithm 1: Generic Conjugate Gradient Solver

3.2 Overlap Opportunities

Our approach to overlapping computations with communications is to break the sublattice into boundary and inside pieces. After that, we first compute $(1 \pm \gamma_\mu)$ projections on the boundary and start send and receive operations. While communications are in progress, everything is computed on the inside nodes of the sublattice. Once receive is complete, we compute the operator on the boundary sites. Such an approach helps to improve temporal locality (and, therefore, cache utilization) at the expence of losing the ability of overlap if one of the sublattice dimensions is 2. However, it is unlikely that we could afford a large enough cluster to be forced into this corner of the parameter space.

3.3 Non-hermitial Matrix

Hermiticity of M is the only obstacle in applying algorithm 1 directly to our problem $M\psi = \eta$. This issue can be easily resolved by multiplying both sides by M^\dagger . However, instead of using algorithm 1 with $A = M^\dagger M$, it is better to keep M and M^\dagger separate—this makes it possible to hide one of the global sum computations, thus improving machine size scaling. Algorithm 2 is what we use in the solver.

Input: M , the matrix
Input: b , the right hand side of the linear equation
Input: x_0 , an initial guess
Input: n , the maximum number of iterations
Input: ϵ , required precision
Output: x , approximate solution
Output: ρ , final residue
Output: k , number of iterations used
begin
 $x \leftarrow x_0$
 $p \leftarrow r \leftarrow b - M^\dagger Mx$
 $\rho \leftarrow \langle r, r \rangle$
 $k \leftarrow 0$
while $\rho > \epsilon$ **or** $k < n$ **do**
 $z \leftarrow Mp$
 $q \leftarrow M^\dagger z$
 $\alpha \leftarrow \rho / \langle z, z \rangle$
 $r \leftarrow r - \alpha q$
 $x \leftarrow x + \alpha p$
 $\gamma \leftarrow \langle r, r \rangle$
if $\gamma < \epsilon$ **then**
 $\rho \leftarrow \gamma$
break
end
 $\beta \leftarrow \gamma / \rho$
 $\rho \leftarrow \gamma$
 $p \leftarrow r + \beta p$
 $k \leftarrow k + 1$
end
return x, ρ, k .
end

Algorithm 2: DWF-ready Gradient Solver.

4 PRECONDITIONING

We use four dimensional preconditioner to improve convergence of the CG. Following Kostas Orginos, let us color the lattice sites according to the parity of $x_0 + x_1 + x_2 + x_3$. Then we can rewrite $\chi = D\psi$ as follows:

$$\begin{pmatrix} \chi_e \\ \chi_o \end{pmatrix} = D\psi = \begin{pmatrix} Q_{ee} & Q_{eo} \\ Q_{oe} & Q_{oo} \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_o \end{pmatrix}$$

From the form of D it follows that all dependance on the gauge field is located in Q_{xy} , and that Q_{xx} does not depend on U . That, in turn, allows us to invert Q_{xx} easily. With this in mind, one writes:

$$\begin{pmatrix} Q_{ee} & Q_{eo} \\ Q_{oe} & Q_{oo} \end{pmatrix} = \begin{pmatrix} Q_{ee} & 0 \\ Q_{oe} & Q_{oo} \end{pmatrix} \begin{pmatrix} 1 & Q_{ee}^{-1}Q_{eo} \\ 0 & 1 - Q_{oo}^{-1}Q_{oe}Q_{ee}^{-1}Q_{eo} \end{pmatrix}$$

Now, to solve the equation

$$D\psi = \eta,$$

one needs to perform the following steps:

1. Compute

$$\phi_o = Q_{oo}^{-1}(\eta_o - Q_{oe}Q_{ee}^{-1}\eta_e)$$

2. Set $M = 1 - Q_{oo}^{-1}Q_{oe}Q_{ee}^{-1}Q_{eo}$ for the following.

3. Compute

$$\varphi_o = M^\dagger \phi_o$$

4. Solve for ψ_o the following equation using Algorithm 2

$$M^\dagger M \psi_o = \varphi_o$$

5. Compute

$$\psi_e = Q_{ee}^{-1}(\eta_e - Q_{eo}\psi_o)$$

Note, that $M^\dagger = 1 - (Q_{eo})^\dagger(Q_{ee}^{-1})^\dagger(Q_{oe})^\dagger(Q_{oo}^{-1})^\dagger = 1 - S_{oe}S_{ee}^{-1}S_{oe}S_{oo}^{-1}$, where

$$\begin{aligned} S_{ee} &= Q_{ee}[\gamma_5 \rightarrow -\gamma_5] \\ S_{oo} &= Q_{oo}[\gamma_5 \rightarrow -\gamma_5] \\ S_{oe} &= Q_{eo}[\gamma_\mu \rightarrow -\gamma_\mu] \\ S_{eo} &= Q_{oe}[\gamma_\mu \rightarrow -\gamma_\mu] \end{aligned}$$

4.1 Q_{xx} inversion

The previous section is based on a tacit assumption that Q_{ee} and Q_{oo} are easy to invert. Here we show that it is so. Let us rewrite

$$\chi_{s,x} = (Q_{ee}\psi)_{s,x} = M_0\psi_{s,x} + (1 + \gamma_5)M_s^{(+)}\psi_{s+1,x} + (1 + \gamma_5)M_s^{(-)}\psi_{s-1,x}$$

as follows:

$$(Q_{ee}\psi)_{s,x} = M_0 \left(\left(\frac{1 + \gamma_5}{2} \right) \left(\psi_{s,x} + \frac{2M_s^{(+)}}{M_0}\psi_{s+1,x} \right) + \left(\frac{1 - \gamma_5}{2} \right) \left(\psi_{s,x} + \frac{2M_s^{(-)}}{M_0}\psi_{s-1,x} \right) \right).$$

Thus,

$$Q_{ee} = \frac{1 + \gamma_5}{2} \begin{pmatrix} a & b & \cdots & 0 & 0 \\ 0 & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & b \\ c & 0 & \cdots & 0 & a \end{pmatrix} + \frac{1 - \gamma_5}{2} \begin{pmatrix} a & 0 & \cdots & 0 & c \\ b & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & 0 \\ 0 & 0 & \cdots & b & a \end{pmatrix} = P_+A + P_-B,$$

where $a = M_0$, $b = 2$, $c = -2m_f$. Computing $Q_{xx}\psi$ and $S_{xx}\psi$ is done below with SSE. Here we compute constant values needed for effective use of SSE:

9 $\langle \text{Compute constant values for } Q_{xx} \text{ and } S_{xx} \rangle \equiv$
 $\{$
 $\quad \text{double } a = M_0;$
 $\quad \text{double } b = 2.0;$
 $\quad \text{double } c = -2 * m_f;$

 $\quad va = \text{vmk1}(a);$
 $\quad vb4 = \text{vmk1}(b);$
 $\quad vcb3 = \text{vmk4}(c, b, b, b);$
 $\quad vb3c = \text{vmk4}(b, b, b, c);$
 $\}$

10 $\langle \text{Global variables} \rangle \equiv$
 $\quad \text{static vReal } vcb3;$
 $\quad \text{static vReal } vb3c;$
 $\quad \text{static vReal } vb4;$
 $\quad \text{static vReal } va;$

Now, since P_{\pm} commute with A and B , $Q_{ee}^{-1} = P_+ A^{-1} + P_- B^{-1}$. Computing A^{-1} and B^{-1} is done by decomposition $A = L_A R_A$, $B = L_B R_B$, where

$$R_A = \begin{pmatrix} a & b & \cdots & 0 & 0 \\ 0 & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & b \\ 0 & 0 & \cdots & 0 & a \end{pmatrix} \quad R_B = \begin{pmatrix} a & 0 & \cdots & 0 & 0 \\ b & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & 0 \\ 0 & 0 & \cdots & b & a \end{pmatrix},$$

and

$$L_A = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & & 0 \\ \vdots & & & & & \vdots \\ c/a & -bc/a^2 & b^2c/a^3 & -b^3c/a^4 & \cdots & 1 + (-b)^{n-1}c/a^n \end{pmatrix}$$

$$L_B = \begin{pmatrix} 1 + (-b)^{n-1}c/a^n & (-b)^{n-2}c/a^{n-1} & \cdots & b^2c/a^3 & -bc/a^2 & c/a \\ 0 & 1 & & 0 & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix}.$$

In these terms,

$$Q_{ee}^{-1} = \frac{1 + \gamma_5}{2} R_A^{-1} L_A^{-1} + \frac{1 - \gamma_5}{2} R_B^{-1} L_B^{-1}.$$

We will also need

$$S_{ee}^{-1} = \frac{1 - \gamma_5}{2} R_A^{-1} L_A^{-1} + \frac{1 + \gamma_5}{2} R_B^{-1} L_B^{-1}.$$

For further reference,

$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

5 CODE

This section contains chunks of the source that go into `dwf.c` source file. We start with the interface functions and elaborate from there.

5.1 Interface Functions

We can not expect the user to call different parts of the interface in an appropriate order. Therefore, successful initialization allows the user to call other interface elements, as well as prevents repeated initializations.

```
11a  <Global variables 10>+≡
      static int initied_p = 0;
```

5.1.1 SSE DWF Initializer

```
11b  <Interface functions 11b>≡
      int
      SSE_DWF_init(const int lattice[DIM+1],
                    SSE_DWF_FP_SIZE fp_size,
                    void *(*allocator)(size_t size),
                    void (*deallocater)(void *))
      {
          if (initied_p)
              return 1; /* error: second init */

          <Check floating point size 12a>
          <Check lattice size 12b>
          <Get network topology 21a>
          <Setup heap management functions 11d>
          <Initialize tables 25b>
          <Allocate fields 38c>
          <Initialize QMP 34c>
          initied_p = 1;
          return 0;

          <Handle init errors 11c>
      }
```

If any error occurs during initialization, we simply unroll state and fail:

```
11c  <Handle init errors 11c>≡
      error:
          SSE_DWF_fini();
          return 1;
```

Check if the user requested special allocation mechanisms:

```
11d  <Setup heap management functions 11d>≡
      if (allocator)
          tmalloc = allocator;
      else
          tmalloc = malloc;

      if (deallocater)
          tfree = deallocater;
      else
          tfree = free;
```

```
11e  <Global variables 10>+≡
      static void *(*tmalloc)(size_t size);
      static void (*tfree)(void *ptr);
```

For now we only support single precision floating point numbers:

```
12a <Check floating point size 12a>≡
    if (fp_size != SSE_DWF_FLOAT)
        goto error;
```

For single precision arithmentics, L_s should be a muplitple of 4.

```
12b <Check lattice size 12b>≡
    if (lattice[DIM] % Vs)
        goto error;
    tlattice[DIM] = lattice[DIM];
```

Otherwise, lattice sizes must be even to allow us to do red/black preconditioning:

```
12c <Check lattice size 12b>+≡
{
    int i;
    for (i = 0; i < DIM; i++) {
        if (lattice[i] & 1)
            goto error;
        tlattice[i] = lattice[i];
    }
}
```

```
12d <Global variables 10>+≡
    static int tlattice[DIM+1];
```

5.1.2 SSE DWF Clean Up

The cleanup routine may be called from partially initialized context, we should be able to do a partial cleanup.

```
12e <Interface functions 11b>+≡
void
SSE_DWF_fini(void)
{
    <Cleanup QMP 37b>
    <Free fields 38d>
    <Free tables 32d>
    initied_p = 0;
}
```

5.1.3 DWF Fermion Allocator

When one needs an SSE DWF fermion, the allocator does the job. Remember, users are stupid enough to call this function in the uninitialized state. It is convenient to break all internal fermions into odd and even parts at this stage.

```
12f <Data types 12f>≡
struct SSE_DWF_Fermion {
    vEvenFermion *even;
    vOddFermion *odd;
};
```

Now, the fermion allocator proper:

```

13a  <Interface functions 11b>+≡
      SSE_DWF_Fermion *
      SSE_DWF_allocate_fermion(void)
      {
          SSE_DWF_Fermion *ptr;

          if (!inited_p)
              return 0;

          ptr = tmalloc(sizeof (*ptr));
          if (ptr == 0)
              return 0;

          ptr->even = allocate_even_fermion();
          if (ptr->even == 0)
              goto error1;

          ptr->odd  = allocate_odd_fermion();
          if (ptr->odd == 0)
              goto error2;

          return ptr;
      error2:
          free16(ptr->even);
      error1:
          tfree(ptr);
          return 0;
      }

```

5.1.4 DWF Fermion Exporter

When we need to create an SSE fermion field and populate it from an outer environment, we use the following procedure

```

13b  <Interface functions 11b>+≡
      SSE_DWF_Fermion *
      SSE_DWF_load_fermion(const void *OuterFermion,
                          void *env,
                          SSE_DWF_fermion_reader reader)
      {
          SSE_DWF_Fermion *ptr = SSE_DWF_allocate_fermion();

          /* Handle both lack of memory and missing initialization */
          if (ptr == 0)
              return 0;

          <Read fermion 23c>

          return ptr;
      }

```

5.1.5 DWF Fermion Importer

For moving data back to the outer environment, the following importer is used:

```
14a  <Interface functions 11b>+≡
      void
      SSE_DWF_save_fermion(void *OuterFermion,
                           void *env,
                           SSE_DWF_fermion_writer writer,
                           SSE_DWF_Fermion *CGfermion)
      {
        if (!inited_p)
          return;

        <Write fermion 24b>
      }
```

5.1.6 DWF Fermion Deallocator

We only free pointers that we allocated. The magic is in `free16()`—it knows about all heap objects allocated by `alloc16()`.

```
14b  <Interface functions 11b>+≡
      void
      SSE_DWF_delete_fermion(SSE_DWF_Fermion *ptr)
      {
        if (!inited_p)
          return;

        free16(ptr->even);
        free16(ptr->odd);
        tfree(ptr);
      }
```

5.1.7 DWF Gauge Exporter

Unlike fermions, gauge fields are 4-d in the solver. Though they are not loaded by SSE memory operations, we still allocate 16-byte aligned memory for them (apparently for no good reason at all.)

```
14c  <Interface functions 11b>+≡
      SSE_DWF_Gauge *
      SSE_DWF_load_gauge(const void *OuterGauge_U,
                         const void *OuterGauge_V,
                         void *env,
                         SSE_DWF_gauge_reader reader)
      {
        SSE_DWF_Gauge *g;

        if (!inited_p)
          return 0;

        g = allocate_gauge_field();
        if (g == 0)
          return 0;

        <Read gauge field 21c>
        return g;
      }
```

Let us also define `SSE_DWF_Gauge` here. We do not need anything fancy for the gauge field:

```
15a  <Data types 12f>+≡
      struct SSE_DWF_Gauge {
          complex v[Nc][Nc];
      };

```

5.1.8 DWF Gauge Deallocator

Gauge deallocator is very much like fermion deallocator. We only keep them separate to help the type system cope with a error making user.

```
15b  <Interface functions 11b>+≡
      void
      SSE_DWF_delete_gauge(SSE_DWF_Gauge *ptr)
      {
          if (!inited_p)
              return;

          free16(ptr);
      }

```

5.1.9 The Solver

Finally, the solver itself. Here we check if the system has been properly initialized and dispatch on the float size (but not now yet.)

```
15c  <Interface functions 11b>+≡
      int
      SSE_DWF_cg_solver(SSE_DWF_Fermion *psi,          /* result */
                        double *out_eps,
                        int *out_iter,
                        const SSE_DWF_Gauge *gauge,
                        double M, double m0,
                        const SSE_DWF_Fermion *x0,      /* guess */
                        const SSE_DWF_Fermion *eta,     /* rhs */
                        double eps, int max_iter)
      {
          int status;

          if (!inited_p)
              return 1;

          U = (SU3 *)gauge;
          <Compute constant values for  $Q_{xx}^{-1}$  and  $S_{xx}^{-1}$  71d>
          <Compute  $\varphi_o$  38a>
          <Solve  $M^\dagger M \psi_o = \varphi_o$  38e>
          <Compute  $\psi_e$  40a>
          return status;
      }

```

Save one argument in many functions:

```
15d  <Global variables 10>+≡
      static SU3 *U;

```

5.1.10 Dirac Operator

It is convenient to have the Dirac operator and its conjugate as separate functions.

$$\chi \leftarrow D_{DW}\psi.$$

```
16a  <Interface functions 11b>+≡
      void
      SSE_DWF_Dirac_Operator(SSE_DWF_Fermion *chi,
                             const SSE_DWF_Gauge *gauge,
                             double M_0, double m_f,
                             const SSE_DWF_Fermion *psi)
      {
        if (!inited_p)
          return;

        U = (SU3 *)gauge;
        <Compute constant values for Qxx and Sxx 9>
        compute_Do(chi->odd, psi->odd, psi->even);
        compute_De(chi->even, psi->even, psi->odd);
      }
```

$$\chi \leftarrow D_{DW}^\dagger\psi.$$

```
16b  <Interface functions 11b>+≡
      void
      SSE_DWF_Dirac_Operator_conjugate(SSE_DWF_Fermion *chi,
                                         const SSE_DWF_Gauge *gauge,
                                         double M_0, double m_f,
                                         const SSE_DWF_Fermion *psi)
      {
        if (!inited_p)
          return;

        U = (SU3 *)gauge;
        <Compute constant values for Qxx and Sxx 9>
        compute_Dco(chi->odd, psi->odd, psi->even);
        compute_Dce(chi->even, psi->even, psi->odd);
      }
```


5.1.11 Little Helpers

$$\psi \leftarrow \varphi + a\eta$$

```

17  <Interface functions 11b>+≡
    void
    SSE_DWF_Add_Fermion(SSE_DWF_Fermion *psi,
                        const SSE_DWF_Fermion *phi,
                        double a,
                        const SSE_DWF_Fermion *eta)
    {
        int i;
        vReal *r, *p, *e;
        vReal A = vmk1(a);

        if (!inited_p)
            return;

        i = odd_even.size * Sv * sizeof(vOddFermion)/sizeof(vReal);
        r = (vReal *)psi->odd;
        p = (vReal *)phi->odd;
        e = (vReal *)eta->odd;
        for (; i--; r++, p++, e++)
            *r = *p + A * *e;

        i = even_odd.size * Sv * sizeof(vEvenFermion)/sizeof(vReal);
        r = (vReal *)psi->even;
        p = (vReal *)phi->even;
        e = (vReal *)eta->even;
        for (; i--; r++, p++, e++)
            *r = *p + A * *e;
    }

```

```

18  <Interface functions 11b>+≡
    void SSE_DWF_Fermion_Dot_Product(double *r_re,
                                     double *r_im,
                                     const SSE_DWF_Fermion *psi,
                                     const SSE_DWF_Fermion *phi)
{
    int i, c, d;
    vFermion *a;
    vFermion *b;
    vReal c_re = vmk1(0.0);
    vReal c_im = c_re;

    i = odd_even.size * Sv;
    a = &psi->odd->f;
    b = &phi->odd->f;
    *r_re = *r_im = 0;
    for (;i--; a++, b++) {
        for (c = 0; c < Nc; c++) {
            for (d = 0; d < Fd; d++) {
                c_re = a->f[d][c].re * b->f[d][c].re
                    + a->f[d][c].im * b->f[d][c].im;
                c_im = a->f[d][c].re * b->f[d][c].im
                    - a->f[d][c].im * b->f[d][c].re;
                *r_re += vsum(c_re);
                *r_im += vsum(c_im);
            }
        }
    }

    i = even_odd.size * Sv;
    a = &psi->even->f;
    b = &phi->even->f;
    for (;i--; a++, b++) {
        for (c = 0; c < Nc; c++) {
            for (d = 0; d < Fd; d++) {
                c_re = a->f[d][c].re * b->f[d][c].re
                    + a->f[d][c].im * b->f[d][c].im;
                c_im = a->f[d][c].re * b->f[d][c].im
                    - a->f[d][c].im * b->f[d][c].re;
                *r_re += vsum(c_re);
                *r_im += vsum(c_im);
            }
        }
    }
    QMP_sum_double(r_re);
    QMP_sum_double(r_im);
}

```

5.2 Memory Allocation

SSE does like properly aligned memory. While automatic variables are aligned by the compiler, extra care is needed when dealing with the heap. The code allocates all its own memory aligned on 16-byte boundary by calling `alloc16()`, and returns the memory through `free16()`.

```
19a  <Static functions 19a>≡
      static void *
      alloc16(int size)
      {
          int xsize = PAD16(size + sizeof (struct memblock));
          struct memblock *p = tmalloc(xsize);

          if (p == 0)
              return p;

          p->data = ALIGN16(&p[1]);
          p->size = size;
          p->next = memblock.next;
          p->prev = &memblock;
          p->next->prev = p;
          p->prev->next = p;

          return p->data;
      }
```

For readability, here are alignment operations:

```
19b  <Macro definitions 19b>≡
      #define PAD16(size) (15+(size))
      #define ALIGN16(addr) ((void *) (~15 & (15 + (size_t)(addr))))
```

For deallocation we need to find an appropriate memory block:

```
19c  <Static functions 19a>+≡
      static void
      free16(void *ptr)
      {
          struct memblock *p;

          if (ptr == 0)
              return;

          for (p = memblock.next; p != &memblock; p = p->next) {
              if (p->data != ptr)
                  continue;
              p->next->prev = p->prev;
              p->prev->next = p->next;
              tfree(p);
              return;
          }
          /* this is BAD: control should reach here! */
      }
```

The head of the memory list is stored in a static variable. Of course, such an implementation makes no threadable, but let us worry about that when the time is right.

```
19d  <Global variables 10>+≡
      static struct memblock memblock = {
          &memblock,
          &memblock,
          NULL,
          0
      };
```

Finally, the datatype for the linked list:

20a $\langle \text{Data types 12f} \rangle + \equiv$

```

    struct memblock {
        struct memblock *next;
        struct memblock *prev;
        void *data;
        size_t size;
    };

```

5.2.1 Field allocators

First, the prototypes:

20b $\langle \text{Static function prototypes 20b} \rangle \equiv$

```

    static vEvenFermion *allocate_even_fermion(void);
    static vOddFermion *allocate_odd_fermion(void);
    static SSE_DWF_Gauge *allocate_gauge_field(void);

```

The only difference between even and odd fermions is (possibly) their size:

20c $\langle \text{Static functions 19a} \rangle + \equiv$

```

    vEvenFermion *
    allocate_even_fermion(void)
    {
        return alloc16(even_odd.size * Sv * sizeof (vFermion));
    }

    vOddFermion *
    allocate_odd_fermion(void)
    {
        return alloc16(odd_even.size * Sv * sizeof (vFermion));
    }

    SSE_DWF_Gauge *
    allocate_gauge_field(void)
    {
        return alloc16(gauge_XYZT * sizeof (SSE_DWF_Gauge));
    }

```

5.3 Probing Cluster Topology

There is no proper way to query QMP about lattice layout. We have to request the minimal meaningful information the library provides and try to repeat outer layer's partitioning of the lattice. There are good chances of success, but this is a potential danger spot.

Here we prepare compute where on the lattice this node is and to build up our understanding of neighbors. Maybe optimistically, we assume that once QMP is initialized, it reports logical dimensions and coordinates properly, so that we do not need to be paranoid about errors here.

```
21a  <Get network topology 21a>≡
    {
        int i, dn;
        const int *xn, *xc;

        if (!QMP_logical_topology_is_declared())
            /* The user must have declared logical topology before */
            goto error;
        dn = QMP_get_logical_number_of_dimensions();
        if (dn > DIM)
            /* Too high dimension of the logical network */
            goto error;

        xn = QMP_get_logical_dimensions();
        xc = QMP_get_logical_coordinates();
        for (i = 0; i < dn; i++) {
            network[i] = xn[i];
            coord[i] = xc[i];
        }

        for (; i < dn; i++) {
            network[i] = 1;
            coord[i] = 0;
        }
    }
```

Some global variables:

```
21b  <Global variables 10>+≡
    static int network[DIM];
    static int coord[DIM];
```

5.4 Moving Data

5.4.1 Reading the Gauge Field

Let us start with reading of the gauge field from the outer environment first. Here we assume that there is an address translation function to help us in talking to the outer layer.

```
21c  <Read gauge field 21c>≡
    {
        int x[DIM], i, d, a, b, p1;

        <Start DIM-d sublattice scan 22b>
        <Load DIM gauge links from U at x 22a>
        <Advance DIM-d index for full sublattice scan 22d>

        for (d = 0; d < DIM; d++)
            <Load gauge boundary in direction d 23a>
    }
```

At a given site, load DIM gauge elements:

```
22a  <Load DIM gauge links from U at x 22a>≡
      p1 = to_Ulinear(x, &bounds, -1);
      for (d = 0; d < DIM; d++) {
        for (a = 0; a < Nc; a++) {
          for (b = 0; b < Nc; b++) {
            g[p1 + d].v[a][b].re = reader(OuterGauge_U, env, x, d, a, b, 0);
            g[p1 + d].v[a][b].im = reader(OuterGauge_U, env, x, d, a, b, 1);
          }
        }
      }
    }
```

To start a scan over the lattice, initialize x and start the loop:

```
22b  <Start DIM-d sublattice scan 22b>≡
      for (i = 0; i < DIM; i++)
        x[i] = bounds.lo[i];
      for (i = 0; i < DIM;) {
```

```
22c  <Start DIM-d sublattice scan locally 22c>≡
      for (i = 0; i < DIM; i++)
        x[i] = bounds->lo[i];
      for (i = 0; i < DIM;) {
```

Once all is done with the site x, we are ready to advance the index:

```
22d  <Advance DIM-d index for full sublattice scan 22d>≡
      for (i = 0; i < DIM; i++) {
        <Advance x at i 22g>
      }
    }

22e  <Advance DIM-d index for full sublattice scan locally 22e>≡
      for (i = 0; i < DIM; i++) {
        <Advance x at i locally 22h>
      }
    }
```

Since we are going to use a DIM-1 dimensional scan as well, let us write it down here:

```
22f  <Advance DIM-d index for DIM-1-d scan 22f>≡
      for (i = 0; i < DIM; i++) {
        if (i == d)
          continue;
        <Advance x at i 22g>
      }
    }
```

Now we can scan DIM-dimensional indices:

```
22g  <Advance x at i 22g>≡
      if (++x[i] == bounds.hi[i])
        x[i] = bounds.lo[i];
      else
        break;

22h  <Advance x at i locally 22h>≡
      if (++x[i] == bounds->hi[i])
        x[i] = bounds->lo[i];
      else
        break;
```

DWF Dirac operator needs backward gauge links. We get them from `OuterGauge_V`. Here we only read the boundary links.

```
23a  <Load gauge boundary in direction d 23a>≡
      {
          if (network[d] == 1)
              continue;

          <Start DIM-d sublattice scan 22b>
          <Load a d gauge link from V at x 23b>
          <Advance DIM-d index for DIM-1-d scan 22f>
      }
```

Now we read a boundary element:

```
23b  <Load a d gauge link from V at x 23b>≡
      p1 = to_Ulinear(x, &bounds, d);
      for (a = 0; a < Nc; a++) {
          for (b = 0; b < Nc; b++) {
              g[p1].v[a][b].re = reader(OuterGauge_V, env, x, d, a, b, 0);
              g[p1].v[a][b].im = reader(OuterGauge_V, env, x, d, a, b, 1);
          }
      }
```

5.4.2 Reading a Fermion

There are but two complications in reading the domain wall fermion. First, this is a good time to break the fermion into red and black pieces. In addition, here we construct SSE fermions.

```
23c  <Read fermion 23c>≡
      {
          int x[DIM+1], i;

          <Start DIM-d sublattice scan 22b>
          <Load an s-line of fermion at x 23d>
          <Advance DIM-d index for full sublattice scan 22d>
      }
```

Data conversion is inherently inefficient. We do not try to overoptimize it here:

```
23d  <Load an s-line of fermion at x 23d>≡
      {
          int p = parity(x);
          int p1 = Sv * to_HFlinear(x, &bounds, -1, 0); /* p is taken care of! */
          vFermion *f = p? &ptr->odd[p1].f: &ptr->even[p1].f;

          for (x[DIM] = 0; x[DIM] < tlattice[DIM]; x[DIM] += Vs, f++) {
              int d;
              for (d = 0; d < Fd; d++) {
                  int c;
                  for (c = 0; c < Nc; c++) {
                      f->f[d][c].re = import_vector(OuterFermion, env, reader,
                                                    x, c, d, 0);
                      f->f[d][c].im = import_vector(OuterFermion, env, reader,
                                                    x, c, d, 1);
                  }
              }
          }
      }
```

A simple packer of Vs elements into a vector:

```

24a  <Static function prototypes 20b>+≡
      static inline vReal
      import_vector(const void *z, void *env, SSE_DWF_fermion_reader reader,
                    int x[DIM+1], int c, int d, int re_im)
      {
        vReal f;
        REAL *v = (REAL *)&f;
        int i, xs;

        for (xs = x[DIM], i = 0; i < Vs; i++, x[DIM]++) {
          *v++ = reader(z, env, x, c, d, re_im);
        }
        x[DIM] = xs;
        return f;
      }

```

5.4.3 Writing a Fermion

Writing a fermion is not much different:

```

24b  <Write fermion 24b>≡
      {
        int x[DIM+1], i;

        <Start DIM-d sublattice scan 22b>
        <Save an s-line of fermion at x 24c>
        <Advance DIM-d index for full sublattice scan 22d>
      }

24c  <Save an s-line of fermion at x 24c>≡
      {
        int p = parity(x);
        int p1 = Sv * to_HFlinear(x, &bounds, -1, 0); /* p is taken care of! */
        vFermion *f = p? &CGfermion->odd[p1].f: &CGfermion->even[p1].f;

        for (x[DIM] = 0; x[DIM] < tlattice[DIM]; x[DIM] += Vs, f++) {
          int d;
          for (d = 0; d < Fd; d++) {
            int c;
            for (c = 0; c < Nc; c++) {
              save_vector(OuterFermion, env, writer, x, c, d, 0,
                          &f->f[d][c].re);
              save_vector(OuterFermion, env, writer, x, c, d, 1,
                          &f->f[d][c].im);
            }
          }
        }
      }

```


Here's another little helper good only for writing back the fermion from SSE to the outer environment:

```
25a  <Static function prototypes 20b>+≡
      static inline void
      save_vector(void *z, void *env, SSE_DWF_fermion_writer writer,
                  int x[DIM+1], int c, int d, int re_im, vReal *f)
      {
        REAL *v = (REAL *)f;
        int i, xs;

        for (xs = x[DIM], i = 0; i < Vs; i++, x[DIM]++) {
          writer(z, env, x, c, d, re_im, *v++);
        }
        x[DIM] = xs;
      }
```

5.5 Solver Initialization

Here are all pieces for setting up the structures needed to run the solver.

5.5.1 Constructing the neighbor tables

```
25b  <Initialize tables 25b>≡
      if (init_tables()) {
        /* Something went wrong in the table construction */
        goto error;
      }
```

The table initializer creates all tables necessary for communication and computation. Memory is allocated here for index arrays.

```
25c  <Static functions 19a>+≡
      static int
      init_tables(void)
      {
        struct neighbor tmp;
        int i, v;

        init_neighbor(&bounds, &neighbor);
        <Compute init sizes 26a>
        tmp = neighbor;
        build_neighbor(&even_odd, &bounds, 0, &tmp);
        build_neighbor(&odd_even, &bounds, 1, &tmp);

        return 0;
      }
```

First, we set global data:

```
25d  <Global variables 10>+≡
      static struct bounds bounds;
      static int gauge_XYZT;
      static int Sv, Sv_1;
```

26a *⟨Compute init sizes 26a⟩*≡

```

Sv = tlattice[DIM] / Vs;
Sv_1 = Sv - 1;
for (v = 1, i = 0; i < DIM; i++) {
    v *= bounds.hi[i] - bounds.lo[i];
}
gauge_XYZT = DIM * v;
for (i = 0; i < DIM; i++) {
    if (network[i] < 2)
        continue;
    gauge_XYZT += v / (bounds.hi[i] - bounds.lo[i]);
}

```

The `struct bounds` helps us to navigate through the local part of the lattice. It is used by the initialization code only.

26b *⟨Data types 12f⟩*+≡

```

struct bounds {
    int lo[DIM];
    int hi[DIM];
};

```

We keep two `struct neighbor`, one for computation on the even sublattice, another—on the odd. In addition to `even_odd` and `odd_even`, we need one more `struct neighbor` to keep the allocated pointers in.

26c *⟨Global variables 10⟩*+≡

```

static struct neighbor neighbor;
static struct neighbor odd_even;
static struct neighbor even_odd;

```

Let us start with computing the boundary of the sublattice

26d *⟨Static function prototypes 20b⟩*+≡

```

static inline int
lattice_start(int lat, int net, int coord)
{
    int q = lat / net;
    int r = lat % net;

    return coord * q + ((coord < r)? coord: r);
}

static inline void
mk_sublattice(struct bounds *bounds,
              int coord[])
{
    int i;

    for (i = 0; i < DIM; i++) {
        bounds->lo[i] = lattice_start(tlattice[i], network[i], coord[i]);
        bounds->hi[i] = lattice_start(tlattice[i], network[i], coord[i] + 1);
    }
}

```

All dynamic data are allocated in `init_neighbor` and are stored in `neighbor`.

26e *⟨Static function prototypes 20b⟩*+≡

```

static void
init_neighbor(struct bounds *bounds, struct neighbor *neighbor);

```

```

27a  <Static functions 19a>+≡
      static void
      init_neighbor(struct bounds *bounds, struct neighbor *neighbor)
      {
          int i;

          mk_sublattice(bounds, coord);
          neighbor->qmp_smask = 0;
          <Compute inside_size and boundary_size 27b>
          <Allocate inside table 27c>
          <Allocate boundary table 27d>
          <Compute send sizes and allocate index tables 27e>
      }

27b  <Compute inside_size and boundary_size 27b>≡
      for (neighbor->size = 1, neighbor->inside_size = 1, i = 0; i < DIM; i++) {
          int ext = bounds->hi[i] - bounds->lo[i];

          neighbor->size *= ext;
          if (network[i] > 1)
              neighbor->inside_size *= ext - 2;
          else
              neighbor->inside_size *= ext;
      }
      neighbor->boundary_size = neighbor->size - neighbor->inside_size;
      neighbor->site = tmalloc(neighbor->size * sizeof (struct site));

27c  <Allocate inside table 27c>≡
      if (neighbor->inside_size)
          neighbor->inside = tmalloc(neighbor->inside_size * sizeof (int));
      else
          neighbor->inside = 0;

27d  <Allocate boundary table 27d>≡
      if (neighbor->boundary_size)
          neighbor->boundary = tmalloc(neighbor->boundary_size * sizeof (struct boundary));
      else
          neighbor->boundary = 0;

27e  <Compute send sizes and allocate index tables 27e>≡
      for (i = 0; i < 2 * DIM; i++) {
          int d = i / 2;

          if (network[d] > 1) {
              neighbor->snd_size[i] = neighbor->size / (bounds->hi[d] - bounds->lo[d]);
              neighbor->snd[i] = tmalloc(neighbor->snd_size[i] * sizeof (int));
          } else {
              neighbor->snd_size[i] = 0;
              neighbor->snd[i] = 0;
          }
      }

```

Here is the definition of the neighbor table we spent soo much time initializing:

```
28a  <Data types 12f>+≡
      struct neighbor {
          int          size;           /* size of site table */
          int          inside_size;    /* number of inside sites */
          int          boundary_size;  /* number of boundary sites */
          int          snd_size[2*DIM]; /* size of send buffers in 8 dirs */
          int          rcv_size[2*DIM]; /* size of receive buffers */
          int          *snd[2*DIM];    /* i->x translation for send buffers */
          int          *inside;        /* i->x translation for inside sites */
          struct boundary *boundary;   /* i->x,mask translation for boundary */
          struct site   *site;         /* x->site translation for sites */
          vHalfFermion  *snd_buf[2*DIM]; /* Send buffers */
          vHalfFermion  *rcv_buf[2*DIM]; /* Receive buffers */

          int          qmp_size[4*DIM]; /* sizes of QMP buffers */
          void          *qmp_xbuf[4*DIM]; /* QMP snd/rcv buffer addresses */
          vHalfFermion  *qmp_buf[4*DIM]; /* send and receive buffers for QMP */
          QMP_msgmem_t  qmp_mm[4*DIM];  /* msgmem's for send and receive */
          int          Nx;              /* number of msecs */

          int          qmp_smask;       /* send flags for qmp_sh[] */
          QMP_msghandle_t qmp_handle;   /* common send & receive handle */
      };
```

For boundary sites we only need 8 bits for the boundary indicators. However, allocating a whole `int` for mask is what the compiler does anyway.

```
28b  <Data types 12f>+≡
      struct boundary {
          int  index; /* x-index of this boundary site */
          int  mask;  /* bitmask of the borders */
      };
```

In the following structure we keep information about links and neighbors of the site. Note, that there is one address for four forward links: they are packed in memory as defined in the comment.

```
28c  <Data types 12f>+≡
      struct site {
          int Uup;      /* up-links are Uup, Uup+1, Uup+2, Uup+3 */
          int Udown[DIM]; /* four down-links */
          int F[2*DIM]; /* eight neighboring fermions on the other sublattice */
      };
```

Now we can define `build_neighbor()`:

```
28d  <Static functions 19a>+≡
      static void
      build_neighbor(struct neighbor *out,
                    struct bounds  *bounds,
                    int            par,
                    struct neighbor *in)
      {
          int i,d, s, p, m;
          int x[DIM];

          <Initialize out and p 29b>
          <Walk through sublattice 29c>
          <Build outside indices 30c>
      }
```

29a \langle Static function prototypes 20b $\rangle + \equiv$

```
static void build_neighbor(struct neighbor *out,
                          struct bounds *bounds,
                          int parity,
                          struct neighbor *in);
```

First part is easy: we start with copying in to out, resetting fields which will be computed shortly and setting p to bounds->lo:

29b \langle Initialize out and p 29b $\rangle \equiv$

```
*out = *in;
out->size = 0;
out->inside_size = 0;
out->boundary_size = 0;
for (d = 0; d < DIM; d++) {
    out->rcv_size[2*d] = out->snd_size[2*d] = 0;
    out->rcv_size[2*d+1] = out->snd_size[2*d+1] = 0;
}
```

This is a good place to reuse our lattice walking chunks.

29c \langle Walk through sublattice 29c $\rangle \equiv$

\langle Start DIM-d sublattice scan locally 22c \rangle

```
s = parity(x);
if (s != par)
    goto next;
 $\langle$ Compute p and m 29d $\rangle$ 
 $\langle$ Setup boundary or inside 29e $\rangle$ 
 $\langle$ Build local neighbors 30b $\rangle$ 
out->size++;
in->site++;
next:
 $\langle$ Advance DIM-d index for full sublattice scan locally 22e $\rangle$ 
```

For p we use a function to compute it from x. As for m, its eight low bits encode if there is a boundary nearby. Note, that even bits corresponds to *step down* and odd bits correspond to *step up*.

29d \langle Compute p and m 29d $\rangle \equiv$

```
p = to_HFlinear(x, bounds, -1, 0);
for (m = 0, d = 0; d < DIM; d++) {
    if (network[d] > 1) {
        if (x[d] == bounds->lo[d])
            m |= 1 << (2 * d);
        if (x[d] + 1 == bounds->hi[d])
            m |= 1 << (2 * d + 1);
    }
}
```

If no boundary was found near p, we put it into inside. Otherwise, p belongs to the boundary.

29e \langle Setup boundary or inside 29e $\rangle \equiv$

```
if (m) {
     $\langle$ Setup boundary 30a $\rangle$ 
} else {
     $\langle$ Setup inside 29f $\rangle$ 
}
```

For the inside, simply add p to the list of sites and advance pointers and counters:

29f \langle Setup inside 29f $\rangle \equiv$

```
*in->inside++ = p;
out->inside_size++;
```

For the boundary, place `p` into `index` and `m` into `mask` and advance pointers. We also take the opportunity to place `p` into send buffers where bits of `m` are set

```
30a  <Setup boundary 30a>≡
      in->boundary->index = p;
      in->boundary->mask = m;
      in->boundary++;
      out->boundary_size++;
      for (d = 0; d < 2*DIM; d++) {
          if ((m & (1 << d)) == 0)
              continue;
          *in->snd[d]++ = p * Sv;
          out->snd_size[d]++;
      }
```

We are ready now to build local neighbors. All gauge fields are local, and we still have `m` to tell if the other sublattice neighbor is local or not.

```
30b  <Build local neighbors 30b>≡
      in->site->Uup = to_Ulinear(x, bounds, -1);
      for (d = 0; d < DIM; d++) {
          in->site->Udown[d] = to_Ulinear(x, bounds, d);
          if ((m & (1 << (2 * d))) == 0)
              in->site->F[2*d] = Sv * to_HFlinear(x, bounds, d, -1);
          if ((m & (1 << (2 * d + 1))) == 0)
              in->site->F[2*d + 1] = Sv * to_HFlinear(x, bounds, d, +1);
      }
```

The only piece left is the one dealing with outside indices. This is a tricky part, but we just happen to have almost enough machinery already to solve it:

```
30c  <Build outside indices 30c>≡
      for (d = 0; d < DIM; d++) {
          if (network[d] < 2)
              continue;
          construct_rec(out, par, bounds, d, +1);
          construct_rec(out, par, bounds, d, -1);
      }
```

We also need a function that will walk through a boundary of a neighbor building the outside part of the `site[]`.F indices.

```
30d  <Static function prototypes 20b>+≡
      static void construct_rec(struct neighbor *out,
                              int par,
                              struct bounds *bounds,
                              int dir,
                              int step);
```

```

31a  <Static functions 19a>+≡
      static void
      construct_rec(struct neighbor *out,
                    int par,
                    struct bounds *bounds,
                    int dir,
                    int step)
      {
          struct bounds xb;
          int xc[DIM], x[DIM];
          int s, d, p, k;
          int dz = dir * 2 + ((step>0)?1:0);

          <Construct the neighbor's network coordinates xc and bounds xb 31b>
          <Construct the initial point of the hypersurface 31c>
          <Walk through the hypersurface 32a>
      }

Constructing the neighbor's network position is straightforward:
31b  <Construct the neighbor's network coordinates xc and bounds xb 31b>≡
      for (d = 0; d < DIM; d++) {
          int v = coord[d] + ((d==dir)?step:0);

          if (v < 0)
              v += network[d];
          if (v >= network[d])
              v -= network[d];
          xc[d] = v;
      }
      mk_sublattice(&xb, xc);

The initial point should be on the surface we are walking:
31c  <Construct the initial point of the hypersurface 31c>≡
      for (d = 0; d < DIM; d++)
          x[d] = ((d == dir) && (step < 0)) ? (xb.hi[d] - 1) : xb.lo[d];

```

Walking through the hypersurface is very much like walking through the sublattice below. There are only two differences: (a) we are walking opposite parity sublattice surface here and, (b) while advancing the point, we should stay on the surface selected above.

```

32a  <Walk through the hypersurface 32a>≡
      /* ZZZ: This needs some cleaning */
      k = 0;
      do {
          for (d = 0, s = par; d < DIM; d++)
              s += x[d];
          if (!(s & 1))
              goto next;

          <Translate x to target p 32b>
          <Insert k into site[p].F[dx] 32c>

      next:
          for (d = 0; d < DIM; d++) {
              if (d == dir)
                  continue;
              if (++x[d] == xb.hi[d])
                  x[d] = xb.lo[d];
              else
                  break;
          }
      } while (d != DIM);
      out->rcv_size[dz^1] = k; /* XXX is it true? */

32b  <Translate x to target p 32b>≡
      p = to_HFlinear(x, bounds, dir, -step);

32c  <Insert k into site[p].F[dx] 32c>≡
      out->site[p].F[dz] = Sv * k++;

Here we do the reverse, namely, free all memory allocated by init_tables():

32d  <Free tables 32d>≡
      {
          int i;

          if (neighbor.site) {
              tfree(neighbor.site);
              neighbor.site = 0;
          }

          if (neighbor.inside) {
              tfree(neighbor.inside);
              neighbor.inside = 0;
          }

          if (neighbor.boundary) {
              tfree(neighbor.boundary);
              neighbor.boundary = 0;
          }

          for (i = 2 * DIM; i--;) {
              if (neighbor.snd[i] == 0)
                  continue;
              tfree(neighbor.snd[i]);
              neighbor.snd[i] = 0;
          }
      }

```


5.5.2 Address translation routines

Let us define a couple of functions for translating 4-d lattice positions into 1-d offsets.

Computing linear position on the sublattice is used often enough to be placed in a function. To avoid writing two very similar functions, we pass two arguments q , and z to specify that q -component of p should adjusted by z . If $q < 0$, q and z are ignored.

```
33a  <Static function prototypes 20b>+≡
      static int
      to_HFlinear(int p[],
                  struct bounds *b,
                  int q,
                  int z)
      {
        int x, d;
        for (x = 0, d = DIM; d--;) {
          int v = p[d] + ((d == q)?z:0);
          int s = b->hi[d] - b->lo[d];
          if (v < 0)
            v += tlattice[d];
          if (v >= tlattice[d])
            v -= tlattice[d];
          x = x * s + v - b->lo[d];
        }
        return x / 2;
      }
```

Computing the index of the gauge link is similar to `to_HFlinear`, except that the extra parameter q tells us which of p should be stepped down by one. If $q < 0$, we are computing forward link position.

```
33b  <Static function prototypes 20b>+≡
      static int
      to_Ulinear(int p[],
                 struct bounds *b,
                 int q)
      {
        int x, d;

        if ((q < 0) || (p[q] > b->lo[q]) || (network[q] < 2)) {
          <Find index of a regular gauge link 33c>
        } else {
          <Find index of a borrowed gauge link 34a>
        }
      }
```

Regular gauge links sits four per site and their indices are easy to compute:

```
33c  <Find index of a regular gauge link 33c>≡
      for (x = 0, d = DIM; d--;) {
        int s = b->hi[d] - b->lo[d];
        int v = p[d] - ((q == d)?1:0);
        if (v < 0)
          v += tlattice[d];
        x = x * s + v - b->lo[d];
      }
      return DIM * x + ((q < 0)?0:q);
```

For borrowed links we need first to skip all regulars and previous faces and then count position on the borrowed 3-face:

```

34a  <Find index of a borrowed gauge link 34a>≡
      int s0, v0;
      for (d = 0, v0 = 1; d < DIM; d++)
          v0 *= b->hi[d] - b->lo[d];
      for (d = 0, s0 = DIM * v0; d < q; d++) {
          if (network[d] < 2)
              continue;
          s0 += v0 / (b->hi[d] - b->lo[d]);
      }
      for (d = DIM, x = 0; d--;) {
          int s = b->hi[d] - b->lo[d];
          int v = p[d];

          if (d == q)
              continue;
          x = x * s + v - b->lo[d];
      }
      return s0 + x;

```

5.6 QMP Initialization

```

34b  <Include files 34b>≡
      #include <qmp.h>

```

Once the tables and sizes are known, allocate all send and receive buffers and register them with QMP.

```

34c  <Initialize QMP 34c>≡
      if (build_buffers(&even_odd)) goto error;
      if (build_buffers(&odd_even)) goto error;

```

There are three cases we need to consider when preparing the communication handles. Note: return 1 if there was trouble.

```

34d  <Static function prototypes 20b>+≡
      static int build_buffers(struct neighbor *nb);

34e  <Static functions 19a>+≡
      static int
      build_buffers(struct neighbor *nb)
      {
          int i, k, Nh;
          QMP_msghandle_t SRh[4*DIM];

          Nh = nb->Nx = 0;
          for (i = 0; i < DIM; i++) {
              switch (network[i]) {
                  case 1: break;
                  case 2:
                      <Clump up and down directions 35a>
                      break;
                  default:
                      /* Order here is important */
                      <Allocate down buffers 35c>
                      <Allocate up buffers 35b>
                      break;
              }
          }
          <Construct the collective handle 36d>
          return 0;
      }

```

If there is only two nodes in a direction, we use only up link to communicate (becasuse there is only one wire between the nodes.)

```
35a  <Clump up and down directions 35a>≡
      k = make_buffer(nb, nb->snd_size[2*i] + nb->snd_size[2*i+1]);
      nb->snd_buf[2*i] = nb->qmp_buf[k];
      nb->snd_buf[2*i+1] = nb->snd_buf[2*i] + Sv * nb->snd_size[2*i];
      Nh = make_send(nb, k, i, +1, SRh, Nh);

      k = make_buffer(nb, nb->rcv_size[2*i] + nb->rcv_size[2*i+1]);
      nb->rcv_buf[2*i+1] = nb->qmp_buf[k]; /* should be opposite to snd_buf[] */
      nb->rcv_buf[2*i] = nb->rcv_buf[2*i+1] + Sv * nb->rcv_size[2*i+1];
      Nh = make_receive(nb, k, i, -1, SRh, Nh); /* -1 fixes a bug in GigE QMP */
```

On a large machine, up and down buffers are separate:

```
35b  <Allocate up buffers 35b>≡
      k = make_buffer(nb, nb->snd_size[2*i+1]);
      nb->snd_buf[2*i+1] = nb->qmp_buf[k];
      Nh = make_send(nb, k, i, +1, SRh, Nh);

      k = make_buffer(nb, nb->rcv_size[2*i+1]);
      nb->rcv_buf[2*i+1] = nb->qmp_buf[k];
      Nh = make_receive(nb, k, i, +1, SRh, Nh);
```

```
35c  <Allocate down buffers 35c>≡
      k = make_buffer(nb, nb->snd_size[2*i]);
      nb->snd_buf[2*i] = nb->qmp_buf[k];
      Nh = make_send(nb, k, i, -1, SRh, Nh);

      k = make_buffer(nb, nb->rcv_size[2*i]);
      nb->rcv_buf[2*i] = nb->qmp_buf[k];
      Nh = make_receive(nb, k, i, -1, SRh, Nh);
```

Allocate a buffer of size vHalfFermion's fit for send and/or receive.

```
35d  <Static function prototypes 20b>+≡
      static int make_buffer(struct neighbor *nb, int size);

35e  <Static functions 19a>+≡
      static int
      make_buffer(struct neighbor *nb, int size)
      {
          int bcount = size * Sv * sizeof (vHalfFermion);
          int N = nb->Nx;

          nb->qmp_size[N] = size;
          sse_aligned_buffer(nb, N, bcount);
          nb->qmp_mm[N] = QMP_declare_msgmem(nb->qmp_buf[N], bcount);
          nb->Nx = N + 1;
          DEBUG_QMP3("(): declare_msgmem(%p,%d)=0x%x\n",
                    nb->qmp_buf[N], bcount, (int)nb->qmp_mm[N])
          return N;
      }
```

Construct a send handle. This function places a send handle into SRh for future construction of the superhandle and sets a bit in qmp_smask.

```
35f  <Static function prototypes 20b>+≡
      static int make_send(struct neighbor *nb, int k, int i, int d,
                          QMP_msghandle_t SRh[4*DIM], int Nsr);
```

```

36a  <Static functions 19a>+≡
      static int
      make_send(struct neighbor *nb, int k, int i, int d,
                QMP_msghandle_t SRh[4*DIM], int Nsr)
      {
          int j = 2 * i + ((d < 0)? 0: 1);

          nb->qmp_smask |= (1 << j);
          SRh[Nsr] = QMP_declare_send_relative(nb->qmp_mm[k], i, d, 1);

          DEBUG_QMP4("(): declare_send_relative(0x%x,%d,%d,1)=0x%x\n",
                    (int)nb->qmp_mm[k], i, d, (int)SRh[Nsr])

          return Nsr+1;
      }

```

Constructing a receive handle is similar. There is no need for a mask bit though.

```

36b  <Static function prototypes 20b>+≡
      static int make_receive(struct neighbor *nb, int k, int i, int d,
                            QMP_msghandle_t SRh[4*DIM], int Nsr);

36c  <Static functions 19a>+≡
      static int
      make_receive(struct neighbor *nb, int k, int i, int d,
                  QMP_msghandle_t SRh[4*DIM], int Nsr)
      {
          SRh[Nsr] = QMP_declare_receive_relative(nb->qmp_mm[k], i, d, 1);

          DEBUG_QMP4("(): declare_receive_relative(0x%x,%d,%d,1)=0x%x\n",
                    (int)nb->qmp_mm[k], i, d, (int)SRh[Nsr])

          return Nsr+1;
      }

```

Finally, aggregate all receive handles:

```

36d  <Construct the collective handle 36d>≡
      if (nb->qmp_smask) {
          nb->qmp_handle = QMP_declare_multiple(SRh, Nh);
          #ifdef DEBUG_QMP
          {
              int i;
              for (i = 0; i < Nh; i++)
                  DEBUG_QMP2("(): declare_multiple([%d]=0x%x)\n", i, (int)SRh[i])
              DEBUG_QMP2("(): declare_multiple(..., %d)=0x%x\n", Nh, (int)nb->qmp_handle)
          }
          #endif
      }

```

SSE likes its memory aligned at 16 bytes. We need to keep that in mind when asking for QMP memory. Note, that this function may be in violation of a strict interpretation of the QMP Specification, but on many SciDAC calls numerous assurances were given that such usage is permissible.

```

36e  <Static function prototypes 20b>+≡
      static void sse_aligned_buffer(struct neighbor *nb, int k, int size);

```

```

37a  <Static functions 19a>+≡
      static void
      sse_aligned_buffer(struct neighbor *nb, int k, int size)
      {
      #ifdef USE_QMP2
          nb->qmp_xbuf[k] = QMP_allocate_aligned_memory(size, 128, 0);
          nb->qmp_buf[k] = QMP_get_memory_pointer(nb->qmp_xbuf[k]);
      #else
          int xcount = size + 15;
          char *ptr = QMP_allocate_aligned_memory(xcount);
          nb->qmp_buf[k] = (void *) (~15 & (15 + (unsigned long)(ptr)));
          nb->qmp_xbuf[k] = ptr;
      #endif
          DEBUG_QMP4("(%p,%d,%d): allocate: 0x%x\n",
                     nb, k, size, (int)nb->qmp_xbuf[k])
          DEBUG_QMP4("(%p,%d,%d): ptr: %p\n",
                     nb, k, size, (int)nb->qmp_buf[k])
      }

```

Freeing QMP structure does the reverse of the allocator:

```

37b  <Cleanup QMP 37b>≡
      free_buffers(&even_odd);
      free_buffers(&odd_even);

```

There are some unsettling omissions in the QMP specification. What follows is based on the tribal wisdom which was not codified.

```

37c  <Static function prototypes 20b>+≡
      static void free_buffers(struct neighbor *nb);

```

```

37d  <Static functions 19a>+≡
      static void
      free_buffers(struct neighbor *nb)
      {
          int i;

          <Free common handle 37e>
          <Free QMP buffers 37f>
      }

```

Here we assume that `QMP_free_msghandle()` knows what to do with a bad handle returned from `QMP_declare_send...()` and `QMP_declare_receive...()`.

The first common wisdom is that `QMP_declare_multiple()` invalidates individual handles. We only need to free one handle in `nb->qmp_handle`:

```

37e  <Free common handle 37e>≡
      QMP_free_msghandle(nb->qmp_handle);
      DEBUG_QMP1("(): free_msghandle(0x%x) / common receive handle\n",
                 (int)nb->qmp_handle)

```

Two steps are needed to deallocate QMP memory:

```

37f  <Free QMP buffers 37f>≡
      for (i = nb->Nx; i--;) {
          QMP_free_msgmem(nb->qmp_mm[i]);
          DEBUG_QMP1("(): free_msgmem(0x%x)\n", (int)nb->qmp_mm[i]);
      #ifdef USE_QMP2
          QMP_free_memory(nb->qmp_xbuf[i]);
      #else /* QMP 1.3 */
          QMP_free_aligned_memory(nb->qmp_xbuf[i]);
      #endif
          DEBUG_QMP1("(): free_memory(0x%x)\n", (int)nb->qmp_xbuf[i]);
      }

```

5.7 Parts of the Solver

Here are three principal parts of the solver. First, we compute the right hand side of the equation to be solved by the CG. Next, there is a solver of a hermitian matrix. Finally, the second half of the solution is computed.

5.7.1 Compute the RHS

Here we perform steps 1–3 of the outline above.

```
38a  <Compute  $\varphi_o$  38a>≡
      compute_Qee1(auxA_e, eta->even);
      compute_Qoe(auxB_o, auxA_e);
      compute_sum_o(auxA_o, eta->odd, -1, auxB_o);
      compute_Qoo1(auxB_o, auxA_o);
      compute_Mx(Phi_o, auxB_o);

38b  <Global variables 10>+≡
      static vOddFermion *auxA_o, *auxB_o, *Phi_o;
      static vEvenFermion *auxA_e;

38c  <Allocate fields 38c>≡
      Phi_o = allocate_odd_fermion(); if (Phi_o == 0) goto error;
      auxA_o = allocate_odd_fermion(); if (auxA_o == 0) goto error;
      auxB_o = allocate_odd_fermion(); if (auxB_o == 0) goto error;
      auxA_e = allocate_even_fermion(); if (auxA_e == 0) goto error;

38d  <Free fields 38d>≡
      if (auxA_e) free16(auxA_e); auxA_e = 0;
      if (auxB_o) free16(auxB_o); auxB_o = 0;
      if (auxA_o) free16(auxA_o); auxA_o = 0;
      if (Phi_o) free16(Phi_o); Phi_o = 0;
```

5.8 Field Operations

Hermitian solver follows:

```
38e  <Solve  $M^\dagger M \psi_o = \varphi_o$  38e>≡
      status = cg(psi->odd, Phi_o, x0->odd, eps, max_iter, out_eps, out_iter);

38f  <Static function prototypes 20b>+≡
      static int cg(vOddFermion *psi,
                    const vOddFermion *b,
                    const vOddFermion *x0,
                    double epsilon, int max_iter,
                    double *out_eps, int *out_iter);
```

```

39a  <Static functions 19a>+≡
      static int
      cg(vOddFermion *x_o,
         const vOddFermion *b,
         const vOddFermion *x0,
         double epsilon, int N,
         double *out_eps, int *out_N)
      {
        double rho, alpha, beta, gamma, norm_z;
        int status = 1;
        int k;

        copy_o(x_o, x0);
        compute_MxM(p_o, &norm_z, x_o);
        compute_sum_oN(r_o, &rho, b, -1, p_o);
        copy_o(p_o, r_o);
        <Finalize <r,r> computation 71e>

        for (k = 0; (rho > epsilon) && (k < N); k++) {
          compute_MxM(q_o, &norm_z, p_o);
          <Finalize <r,r> computation 71e>
          alpha = rho / norm_z;
          compute_sum2_oN(r_o, &gamma, -alpha, q_o);
          compute_sum2_o(x_o, alpha, p_o);
          <Finalize <r,r> computation 71e>
          if (gamma <= epsilon) {
            rho = gamma;
            status = 0;
            break;
          }
          beta = gamma / rho;
          rho = gamma;
          compute_sum2x_o(p_o, r_o, beta);
        }
        *out_N = k;
        *out_eps = rho;

        return status;
      }

Temporaries used by the CG
39b  <Global variables 10>+≡
      static vOddFermion *r_o, *p_o, *q_o;

39c  <Allocate fields 38c>+≡
      r_o = allocate_odd_fermion(); if (r_o == 0) goto error;
      p_o = allocate_odd_fermion(); if (p_o == 0) goto error;
      q_o = allocate_odd_fermion(); if (q_o == 0) goto error;

39d  <Free fields 38d>+≡
      if (r_o) free16(r_o); r_o = 0;
      if (p_o) free16(p_o); p_o = 0;
      if (q_o) free16(q_o); q_o = 0;

```

5.8.1 Computing the even part of the result

Again, this is simpling performing step 5 of the outline above:

```

40a  <Compute  $\psi_e$  40a>≡
      compute_Qeo(auxA_e, psi->odd);
      compute_sum_e(auxB_e, eta->even, -1, auxA_e);
      compute_Qee1(psi->even, auxB_e);

40b  <Global variables 10>+≡
      static vEvenFermion *auxB_e;

40c  <Allocate fields 38c>+≡
      auxB_e = allocate_even_fermion();  if (auxB_e == 0) goto error;

40d  <Free fields 38d>+≡
      if (auxB_e) free16(auxB_e); auxB_e = 0;

```

5.8.2 copy_o(d, s) or $d \leftarrow s$

This is a copies $d \leftarrow s$. Since it is used outside of the cg loop, we do not worry too much about efficiency here. Hence, cache pollution.

```

40e  <Static function prototypes 20b>+≡
      static void copy_o(vOddFermion *dst, const vOddFermion *src);

40f  <Static functions 19a>+≡
      static void
      copy_o(vOddFermion *dst, const vOddFermion *src)
      {
          int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);
          vReal *d = (vReal *)dst;
          const vReal *s = (const vReal *)src;

          for ( ;i--;)
              *d++ = *s++;
      }

```

5.8.3 compute_sum2_o(d,alpha,s), or $d \leftarrow d + \alpha s$

This is a function we can not speedup much: too many bytes are needed per operation. In principle, one can play with uncached loads and stores, but let us leave that for later.

```

40g  <Static function prototypes 20b>+≡
      static void compute_sum2_o(vOddFermion *dst, double alpha, const vOddFermion *src);

40h  <Static functions 19a>+≡
      static void
      compute_sum2_o(vOddFermion *dst, double alpha, const vOddFermion *src)
      {
          int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);
          vReal a = vmk1(alpha);
          vReal *d = (vReal *)dst;
          const vReal *s = (const vReal *)src;

          for ( ;i--;)
              *d++ += a * *s++;
      }

```


5.8.4 compute_sum2x_o(d,s,alpha), or $d \leftarrow \alpha d + s$

Almost the same as the previous one, but scaling is applied to another summand.

```

41a  <Static function prototypes 20b>+≡
      static void compute_sum2x_o(vOddFermion *dst, const vOddFermion *src, double alpha);

41b  <Static functions 19a>+≡
      static void
      compute_sum2x_o(vOddFermion *dst, const vOddFermion *src, double alpha)
      {
        int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);
        vReal a = vmk1(alpha);
        vReal *d = (vReal *)dst;
        const vReal *s = (const vReal *)src;

        for (;i--; d++)
          *d = a * *d + *s++;
      }

```

5.8.5 compute_sum_x(d,x,alpha,y) or $q \leftarrow x + \alpha y$

Next are a pair of general sums with the destination distinct from the sources. Do we really need separate functions for these?

```

41c  <Static function prototypes 20b>+≡
      static void compute_sum_e(vEvenFermion *d,
                                const vEvenFermion *x, double alpha, const vEvenFermion *y);
      static void compute_sum_o(vOddFermion *d,
                                const vOddFermion *x, double alpha, const vOddFermion *y);

41d  <Static functions 19a>+≡
      static void
      compute_sum_e(vEvenFermion *d,
                    const vEvenFermion *x, double alpha, const vEvenFermion *y)
      {
        const vReal *X = (const vReal *)x;
        const vReal *Y = (const vReal *)y;
        vReal *D = (vReal *)d;
        vReal a = vmk1(alpha);
        int i = even_odd.size * Sv * sizeof (vEvenFermion) / sizeof (vReal);

        for (;i--;)
          *D++ = *X++ + a * *Y++;
      }

41e  <Static functions 19a>+≡
      static void
      compute_sum_o(vOddFermion *d,
                    const vOddFermion *x, double alpha, const vOddFermion *y)
      {
        const vReal *X = (const vReal *)x;
        const vReal *Y = (const vReal *)y;
        vReal *D = (vReal *)d;
        vReal a = vmk1(alpha);
        int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);

        for (;i--;)
          *D++ = *X++ + a * *Y++;
      }

```

5.8.6 compute_sum_oN(d,norm,x,alpha,y), or $d \leftarrow x + \alpha y$ and $r \leftarrow \langle d, d \rangle$

There are two remaining sums which compute a sum of two fermions and the norm of the result at the same time.

```

42a  <Static function prototypes 20b>+≡
      static void compute_sum_oN(vOddFermion *d, double *norm,
                                const vOddFermion *x, double alpha, const vOddFermion *y);

42b  <Static functions 19a>+≡
      static void
      compute_sum_oN(vOddFermion *d, double *norm,
                    const vOddFermion *x, double alpha, const vOddFermion *y)
      {
        const vReal *X = (const vReal *)x;
        const vReal *Y = (const vReal *)y;
        vReal *D = (vReal *)d;
        vReal a = vmk1(alpha);
        vReal s = vmk1(0.0);
        vReal v;
        int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);

        for (;i--;) {
          v = *X++ + a * *Y++;
          s += v * v;
          *D++ = v;
        }
        *norm = vsum(s);
        <Start <r,r> computation 71h>
      }

42c  <Static function prototypes 20b>+≡
      static void compute_sum2_oN(vOddFermion *d, double *norm,
                                double alpha, const vOddFermion *y);

42d  <Static functions 19a>+≡
      static void
      compute_sum2_oN(vOddFermion *d, double *norm,
                    double alpha, const vOddFermion *y)
      {
        const vReal *Y = (const vReal *)y;
        vReal *D = (vReal *)d;
        vReal a = vmk1(alpha);
        vReal s = vmk1(0.0);
        vReal v;
        int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);

        for (;i--;) {
          v = *D + a * *Y++;
          s += v * v;
          *D++ = v;
        }
        *norm = vsum(s);
        <Start <r,r> computation 71h>
      }

```

5.8.7 compute_MxM(eta,norm,psi), or $\eta \leftarrow M^\dagger M \psi$ and friends

Last three easy pieces.

```
43a  <Static function prototypes 20b>+≡
      static void compute_MxM(vOddFermion *eta, double *norm,
                              const vOddFermion *psi);
      static void compute_M(vOddFermion *eta, double *norm,
                              const vOddFermion *psi);
      static void compute_Mx(vOddFermion *eta,
                              const vOddFermion *psi);
```

```
43b  <Static functions 19a>+≡
      static void
      compute_MxM(vOddFermion *eta, double *norm,
                  const vOddFermion *psi)
      {
          compute_M(auxB_o, norm, psi);
          compute_Mx(eta, auxB_o);
      }
```

Computation of M starts the global sum which will be completed separately.

```
43c  <Static functions 19a>+≡
      static void compute_M(vOddFermion *eta, double *norm,
                              const vOddFermion *psi)
      {
          compute_Qee1Qeo(auxA_e, psi);
          compute_1Qoo1Qoe(eta, norm, psi, auxA_e);
      }
```

For M^\dagger the order of factors differs from optimal. For now we have to live with the inefficiency here.

```
43d  <Static functions 19a>+≡
      static void compute_Mx(vOddFermion *eta,
                              const vOddFermion *psi)
      {
          compute_Soo1(auxA_o, psi);
          compute_See1Seo(auxA_e, auxA_o);
          compute_1Soe(eta, psi, auxA_e);
      }
```

5.8.8 compute_Qee1(eta,psi), or $\eta \leftarrow Q_{ee}^{-1} \psi$

Some code savings are still possible, since compute_Qee1() may differ from compute_Qoo1() by the number of sites only.

```
43e  <Static function prototypes 20b>+≡
      static void compute_Qxx1(vFermion *eta, const vFermion *psi, int xyzt);
      static void inline compute_Qee1(vEvenFermion *eta, const vEvenFermion *psi)
      {
          compute_Qxx1(&eta->f, &psi->f, even_odd.size);
      }
```

5.8.9 compute_Qoo1(eta,psi), or $\eta \leftarrow Q_{oo}^{-1} \psi$

```
43f  <Static function prototypes 20b>+≡
      static void inline compute_Qoo1(vOddFermion *eta, const vOddFermion *psi)
      {
          compute_Qxx1(&eta->f, &psi->f, odd_even.size);
      }
```

5.8.10 compute_Qxx1(eta,psi), or $\eta \leftarrow Q_{xx}^{-1}\psi$

```

44a  <Static functions 19a>+≡
      static void
      compute_Qxx1(vFermion *chi, const vFermion *psi, int size)
      {
          const vFermion *qs, *qx5;
          <Q common locals 70c>
          <Qxx locals 47c>

          for (i = 0; i < size; i++) {
              xyzt5 = i * Sv;
              <Compute rx5 71b>
              <Compute qx5 71c>
              <Compute  $Q_{xx}^{-1}$  part on the s-chain 45a>
          }
      }

```

5.8.11 compute_Soo1(eta,psi), or $\eta \leftarrow S_{oo}^{-1}\psi$

```

44b  <Static function prototypes 20b>+≡
      static void compute_Soo1(vOddFermion *eta, const vOddFermion *psi);

44c  <Static functions 19a>+≡
      static void
      compute_Soo1(vOddFermion *Chi, const vOddFermion *Psi)
      {
          vFermion *chi = &Chi->f;
          const vFermion *psi = &Psi->f;
          int size = odd_even.size;
          const vFermion *qs, *qx5;
          <Q common locals 70c>
          <Qxx locals 47c>

          for (i = 0; i < size; i++) {
              xyzt5 = i * Sv;
              <Compute rx5 71b>
              <Compute qx5 71c>
              <Compute  $S_{xx}^{-1}$  part on the s-chain 45b>
          }
      }

```

5.8.12 Q_{xx}^{-1} and S_{xx}^{-1} on a single s -chain

Therefore,

45a $\langle \text{Compute } Q_{xx}^{-1} \text{ part on the } s\text{-chain 45a} \rangle \equiv$
 $\langle \text{Compute } A^{-1}\psi \text{ on the upper two components 45c} \rangle$
 $\langle \text{Compute } B^{-1}\psi \text{ on the lower two components 45f} \rangle$

45b $\langle \text{Compute } S_{xx}^{-1} \text{ part on the } s\text{-chain 45b} \rangle \equiv$
 $\langle \text{Compute } A^{-1}\psi \text{ on the lower two components 45d} \rangle$
 $\langle \text{Compute } B^{-1}\psi \text{ on the upper two components 45e} \rangle$

And

45c $\langle \text{Compute } A^{-1}\psi \text{ on the upper two components 45c} \rangle \equiv$
 $\langle \text{Compute } L_A^{-1} \text{ on the upper components 47d} \rangle$
 $\langle \text{Compute } R_A^{-1} \text{ on the upper components 50b} \rangle$

45d $\langle \text{Compute } A^{-1}\psi \text{ on the lower two components 45d} \rangle \equiv$
 $\langle \text{Compute } L_A^{-1} \text{ on the lower components 48b} \rangle$
 $\langle \text{Compute } R_A^{-1} \text{ on the lower components 50c} \rangle$

45e $\langle \text{Compute } B^{-1}\psi \text{ on the upper two components 45e} \rangle \equiv$
 $\langle \text{Compute } L_B^{-1} \text{ on the upper components 49a} \rangle$
 $\langle \text{Compute } R_B^{-1} \text{ on the upper components 50d} \rangle$

45f $\langle \text{Compute } B^{-1}\psi \text{ on the lower two components 45f} \rangle \equiv$
 $\langle \text{Compute } L_B^{-1} \text{ on the lower components 49b} \rangle$
 $\langle \text{Compute } R_B^{-1} \text{ on the lower components 50e} \rangle$

For both Q_{xx}^{-1} and S_{xx}^{-1} we need to compute R_A and R_B . This can be done iteratively:

$$y_k^{(A)} = \begin{cases} \frac{1}{a}z_k, & \text{if } k = n-1 \\ \frac{1}{a}z_k - \frac{b}{a}y_{k+1}^{(A)}, & \text{otherwise} \end{cases}$$

$$y_k^{(B)} = \begin{cases} \frac{1}{a}z_0, & \text{if } k = 0 \\ \frac{1}{a}z_k - \frac{b}{a}y_{k-1}^{(B)}, & \text{otherwise} \end{cases}$$

It turns out, that these computations are faster on the regular FP instructions than on SSE. For this reason corresponding parts for L_X^{-1} depend on the memory layout of `vReal`.

Let us compute constant pieces first. Division is slow, so we compute $1/a$ and $-b/a$ once and for all:

45g $\langle \text{Global variables 10} \rangle + \equiv$
`static REAL inv_a;`
`static REAL b_over_a;`

45h $\langle \text{Compute values from } a, b \text{ and } c \text{ 45h} \rangle \equiv$
`inv_a = 1.0 / a;`
`b_over_a = -b * inv_a;`

Computing $z^{(A)} = L_A^{-1}x$ and $z^{(B)} = L_B^{-1}x$ is easy:

$$z_k^{(A)} = \begin{cases} -\sum_{j=0}^{n-2} \frac{(-b)^j c/a^{j+1}}{1+(-b)^{n-1}c/a^n} x_j + \frac{1}{1+(-b)^{n-1}c/a^n} x_{n-1}, & \text{if } k = n-1 \\ x_k, & \text{otherwise} \end{cases}$$

$$z_k^{(B)} = \begin{cases} \frac{1}{1+(-b)^{n-1}c/a^n} x_0 - \sum_{j=1}^{n-1} \frac{(-b)^{n-1-j} c/a^{n-j}}{1+(-b)^{n-1}c/a^n} x_j, & \text{if } k = 0 \\ x_k, & \text{otherwise} \end{cases}$$

We need to rewrite these expressions in a form suitable for SSE. Let us write

$$z_{n-1}^{(A)} = z_a^{(A)} + z_b^{(A)} + z_c^{(A)} + z_d^{(A)}$$

$$z_0^{(B)} = z_a^{(B)} + z_b^{(B)} + z_c^{(B)} + z_d^{(B)}$$

where

$$z_a^{(A)} = \sum_{j=0}^{n/4-1} \frac{-b^{4j} c/a^{4j+1}}{1+(-b)^{n-1}c/a^n} x_{4j}$$

$$z_b^{(A)} = \sum_{j=0}^{n/4-1} \frac{b^{4j+1} c/a^{4j+2}}{1+(-b)^{n-1}c/a^n} x_{4j+1}$$

$$z_c^{(A)} = \sum_{j=0}^{n/4-1} \frac{-b^{4j+2} c/a^{4j+3}}{1+(-b)^{n-1}c/a^n} x_{4j+2}$$

$$z_d^{(A)} = \sum_{j=0}^{n/4-2} \frac{b^{4j+3} c/a^{4j+4}}{1+(-b)^{n-1}c/a^n} x_{4j+3} + \frac{1}{1+(-b)^{n-1}c/a^n} x_{n-1}$$

$$z_a^{(B)} = \frac{1}{1+(-b)^{n-1}c/a^n} x_0 + \sum_{j=1}^{n/4-1} \frac{b^{n-1-4j} c/a^{n-4j}}{1+(-b)^{n-1}c/a^n} x_{4j}$$

$$z_b^{(B)} = \sum_{j=0}^{n/4-1} \frac{-b^{n-2-4j} c/a^{n-4j-1}}{1+(-b)^{n-1}c/a^n} x_{4j+1}$$

$$z_c^{(B)} = \sum_{j=0}^{n/4-1} \frac{b^{n-3-4j} c/a^{n-4j-2}}{1+(-b)^{n-1}c/a^n} x_{4j+2}$$

$$z_d^{(B)} = \sum_{j=0}^{n/4-1} \frac{-b^{n-4-4j} c/a^{n-4j-3}}{1+(-b)^{n-1}c/a^n} x_{4j+3}$$

These sums could be effectively computed with SSE, because their structures match DWF memory layout. Here are constants needed to compute $z^{(A)}$ and $z^{(B)}$:

```

46a  <Global variables 10>+=
      static vReal vfx_A;
      static vReal vfx_B;
      static vReal vab;
      static REAL c0;

46b  <Compute values from a, b and c 45h>+=
      c0 = 1./(1.-c/b*pow(b/a, (double)Sv*Vs));
      vab = vmk1(pow(b/a, (double)Vs));
      vfx_A = vmk4(-c0*c/a, c0*c*b/(a*a), -c0*c*b*b/(a*a*a), c0*c*b*b*b/(a*a*a*a));
      vfx_B = vmk4(c0*c*b*b*b/(a*a*a*a), -c0*c*b*b/(a*a*a), c0*c*b/(a*a), -c0*c/a);

```

We need `math.h` for the prototype of `pow()`:

```

46c  <Include files 34b>+=
      #include <math.h>

```

5.8.13 Compute L_A^{-1} and L_B^{-1}

There are two cases:

1. L_X^{-1} is computed as part of standalone diagonal piece. In this case, the computation is done from q to r and L_X^{-1} copies elements as needed.
2. Q_{xx}^{-1} is part of combined operator. In this case q is aliased to r , and no copy is performed.

Before spelling out the details, let us define a few handy macros:

```

47a  <Check xx-aliasing of q 47a>≡
      #if defined(qs)
      #define QSETUP(s)
      #define Q2R(d,pt)
      #else
      #define QSETUP(s) qs = &qx5[s];
      #define Q2R(d,pt) rs->f[d][c].pt = qs->f[d][c].pt;
      #endif

47b  <End xx-aliasing of q 47b>≡
      #undef QSETUP
      #undef Q2R

```

For completeness, here are definitions of variables used in the pieces bellow:

```

47c  <Qxx locals 47c>≡
      vReal fx;
      vHalfFermion zV;
      vcomplex zn;
      complex zX[Fd/2][Nc];

47d  <Compute  $L_A^{-1}$  on the upper components 47d>≡
      vhfzero(&zV);
      fx = vfx_A;
      <Check xx-aliasing of q 47a>
      for (s = 0; s < Sv_1; s++, fx = fx * vab) {
          rs = &rx5[s];
          QSETUP(s)
          <Compute  $zV \leftarrow zV + fx * qs^{up}$  48a>
      }
      rs = &rx5[Sv_1];
      QSETUP(Sv_1)
      vput_3(&fx, c0);
      <Compute  $zV \leftarrow zV + fx * qs^{up}$  48a>
      for (c = 0; c < Nc; c++) {
          <Compute wall value in zX[c] 50a>

          zn.re = qs->f[0][c].re;      zn.im = qs->f[0][c].im;
          vput_3(&zn.re, zX[0][c].re); vput_3(&zn.im, zX[0][c].im);
          rs->f[0][c].re = zn.re;      rs->f[0][c].im = zn.im;

          zn.re = qs->f[1][c].re;      zn.im = qs->f[1][c].im;
          vput_3(&zn.re, zX[1][c].re); vput_3(&zn.im, zX[1][c].im);
          rs->f[1][c].re = zn.re;      rs->f[1][c].im = zn.im;
      }
      <End xx-aliasing of q 47b>

```

This piece is used twice: once in the loop over L_s , and the second time after correcting s_3 :

```
48a  ⟨Compute  $zV \leftarrow zV + fx * qs^{up}$  48a⟩≡
      for (c = 0; c < Nc; c++) {
          zV.f[0][c].re += fx * qs->f[0][c].re; Q2R(0,re)
          zV.f[0][c].im += fx * qs->f[0][c].im; Q2R(0,im)
          zV.f[1][c].re += fx * qs->f[1][c].re; Q2R(1,re)
          zV.f[1][c].im += fx * qs->f[1][c].im; Q2R(1,im)
      }
```

The only difference between L_A^{-1} on lower components is the source of the data and the destination of the result. We have to repeat most of the above pieces though.

```
48b  ⟨Compute  $L_A^{-1}$  on the lower components 48b⟩≡
      vhfzero(&zV);
      fx = vfx_A;
      ⟨Check  $xx$ -aliasing of  $q$  47a⟩
      for (s = 0; s < Sv_1; s++, fx = fx * vab) {
          rs = &rx5[s];
          QSETUP(s)
          ⟨Compute  $zV \leftarrow zV + fx * qs^{down}$  48c⟩
      }
      rs = &rx5[Sv_1];
      QSETUP(Sv_1)
      vput_3(&fx, c0);
      ⟨Compute  $zV \leftarrow zV + fx * qs^{down}$  48c⟩
      for (c = 0; c < Nc; c++) {
          ⟨Compute wall value in  $zX[c]$  50a⟩

          zn.re = qs->f[2][c].re;      zn.im = qs->f[2][c].im;
          vput_3(&zn.re, zX[0][c].re); vput_3(&zn.im, zX[0][c].im);
          rs->f[2][c].re = zn.re;      rs->f[2][c].im = zn.im;

          zn.re = qs->f[3][c].re;      zn.im = qs->f[3][c].im;
          vput_3(&zn.re, zX[1][c].re); vput_3(&zn.im, zX[1][c].im);
          rs->f[3][c].re = zn.re;      rs->f[3][c].im = zn.im;
      }
      ⟨End  $xx$ -aliasing of  $q$  47b⟩

48c  ⟨Compute  $zV \leftarrow zV + fx * qs^{down}$  48c⟩≡
      for (c = 0; c < Nc; c++) {
          zV.f[0][c].re += fx * qs->f[2][c].re; Q2R(2,re)
          zV.f[0][c].im += fx * qs->f[2][c].im; Q2R(2,im)
          zV.f[1][c].re += fx * qs->f[3][c].re; Q2R(3,re)
          zV.f[1][c].im += fx * qs->f[3][c].im; Q2R(3,im)
      }
```


For L_B^{-1} the difference is in the direction of the sweep along the s -chain:

```

49a  ⟨Compute  $L_B^{-1}$  on the upper components 49a⟩≡
    vhfzero(&zV);
    fx = vfx_B;
    ⟨Check  $xx$ -aliasing of  $q$  47a⟩
    for (s = Sv; --s; fx = fx * vab) {
        rs = &rx5[s];
        QSETUP(s)
        ⟨Compute  $zV \leftarrow zV + fx * qs^{up}$  48a⟩
    }
    rs = &rx5[0];
    QSETUP(0)
    vput_0(&fx, c0);
    ⟨Compute  $zV \leftarrow zV + fx * qs^{up}$  48a⟩
    for (c = 0; c < Nc; c++) {
        ⟨Compute wall value in  $zX[c]$  50a⟩

        zn.re = qs->f[0][c].re;    zn.im = qs->f[0][c].im;
        vput_0(&zn.re, zX[0][c].re); vput_0(&zn.im, zX[0][c].im);
        rs->f[0][c].re = zn.re;    rs->f[0][c].im = zn.im;

        zn.re = qs->f[1][c].re;    zn.im = qs->f[1][c].im;
        vput_0(&zn.re, zX[1][c].re); vput_0(&zn.im, zX[1][c].im);
        rs->f[1][c].re = zn.re;    rs->f[1][c].im = zn.im;
    }
    ⟨End  $xx$ -aliasing of  $q$  47b⟩

```

Again, some repetition is needed for the lower component case:

```

49b  ⟨Compute  $L_B^{-1}$  on the lower components 49b⟩≡
    vhfzero(&zV);
    fx = vfx_B;
    ⟨Check  $xx$ -aliasing of  $q$  47a⟩
    for (s = Sv; --s; fx = fx * vab) {
        rs = &rx5[s];
        QSETUP(s)
        ⟨Compute  $zV \leftarrow zV + fx * qs^{down}$  48c⟩
    }
    rs = &rx5[0];
    QSETUP(0)
    vput_0(&fx, c0);
    ⟨Compute  $zV \leftarrow zV + fx * qs^{down}$  48c⟩
    for (c = 0; c < Nc; c++) {
        ⟨Compute wall value in  $zX[c]$  50a⟩

        zn.re = qs->f[2][c].re;    zn.im = qs->f[2][c].im;
        vput_0(&zn.re, zX[0][c].re); vput_0(&zn.im, zX[0][c].im);
        rs->f[2][c].re = zn.re;    rs->f[2][c].im = zn.im;

        zn.re = qs->f[3][c].re;    zn.im = qs->f[3][c].im;
        vput_0(&zn.re, zX[1][c].re); vput_0(&zn.im, zX[1][c].im);
        rs->f[3][c].re = zn.re;    rs->f[3][c].im = zn.im;
    }
    ⟨End  $xx$ -aliasing of  $q$  47b⟩

```

By now, we have four partial sums which must be combined into z_{n-1} :

```
50a  ⟨Compute wall value in  $\mathbf{zX}[c]$  50a⟩≡
       $\mathbf{zX}[0][c].\text{re} = \text{vsum}(\mathbf{zV.f}[0][c].\text{re});$ 
       $\mathbf{zX}[0][c].\text{im} = \text{vsum}(\mathbf{zV.f}[0][c].\text{im});$ 
       $\mathbf{zX}[1][c].\text{re} = \text{vsum}(\mathbf{zV.f}[1][c].\text{re});$ 
       $\mathbf{zX}[1][c].\text{im} = \text{vsum}(\mathbf{zV.f}[1][c].\text{im});$ 
```

5.8.14 Compute R_A^{-1} and R_B^{-1}

Since R_X^{-1} is always computed after L_X^{-1} , it takes its source from **rs** and places the result back into **rs**. For R_X^{-1} , again combinations of *A* and *B* and upper and lower parts require some cut, paste and edit.

```
50b  ⟨Compute  $R_A^{-1}$  on the upper components 50b⟩≡
      ⟨Init out of bound  $y$  51a⟩
      for (s = Sv; s--;) {
        rs = &rx5[s];
        for (c = 0; c < Nc; c++) {
          ⟨Compute  $y_{k,[0]}^{(A)}$  51b⟩
          ⟨Compute  $y_{k,[1]}^{(A)}$  51c⟩
        }
      }
```

```
50c  ⟨Compute  $R_A^{-1}$  on the lower components 50c⟩≡
      ⟨Init out of bound  $y$  51a⟩
      for (s = Sv; s--;) {
        rs = &rx5[s];
        for (c = 0; c < Nc; c++) {
          ⟨Compute  $y_{k,[2]}^{(A)}$  51d⟩
          ⟨Compute  $y_{k,[3]}^{(A)}$  52a⟩
        }
      }
```

```
50d  ⟨Compute  $R_B^{-1}$  on the upper components 50d⟩≡
      ⟨Init out of bound  $y$  51a⟩
      for (s = 0; s < Sv; s++) {
        rs = &rx5[s];
        for (c = 0; c < Nc; c++) {
          ⟨Compute  $y_{k,[0]}^{(B)}$  52b⟩
          ⟨Compute  $y_{k,[1]}^{(B)}$  52c⟩
        }
      }
```

```
50e  ⟨Compute  $R_B^{-1}$  on the lower components 50e⟩≡
      ⟨Init out of bound  $y$  51a⟩
      for (s = 0; s < Sv; s++) {
        rs = &rx5[s];
        for (c = 0; c < Nc; c++) {
          ⟨Compute  $y_{k,[2]}^{(B)}$  53a⟩
          ⟨Compute  $y_{k,[3]}^{(B)}$  53b⟩
        }
      }
```

We do not handle boundary cases in a special way. Instead, the previos value of y is stored in **yOut**:

```
50f  ⟨Qxx locals 47c⟩+≡
      complex yOut[Fd/2][Nc];
```

51a $\langle \text{Init out of bound } y \text{ 51a} \rangle \equiv$
`yOut[0][0].re = yOut[0][0].im = 0;
yOut[0][1].re = yOut[0][1].im = 0;
yOut[0][2].re = yOut[0][2].im = 0;
yOut[1][0].re = yOut[1][0].im = 0;
yOut[1][1].re = yOut[1][1].im = 0;
yOut[1][2].re = yOut[1][2].im = 0;`

Now, the magic of copy paste:

51b $\langle \text{Compute } y_{k,[0]}^{(A)} \text{ 51b} \rangle \equiv$

```
{
    REAL *rs0re = (REAL *)&rs->f[0][c].re;
    REAL *rs0im = (REAL *)&rs->f[0][c].im;

    rs0re[3] = inv_a * rs0re[3] + b_over_a * yOut[0][c].re;
    rs0re[2] = inv_a * rs0re[2] + b_over_a * rs0re[3];
    rs0re[1] = inv_a * rs0re[1] + b_over_a * rs0re[2];
    yOut[0][c].re = rs0re[0] = inv_a * rs0re[0] + b_over_a * rs0re[1];

    rs0im[3] = inv_a * rs0im[3] + b_over_a * yOut[0][c].im;
    rs0im[2] = inv_a * rs0im[2] + b_over_a * rs0im[3];
    rs0im[1] = inv_a * rs0im[1] + b_over_a * rs0im[2];
    yOut[0][c].im = rs0im[0] = inv_a * rs0im[0] + b_over_a * rs0im[1];
}
```

51c $\langle \text{Compute } y_{k,[1]}^{(A)} \text{ 51c} \rangle \equiv$

```
{
    REAL *rs1re = (REAL *)&rs->f[1][c].re;
    REAL *rs1im = (REAL *)&rs->f[1][c].im;

    rs1re[3] = inv_a * rs1re[3] + b_over_a * yOut[1][c].re;
    rs1re[2] = inv_a * rs1re[2] + b_over_a * rs1re[3];
    rs1re[1] = inv_a * rs1re[1] + b_over_a * rs1re[2];
    yOut[1][c].re = rs1re[0] = inv_a * rs1re[0] + b_over_a * rs1re[1];

    rs1im[3] = inv_a * rs1im[3] + b_over_a * yOut[1][c].im;
    rs1im[2] = inv_a * rs1im[2] + b_over_a * rs1im[3];
    rs1im[1] = inv_a * rs1im[1] + b_over_a * rs1im[2];
    yOut[1][c].im = rs1im[0] = inv_a * rs1im[0] + b_over_a * rs1im[1];
}
```

51d $\langle \text{Compute } y_{k,[2]}^{(A)} \text{ 51d} \rangle \equiv$

```
{
    REAL *rs2re = (REAL *)&rs->f[2][c].re;
    REAL *rs2im = (REAL *)&rs->f[2][c].im;

    rs2re[3] = inv_a * rs2re[3] + b_over_a * yOut[0][c].re;
    rs2re[2] = inv_a * rs2re[2] + b_over_a * rs2re[3];
    rs2re[1] = inv_a * rs2re[1] + b_over_a * rs2re[2];
    yOut[0][c].re = rs2re[0] = inv_a * rs2re[0] + b_over_a * rs2re[1];

    rs2im[3] = inv_a * rs2im[3] + b_over_a * yOut[0][c].im;
    rs2im[2] = inv_a * rs2im[2] + b_over_a * rs2im[3];
    rs2im[1] = inv_a * rs2im[1] + b_over_a * rs2im[2];
    yOut[0][c].im = rs2im[0] = inv_a * rs2im[0] + b_over_a * rs2im[1];
}
```

52a $\langle \text{Compute } y_{k,[3]}^{(A)} \text{ 52a} \rangle \equiv$

```

{
  REAL *rs3re = (REAL *)&rs->f[3][c].re;
  REAL *rs3im = (REAL *)&rs->f[3][c].im;

  rs3re[3] = inv_a * rs3re[3] + b_over_a * yOut[1][c].re;
  rs3re[2] = inv_a * rs3re[2] + b_over_a * rs3re[3];
  rs3re[1] = inv_a * rs3re[1] + b_over_a * rs3re[2];
  yOut[1][c].re = rs3re[0] = inv_a * rs3re[0] + b_over_a * rs3re[1];

  rs3im[3] = inv_a * rs3im[3] + b_over_a * yOut[1][c].im;
  rs3im[2] = inv_a * rs3im[2] + b_over_a * rs3im[3];
  rs3im[1] = inv_a * rs3im[1] + b_over_a * rs3im[2];
  yOut[1][c].im = rs3im[0] = inv_a * rs3im[0] + b_over_a * rs3im[1];
}

```

52b $\langle \text{Compute } y_{k,[0]}^{(B)} \text{ 52b} \rangle \equiv$

```

{
  REAL *rs0re = (REAL *)&rs->f[0][c].re;
  REAL *rs0im = (REAL *)&rs->f[0][c].im;

  rs0re[0] = inv_a * rs0re[0] + b_over_a * yOut[0][c].re;
  rs0re[1] = inv_a * rs0re[1] + b_over_a * rs0re[0];
  rs0re[2] = inv_a * rs0re[2] + b_over_a * rs0re[1];
  yOut[0][c].re = rs0re[3] = inv_a * rs0re[3] + b_over_a * rs0re[2];

  rs0im[0] = inv_a * rs0im[0] + b_over_a * yOut[0][c].im;
  rs0im[1] = inv_a * rs0im[1] + b_over_a * rs0im[0];
  rs0im[2] = inv_a * rs0im[2] + b_over_a * rs0im[1];
  yOut[0][c].im = rs0im[3] = inv_a * rs0im[3] + b_over_a * rs0im[2];
}

```

52c $\langle \text{Compute } y_{k,[1]}^{(B)} \text{ 52c} \rangle \equiv$

```

{
  REAL *rs1re = (REAL *)&rs->f[1][c].re;
  REAL *rs1im = (REAL *)&rs->f[1][c].im;

  rs1re[0] = inv_a * rs1re[0] + b_over_a * yOut[1][c].re;
  rs1re[1] = inv_a * rs1re[1] + b_over_a * rs1re[0];
  rs1re[2] = inv_a * rs1re[2] + b_over_a * rs1re[1];
  yOut[1][c].re = rs1re[3] = inv_a * rs1re[3] + b_over_a * rs1re[2];

  rs1im[0] = inv_a * rs1im[0] + b_over_a * yOut[1][c].im;
  rs1im[1] = inv_a * rs1im[1] + b_over_a * rs1im[0];
  rs1im[2] = inv_a * rs1im[2] + b_over_a * rs1im[1];
  yOut[1][c].im = rs1im[3] = inv_a * rs1im[3] + b_over_a * rs1im[2];
}

```

53a $\langle \text{Compute } y_{k,[2]}^{(B)} \text{ 53a} \rangle \equiv$

```

{
    REAL *rs2re = (REAL *)&rs->f[2][c].re;
    REAL *rs2im = (REAL *)&rs->f[2][c].im;

    rs2re[0] = inv_a * rs2re[0] + b_over_a * yOut[0][c].re;
    rs2re[1] = inv_a * rs2re[1] + b_over_a * rs2re[0];
    rs2re[2] = inv_a * rs2re[2] + b_over_a * rs2re[1];
    yOut[0][c].re = rs2re[3] = inv_a * rs2re[3] + b_over_a * rs2re[2];

    rs2im[0] = inv_a * rs2im[0] + b_over_a * yOut[0][c].im;
    rs2im[1] = inv_a * rs2im[1] + b_over_a * rs2im[0];
    rs2im[2] = inv_a * rs2im[2] + b_over_a * rs2im[1];
    yOut[0][c].im = rs2im[3] = inv_a * rs2im[3] + b_over_a * rs2im[2];
}

```

53b $\langle \text{Compute } y_{k,[3]}^{(B)} \text{ 53b} \rangle \equiv$

```

{
    REAL *rs3re = (REAL *)&rs->f[3][c].re;
    REAL *rs3im = (REAL *)&rs->f[3][c].im;

    rs3re[0] = inv_a * rs3re[0] + b_over_a * yOut[1][c].re;
    rs3re[1] = inv_a * rs3re[1] + b_over_a * rs3re[0];
    rs3re[2] = inv_a * rs3re[2] + b_over_a * rs3re[1];
    yOut[1][c].re = rs3re[3] = inv_a * rs3re[3] + b_over_a * rs3re[2];

    rs3im[0] = inv_a * rs3im[0] + b_over_a * yOut[1][c].im;
    rs3im[1] = inv_a * rs3im[1] + b_over_a * rs3im[0];
    rs3im[2] = inv_a * rs3im[2] + b_over_a * rs3im[1];
    yOut[1][c].im = rs3im[3] = inv_a * rs3im[3] + b_over_a * rs3im[2];
}

```

5.8.15 Standalone off-diagonal pieces

First, simple off-diagonal parts.

53c $\langle \text{Static function prototypes 20b} \rangle + \equiv$

```

static void compute_Qxy(vFermion *d, const vFermion *s, struct neighbor *nb);

```

5.8.16 `compute_Qoe(d,s)` or $d \leftarrow Q_{eo}s$

53d $\langle \text{Static function prototypes 20b} \rangle + \equiv$

```

static void inline compute_Qoe(vOddFermion *d, const vEvenFermion *s)
{
    compute_Qxy(&d->f, &s->f, &odd_even);
}

```

5.8.17 `compute_Qeo(d,s)` or $d \leftarrow Q_{oe}s$

53e $\langle \text{Static function prototypes 20b} \rangle + \equiv$

```

static void inline compute_Qeo(vEvenFermion *d, const vOddFermion *s)
{
    compute_Qxy(&d->f, &s->f, &even_odd);
}

```

5.8.18 compute_1Soe(d,q,s) or $d \leftarrow q - S_{eo}s$

54a $\langle \text{Static function prototypes 20b} \rangle + \equiv$

```
static void compute_1Sxy(vFermion *d,
                        const vFermion *q,
                        const vFermion *s,
                        struct neighbor *nb);
static void inline compute_1Soe(vOddFermion *d,
                               const vOddFermion *q,
                               const vEvenFermion *s)
{
    compute_1Sxy(&d->f, &q->f, &s->f, &odd_even);
}
```

5.8.19 compute_Qxy(chi,psi), or $\chi \leftarrow Q_{xy}\psi$

54b $\langle \text{Static functions 19a} \rangle + \equiv$

```
static void
compute_Qxy(vFermion *chi,
            const vFermion *psi,
            struct neighbor *nb)
{
     $\langle Q \text{ common locals 70c} \rangle$ 
     $\langle Qxy \text{ locals 70d} \rangle$ 

     $\langle \text{Setup } xy\text{-aliasing of } q \text{ 57b} \rangle$ 
     $\langle \text{Compute projections for } Q \text{ send 58c} \rangle$ 
     $\langle \text{Start sends and receives 71f} \rangle$ 
     $\langle \text{Compute inside part for } Q_{xy} \text{ 60d} \rangle$ 
     $\langle \text{Finish sends and receives 71g} \rangle$ 
     $\langle \text{Compute boundary part for } Q_{xy} \text{ 60e} \rangle$ 
     $\langle \text{Finish } xy\text{-aliasing of } q \text{ 57c} \rangle$ 
}
```

5.8.20 compute_1Sxy(chi,eta,psi), or $\chi \leftarrow \eta - S_{xy}\psi$

For other functions, little need to be changes at this granularity. E.g., here is the final part of M^\dagger :

54c $\langle \text{Static functions 19a} \rangle + \equiv$

```
static void
compute_1Sxy(vFermion *chi,
            const vFermion *eta,
            const vFermion *psi,
            struct neighbor *nb)
{
     $\langle Q \text{ common locals 70c} \rangle$ 
     $\langle Qxy \text{ locals 70d} \rangle$ 

     $\langle \text{Setup } xy\text{-aliasing of } q \text{ 57b} \rangle$ 
     $\langle \text{Compute projections for } S \text{ send 59a} \rangle$ 
     $\langle \text{Start sends and receives 71f} \rangle$ 
     $\langle \text{Compute inside part for } 1 - S_{xy} \text{ 62e} \rangle$ 
     $\langle \text{Finish sends and receives 71g} \rangle$ 
     $\langle \text{Compute boundary part for } 1 - S_{xy} \text{ 63a} \rangle$ 
     $\langle \text{Finish } xy\text{-aliasing of } q \text{ 57c} \rangle$ 
}
```

5.8.21 `compute_Qxx1Qxy(chi,psi)`, or $\chi \leftarrow Q_{xx}^{-1}Q_{xy}\psi$

```
55a  <Static functions 19a>+≡
      static void
      compute_Qxx1Qxy(vFermion *chi,
                      const vFermion *psi,
                      struct neighbor *nb)
      {
        <Q common locals 70c>
        <Qxy locals 70d>
        <Qxx locals 47c>

        <Setup xy-aliasing of q 57b>
        <Compute projections for Q send 58c>
        <Start sends and receives 71f>
        <Compute inside part for  $Q_{xx}^{-1}Q_{xy}$  65a>
        <Finish sends and receives 71g>
        <Compute boundary part for  $Q_{xx}^{-1}Q_{xy}$  65b>
        <Finish xy-aliasing of q 57c>
      }
```

5.8.22 `compute_Sxx1Sxy(chi,psi)`, or $\chi \leftarrow S_{xx}^{-1}S_{xy}\psi$

```
55b  <Static functions 19a>+≡
      static void
      compute_Sxx1Sxy(vFermion *chi,
                      const vFermion *psi,
                      struct neighbor *nb)
      {
        <Q common locals 70c>
        <Qxy locals 70d>
        <Qxx locals 47c>

        <Setup xy-aliasing of q 57b>
        <Compute projections for S send 59a>
        <Start sends and receives 71f>
        <Compute inside part for  $S_{xx}^{-1}S_{xy}$  65c>
        <Finish sends and receives 71g>
        <Compute boundary part for  $S_{xx}^{-1}S_{xy}$  65d>
        <Finish xy-aliasing of q 57c>
      }
```

5.8.23 `compute_1Qxx1Qxy(chi,norm,eta,psi)`, or $\chi \leftarrow \eta - Q_{xx}^{-1}Q_{xy}\psi$ and $r \leftarrow \langle \chi, \chi \rangle$

56a `<Static functions 19a>+≡`
`static void`
`compute_1Qxx1Qxy(vFermion *chi,`
`double *norm,`
`const vFermion *eta,`
`const vFermion *psi,`
`struct neighbor *nb)`
`{`
`<Q common locals 70c>`
`<Qxy locals 70d>`
`<Qxx locals 47c>`
`vReal vv;`
`vReal nv;`
`*norm = 0;`

`<Setup xy-aliasing of q 57b>`
`<Compute projections for Q send 58c>`
`<Start sends and receives 71f>`
`<Compute inside part for $1 - Q_{xx}^{-1}Q_{xy}$ 66b>`
`<Finish sends and receives 71g>`
`<Compute boundary part for $1 - Q_{xx}^{-1}Q_{xy}$ 66c>`
`<Start $\langle r, r \rangle$ computation 71h>`
`<Finish xy-aliasing of q 57c>`
`}`

5.8.24 `compute_Dx(chi,eta,psi)`, or $\chi_x \leftarrow Q_{xx}\eta_x + Q_{xy}\psi_y$

56b `<Static functions 19a>+≡`
`static void`
`compute_Dx(vFermion *chi,`
`const vFermion *eta,`
`const vFermion *psi,`
`struct neighbor *nb)`
`{`
`<Q common locals 70c>`
`<Qxy locals 70d>`
`<Dxx locals 69c>`

`<Setup xy-aliasing of q 57b>`
`<Compute projections for Q send 58c>`
`<Start sends and receives 71f>`
`<Compute inside part for $Q_{xx}\eta + Q_{xy}\psi$ 67b>`
`<Finish sends and receives 71g>`
`<Compute boundary part for $Q_{xx}\eta + Q_{xy}\psi$ 67c>`
`<Finish xy-aliasing of q 57c>`
`}`

5.8.25 `compute_Dcx(chi,eta,psi)`, or $\chi_x \leftarrow S_{xx}\eta_x + S_{xy}\psi_y$

```
57a  <Static functions 19a>+≡
      static void
      compute_Dcx(vFermion *chi,
                  const vFermion *eta,
                  const vFermion *psi,
                  struct neighbor *nb)
      {
        <Q common locals 70c>
        <Qxy locals 70d>
        <Dxx locals 69c>

        <Setup xy-aliasing of q 57b>
        <Compute projections for S send 59a>
        <Start sends and receives 71f>
        <Compute inside part for  $S_{xx}\eta + S_{xy}\psi$  67e>
        <Finish sends and receives 71g>
        <Compute boundary part for  $S_{xx}\eta + S_{xy}\psi$  68a>
        <Finish xy-aliasing of q 57c>
      }
```

5.8.26 Aliasing macros

Remember, that Z_{xy} always puts result into `q`. For standalone diagonal pieces a couple of `define`'s help to manage `__restrict__` pointers properly.

```
57b  <Setup xy-aliasing of q 57b>≡
      #define qx5 rx5
      #define qs rs

57c  <Finish xy-aliasing of q 57c>≡
      #undef qs
      #undef qx5
```

5.8.27 `compute_De(chi,eta,psi)`, or $\chi \leftarrow Q_{ee}\eta + Q_{eo}\psi$

```
57d  <Static function prototypes 20b>+≡
      static void compute_Dx(vFermion *chi,
                            const vFermion *eta,
                            const vFermion *psi,
                            struct neighbor *nb);
      static void inline compute_De(vEvenFermion *chi,
                                    const vEvenFermion *eta,
                                    const vOddFermion *psi)
      {
        compute_Dx(&chi->f, &eta->f, &psi->f, &even_odd);
      }
```

5.8.28 `compute_Do(chi,eta,psi)`, or $\chi \leftarrow Q_{oo}\eta + Q_{oe}\psi$

```
57e  <Static function prototypes 20b>+≡
      static void inline compute_Do(vOddFermion *chi,
                                    const vOddFermion *eta,
                                    const vEvenFermion *psi)
      {
        compute_Dx(&chi->f, &eta->f, &psi->f, &odd_even);
      }
```

5.8.29 `compute_Dce(chi,eta,psi)`, or $\chi \leftarrow S_{ee}\eta + S_{eo}\psi$

```
58a  <Static function prototypes 20b>+≡
      static void compute_Dcx(vFermion *chi,
                             const vFermion *eta,
                             const vFermion *psi,
                             struct neighbor *nb);
      static void inline compute_Dce(vEvenFermion *chi,
                                     const vEvenFermion *eta,
                                     const vOddFermion *psi)
      {
        compute_Dcx(&chi->f, &eta->f, &psi->f, &even_odd);
      }
```

5.8.30 `compute_Dco(chi,eta,psi)`, or $\chi \leftarrow S_{oo}\eta + S_{oe}\psi$

```
58b  <Static function prototypes 20b>+≡
      static void inline compute_Dco(vOddFermion *chi,
                                     const vOddFermion *eta,
                                     const vEvenFermion *psi)
      {
        compute_Dcx(&chi->f, &eta->f, &psi->f, &odd_even);
      }
```

5.8.31 Projections to be sent

Next we compute $(1 \pm \gamma_\mu)$ projections to be sent to our neighbors. There are two cases here, one of Q_{xy} and another for S_{xy} . In principle, we might have handled both of them with some jungling of the `struct neighbor` tables, but let us go a simple if extensive way for now.

Notice that projections are done in the opposite direction than for inside sites. This is because the receiving node looks backward where we send forward.

```
58c  <Compute projections for Q send 58c>≡
      {
        int k, i, s, c, *src;
        const vFermion *f;
        vHalfFermion *g;

        k = 0; <Construct (1 +  $\gamma_0$ ) send k-buffer 59b>
        k = 1; <Construct (1 -  $\gamma_0$ ) send k-buffer 59c>
        k = 2; <Construct (1 +  $\gamma_1$ ) send k-buffer 59d>
        k = 3; <Construct (1 -  $\gamma_1$ ) send k-buffer 59e>
        k = 4; <Construct (1 +  $\gamma_2$ ) send k-buffer 59f>
        k = 5; <Construct (1 -  $\gamma_2$ ) send k-buffer 60a>
        k = 6; <Construct (1 +  $\gamma_3$ ) send k-buffer 60b>
        k = 7; <Construct (1 -  $\gamma_3$ ) send k-buffer 60c>
      }
```

```

59a   $\langle \text{Compute projections for } S \text{ send 59a} \rangle \equiv$ 
    {
        int k, i, s, c, *src;
        const vFermion *f;
        vHalfFermion *g;

        k = 0;  $\langle \text{Construct } (1 - \gamma_0) \text{ send } k\text{-buffer 59c} \rangle$ 
        k = 1;  $\langle \text{Construct } (1 + \gamma_0) \text{ send } k\text{-buffer 59b} \rangle$ 
        k = 2;  $\langle \text{Construct } (1 - \gamma_1) \text{ send } k\text{-buffer 59e} \rangle$ 
        k = 3;  $\langle \text{Construct } (1 + \gamma_1) \text{ send } k\text{-buffer 59d} \rangle$ 
        k = 4;  $\langle \text{Construct } (1 - \gamma_2) \text{ send } k\text{-buffer 60a} \rangle$ 
        k = 5;  $\langle \text{Construct } (1 + \gamma_2) \text{ send } k\text{-buffer 59f} \rangle$ 
        k = 6;  $\langle \text{Construct } (1 - \gamma_3) \text{ send } k\text{-buffer 60c} \rangle$ 
        k = 7;  $\langle \text{Construct } (1 + \gamma_3) \text{ send } k\text{-buffer 60b} \rangle$ 
    }

59b   $\langle \text{Construct } (1 + \gamma_0) \text{ send } k\text{-buffer 59b} \rangle \equiv$ 
    for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
            for (c = 0; c < Nc; c++) {
                 $\langle \text{Build } (1 + \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 5a} \rangle$ 
            }
        }
    }

59c   $\langle \text{Construct } (1 - \gamma_0) \text{ send } k\text{-buffer 59c} \rangle \equiv$ 
    for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
            for (c = 0; c < Nc; c++) {
                 $\langle \text{Build } (1 - \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 5c} \rangle$ 
            }
        }
    }

59d   $\langle \text{Construct } (1 + \gamma_1) \text{ send } k\text{-buffer 59d} \rangle \equiv$ 
    for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
            for (c = 0; c < Nc; c++) {
                 $\langle \text{Build } (1 + \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 5e} \rangle$ 
            }
        }
    }

59e   $\langle \text{Construct } (1 - \gamma_1) \text{ send } k\text{-buffer 59e} \rangle \equiv$ 
    for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
            for (c = 0; c < Nc; c++) {
                 $\langle \text{Build } (1 - \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 5g} \rangle$ 
            }
        }
    }

59f   $\langle \text{Construct } (1 + \gamma_2) \text{ send } k\text{-buffer 59f} \rangle \equiv$ 
    for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
            for (c = 0; c < Nc; c++) {
                 $\langle \text{Build } (1 + \gamma_2) \text{ projection of } *f \text{ in } *g \text{ 6a} \rangle$ 
            }
        }
    }

```

```

60a  <Construct (1 -  $\gamma_2$ ) send k-buffer 60a>≡
      for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
          for (c = 0; c < Nc; c++) {
            <Build (1 -  $\gamma_2$ ) projection of *f in *g 6c>
          }
        }
      }

60b  <Construct (1 +  $\gamma_3$ ) send k-buffer 60b>≡
      for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
          for (c = 0; c < Nc; c++) {
            <Build (1 +  $\gamma_3$ ) projection of *f in *g 6e>
          }
        }
      }

60c  <Construct (1 -  $\gamma_3$ ) send k-buffer 60c>≡
      for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
        for (s = Sv, f = &psi[*src]; s--; g++, f++) {
          for (c = 0; c < Nc; c++) {
            <Build (1 -  $\gamma_3$ ) projection of *f in *g 6g>
          }
        }
      }

```

5.8.32 Parts of $Q_{xy}\psi$

Let us start with the simplest of the five operators we need.

```

60d  <Compute inside part for  $Q_{xy}$  60d>≡
      for (i = 0; i < nb->inside_size; i++) {
        xyzt = nb->inside[i];
        xyzt5 = xyzt * Sv;
        <Extract 1-d addresses 71a>
        <Build SSE SU(3) objects 69d>
        <Compute  $Q_{xy}$  part on the inside s-chain 60f>
      }

60e  <Compute boundary part for  $Q_{xy}$  60e>≡
      for (i = 0; i < nb->boundary_size; i++) {
        int m = nb->boundary[i].mask;

        xyzt = nb->boundary[i].index;
        xyzt5 = xyzt * Sv;
        <Extract 1-d addresses 71a>
        <Build SSE SU(3) objects 69d>
        <Compute  $Q_{xy}$  part on the boundary s-chain 61a>
      }

60f  <Compute  $Q_{xy}$  part on the inside s-chain 60f>≡
      for (s = 0; s < Sv; s++) {
        <Compute Q inside  $\gamma$ -projections 61b>
        <Inside multiply by Vs 62b>
        <Compute Q  $\gamma$ -unprojections and sum the results 62a>
      }

```

```

61a  <Compute  $Q_{xy}$  part on the boundary  $s$ -chain 61a>≡
      for (s = 0; s < Sv; s++) {
        <Compute  $Q$  boundary  $\gamma$ -projections 61c>
        <Boundary multiply by  $V$ s 62c>
        <Compute  $Q$   $\gamma$ -unprojections and sum the results 62a>
      }

61b  <Compute  $Q$  inside  $\gamma$ -projections 61b>≡
      <Construct neighbor pointers 70a>
      for (c = 0; c < Nc; c++) {
        k=0; f=&psi[ps[0]]; g=&gg[0]; <Build  $(1 - \gamma_0)$  projection of *f in *g 5c>
        k=1; f=&psi[ps[1]]; g=&gg[1]; <Build  $(1 + \gamma_0)$  projection of *f in *g 5a>
        k=2; f=&psi[ps[2]]; g=&gg[2]; <Build  $(1 - \gamma_1)$  projection of *f in *g 5g>
        k=3; f=&psi[ps[3]]; g=&gg[3]; <Build  $(1 + \gamma_1)$  projection of *f in *g 5e>
        k=4; f=&psi[ps[4]]; g=&gg[4]; <Build  $(1 - \gamma_2)$  projection of *f in *g 6c>
        k=5; f=&psi[ps[5]]; g=&gg[5]; <Build  $(1 + \gamma_2)$  projection of *f in *g 6a>
        k=6; f=&psi[ps[6]]; g=&gg[6]; <Build  $(1 - \gamma_3)$  projection of *f in *g 6g>
        k=7; f=&psi[ps[7]]; g=&gg[7]; <Build  $(1 + \gamma_3)$  projection of *f in *g 6e>
      }

61c  <Compute  $Q$  boundary  $\gamma$ -projections 61c>≡
      <Construct neighbor pointers 70a>
      for (c = 0; c < 3; c++) {
        if ((m & 0x01) == 0) {
          k=0; f=&psi[ps[0]]; g=&gg[0]; <Build  $(1 - \gamma_0)$  projection of *f in *g 5c>
        }
        if ((m & 0x02) == 0) {
          k=1; f=&psi[ps[1]]; g=&gg[1]; <Build  $(1 + \gamma_0)$  projection of *f in *g 5a>
        }
        if ((m & 0x04) == 0) {
          k=2; f=&psi[ps[2]]; g=&gg[2]; <Build  $(1 - \gamma_1)$  projection of *f in *g 5g>
        }
        if ((m & 0x08) == 0) {
          k=3; f=&psi[ps[3]]; g=&gg[3]; <Build  $(1 + \gamma_1)$  projection of *f in *g 5e>
        }
        if ((m & 0x10) == 0) {
          k=4; f=&psi[ps[4]]; g=&gg[4]; <Build  $(1 - \gamma_2)$  projection of *f in *g 6c>
        }
        if ((m & 0x20) == 0) {
          k=5; f=&psi[ps[5]]; g=&gg[5]; <Build  $(1 + \gamma_2)$  projection of *f in *g 6a>
        }
        if ((m & 0x40) == 0) {
          k=6; f=&psi[ps[6]]; g=&gg[6]; <Build  $(1 - \gamma_3)$  projection of *f in *g 6g>
        }
        if ((m & 0x80) == 0) {
          k=7; f=&psi[ps[7]]; g=&gg[7]; <Build  $(1 + \gamma_3)$  projection of *f in *g 6e>
        }
      }
}

```

62a \langle Compute Q γ -unprojections and sum the results 62a $\rangle \equiv$

```

rs = &rx5[s];
for (c = 0; c < Nc; c++) {
    k = 7;  $\langle$ Unproject  $(1 + \gamma_3)$  link 6f $\rangle$ 
    k = 6;  $\langle$ Unproject and accumulate  $(1 - \gamma_3)$  link 6h $\rangle$ 
    k = 3;  $\langle$ Unproject and accumulate  $(1 + \gamma_1)$  link 5f $\rangle$ 
    k = 2;  $\langle$ Unproject and accumulate  $(1 - \gamma_1)$  link 5h $\rangle$ 
    k = 1;  $\langle$ Unproject and accumulate  $(1 + \gamma_0)$  link 5b $\rangle$ 
    k = 0;  $\langle$ Unproject and accumulate  $(1 - \gamma_0)$  link 5d $\rangle$ 
    k = 5;  $\langle$ Unproject and accumulate  $(1 + \gamma_2)$  link 6b $\rangle$ 
    k = 4;  $\langle$ Unproject and accumulate  $(1 - \gamma_2)$  link 6d $\rangle$ 
}

```

Now we have everything we need to compute $U(1 \pm \gamma_\mu)\psi$ pieces:

62b \langle Inside multiply by V s 62b $\rangle \equiv$

```

for (d = 0; d < 2*DIM; d++) {
    vHalfFermion * __restrict__ h = &hh[d];
    vSU3 *u = &V[d];
    g = &gg[d];
     $\langle$ Multiply *u by *g and store the result in *h 62d $\rangle$ 
}

```

If the neighbor is on another node, it is in the receive buffer by now.

62c \langle Boundary multiply by V s 62c $\rangle \equiv$

```

for (d = 0; d < 2*DIM; d++) {
    vHalfFermion * __restrict__ h = &hh[d];
    vSU3 *u = &V[d];
    g = (m & (1 << d))? &nb->rcv_buf[d][ps[d]]: &gg[d];
     $\langle$ Multiply *u by *g and store the result in *h 62d $\rangle$ 
}

```

62d \langle Multiply *u by *g and store the result in *h 62d $\rangle \equiv$

```

for (c = 0; c < Nc; c++) {
    h->f[0][c].re=u->v[c][0].re*g->f[0][0].re-u->v[c][0].im*g->f[0][0].im
        +u->v[c][1].re*g->f[0][1].re-u->v[c][1].im*g->f[0][1].im
        +u->v[c][2].re*g->f[0][2].re-u->v[c][2].im*g->f[0][2].im;
    h->f[0][c].im=u->v[c][0].im*g->f[0][0].re+u->v[c][0].re*g->f[0][0].im
        +u->v[c][1].im*g->f[0][1].re+u->v[c][1].re*g->f[0][1].im
        +u->v[c][2].im*g->f[0][2].re+u->v[c][2].re*g->f[0][2].im;
    h->f[1][c].re=u->v[c][0].re*g->f[1][0].re-u->v[c][0].im*g->f[1][0].im
        +u->v[c][1].re*g->f[1][1].re-u->v[c][1].im*g->f[1][1].im
        +u->v[c][2].re*g->f[1][2].re-u->v[c][2].im*g->f[1][2].im;
    h->f[1][c].im=u->v[c][0].im*g->f[1][0].re+u->v[c][0].re*g->f[1][0].im
        +u->v[c][1].im*g->f[1][1].re+u->v[c][1].re*g->f[1][1].im
        +u->v[c][2].im*g->f[1][2].re+u->v[c][2].re*g->f[1][2].im;
}

```

5.8.33 Parts of $\eta - S_{xy}\psi$

62e \langle Compute inside part for $1 - S_{xy}$ 62e $\rangle \equiv$

```

for (i = 0; i < nb->inside_size; i++) {
    const vFermion *ex5, *es;

    xyzt = nb->inside[i];
    xyzt5 = xyzt * Sv;
     $\langle$ Extract 1-d addresses 71a $\rangle$ 
    ex5 = &eta[xyzt5];
     $\langle$ Build SSE  $SU(3)$  objects 69d $\rangle$ 
     $\langle$ Compute  $1 - S_{xy}$  part on the inside s-chain 63b $\rangle$ 
}

```

```

63a  <Compute boundary part for  $1 - S_{xy}$  63a>≡
      for (i = 0; i < nb->boundary_size; i++) {
          const vFermion *ex5, *es;
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 71a>
          ex5 = &eta[xyzt5];
          <Build SSE SU(3) objects 69d>
          <Compute  $1 - S_{xy}$  part on the boundary s-chain 63c>
      }

63b  <Compute  $1 - S_{xy}$  part on the inside s-chain 63b>≡
      for (s = 0; s < Sv; s++) {
          <Compute S inside  $\gamma$ -projections 63d>
          <Inside multiply by Vs 62b>
          <Compute  $1 - S$   $\gamma$ -unprojections and sum the results 64b>
      }

63c  <Compute  $1 - S_{xy}$  part on the boundary s-chain 63c>≡
      for (s = 0; s < Sv; s++) {
          <Compute S boundary  $\gamma$ -projections 64a>
          <Boundary multiply by Vs 62c>
          <Compute  $1 - S$   $\gamma$ -unprojections and sum the results 64b>
      }

63d  <Compute S inside  $\gamma$ -projections 63d>≡
      <Construct neighbor pointers 70a>
      for (c = 0; c < Nc; c++) {
          k=0; f=&psi[ps[0]]; g=&gg[0]; <Build  $(1 + \gamma_0)$  projection of *f in *g 5a>
          k=1; f=&psi[ps[1]]; g=&gg[1]; <Build  $(1 - \gamma_0)$  projection of *f in *g 5c>
          k=2; f=&psi[ps[2]]; g=&gg[2]; <Build  $(1 + \gamma_1)$  projection of *f in *g 5e>
          k=3; f=&psi[ps[3]]; g=&gg[3]; <Build  $(1 - \gamma_1)$  projection of *f in *g 5g>
          k=4; f=&psi[ps[4]]; g=&gg[4]; <Build  $(1 + \gamma_2)$  projection of *f in *g 6a>
          k=5; f=&psi[ps[5]]; g=&gg[5]; <Build  $(1 - \gamma_2)$  projection of *f in *g 6c>
          k=6; f=&psi[ps[6]]; g=&gg[6]; <Build  $(1 + \gamma_3)$  projection of *f in *g 6e>
          k=7; f=&psi[ps[7]]; g=&gg[7]; <Build  $(1 - \gamma_3)$  projection of *f in *g 6g>
      }

```

64a \langle Compute S boundary γ -projections 64a $\rangle \equiv$

```

 $\langle$  Construct neighbor pointers 70a  $\rangle$ 
for (c = 0; c < Nc; c++) {
    if ((m & 0x01) == 0) {
        k=0; f=&psi[ps[0]]; g=&gg[0];  $\langle$  Build  $(1 + \gamma_0)$  projection of *f in *g 5a  $\rangle$ 
    }
    if ((m & 0x02) == 0) {
        k=1; f=&psi[ps[1]]; g=&gg[1];  $\langle$  Build  $(1 - \gamma_0)$  projection of *f in *g 5c  $\rangle$ 
    }
    if ((m & 0x04) == 0) {
        k=2; f=&psi[ps[2]]; g=&gg[2];  $\langle$  Build  $(1 + \gamma_1)$  projection of *f in *g 5e  $\rangle$ 
    }
    if ((m & 0x08) == 0) {
        k=3; f=&psi[ps[3]]; g=&gg[3];  $\langle$  Build  $(1 - \gamma_1)$  projection of *f in *g 5g  $\rangle$ 
    }
    if ((m & 0x10) == 0) {
        k=4; f=&psi[ps[4]]; g=&gg[4];  $\langle$  Build  $(1 + \gamma_2)$  projection of *f in *g 6a  $\rangle$ 
    }
    if ((m & 0x20) == 0) {
        k=5; f=&psi[ps[5]]; g=&gg[5];  $\langle$  Build  $(1 - \gamma_2)$  projection of *f in *g 6c  $\rangle$ 
    }
    if ((m & 0x40) == 0) {
        k=6; f=&psi[ps[6]]; g=&gg[6];  $\langle$  Build  $(1 + \gamma_3)$  projection of *f in *g 6e  $\rangle$ 
    }
    if ((m & 0x80) == 0) {
        k=7; f=&psi[ps[7]]; g=&gg[7];  $\langle$  Build  $(1 - \gamma_3)$  projection of *f in *g 6g  $\rangle$ 
    }
}

```

64b \langle Compute $1 - S$ γ -unprojections and sum the results 64b $\rangle \equiv$

```

rs = &rx5[s];
es = &ex5[s];
for (c = 0; c < Nc; c++) {
    k = 6;  $\langle$  Unproject  $(1 + \gamma_3)$  link 6f  $\rangle$ 
    k = 7;  $\langle$  Unproject and accumulate  $(1 - \gamma_3)$  link 6h  $\rangle$ 
    k = 2;  $\langle$  Unproject and accumulate  $(1 + \gamma_1)$  link 5f  $\rangle$ 
    k = 3;  $\langle$  Unproject and accumulate  $(1 - \gamma_1)$  link 5h  $\rangle$ 
    k = 1;  $\langle$  Unproject and accumulate  $(1 - \gamma_0)$  link 5d  $\rangle$ 
    k = 0;  $\langle$  Unproject and accumulate  $(1 + \gamma_0)$  link 5b  $\rangle$ 
    k = 5;  $\langle$  Unproject and accumulate  $(1 - \gamma_2)$  link 6d  $\rangle$ 
    k = 4;  $\langle$  Unproject and accumulate  $(1 + \gamma_2)$  link 6b  $\rangle$ 
     $\langle$  Compute  $(*rs) \leftarrow \eta - (*rs)$  for color c 64c  $\rangle$ 
}

```

64c \langle Compute $(*rs) \leftarrow \eta - (*rs)$ for color c 64c $\rangle \equiv$

```

rs->f[0][c].re = es->f[0][c].re - rs->f[0][c].re;
rs->f[0][c].im = es->f[0][c].im - rs->f[0][c].im;
rs->f[1][c].re = es->f[1][c].re - rs->f[1][c].re;
rs->f[1][c].im = es->f[1][c].im - rs->f[1][c].im;
rs->f[2][c].re = es->f[2][c].re - rs->f[2][c].re;
rs->f[2][c].im = es->f[2][c].im - rs->f[2][c].im;
rs->f[3][c].re = es->f[3][c].re - rs->f[3][c].re;
rs->f[3][c].im = es->f[3][c].im - rs->f[3][c].im;

```


5.8.34 Parts of $Q_{xx}^{-1}Q_{xy}\psi$

```

65a  <Compute inside part for  $Q_{xx}^{-1}Q_{xy}$  65a>≡
      for (i = 0; i < nb->inside_size; i++) {
          xyzt = nb->inside[i];
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 71a>
          <Build SSE SU(3) objects 69d>
          <Compute  $Q_{xy}$  part on the inside s-chain 60f>
          <Compute  $Q_{xx}^{-1}$  part on the s-chain 45a>
      }

65b  <Compute boundary part for  $Q_{xx}^{-1}Q_{xy}$  65b>≡
      for (i = 0; i < nb->boundary_size; i++) {
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 71a>
          <Build SSE SU(3) objects 69d>
          <Compute  $Q_{xy}$  part on the boundary s-chain 61a>
          <Compute  $Q_{xx}^{-1}$  part on the s-chain 45a>
      }

```

5.8.35 Parts of $S_{xx}^{-1}S_{xy}\psi$

```

65c  <Compute inside part for  $S_{xx}^{-1}S_{xy}$  65c>≡
      for (i = 0; i < nb->inside_size; i++) {
          xyzt = nb->inside[i];
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 71a>
          <Build SSE SU(3) objects 69d>
          <Compute  $S_{xy}$  part on the inside s-chain 65e>
          <Compute  $S_{xx}^{-1}$  part on the s-chain 45b>
      }

65d  <Compute boundary part for  $S_{xx}^{-1}S_{xy}$  65d>≡
      for (i = 0; i < nb->boundary_size; i++) {
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 71a>
          <Build SSE SU(3) objects 69d>
          <Compute  $S_{xy}$  part on the boundary s-chain 65f>
          <Compute  $S_{xx}^{-1}$  part on the s-chain 45b>
      }

65e  <Compute  $S_{xy}$  part on the inside s-chain 65e>≡
      for (s = 0; s < Sv; s++) {
          <Compute S inside  $\gamma$ -projections 63d>
          <Inside multiply by Vs 62b>
          <Compute S  $\gamma$ -unprojections and sum the results 66a>
      }

65f  <Compute  $S_{xy}$  part on the boundary s-chain 65f>≡
      for (s = 0; s < Sv; s++) {
          <Compute S boundary  $\gamma$ -projections 64a>
          <Boundary multiply by Vs 62c>
          <Compute S  $\gamma$ -unprojections and sum the results 66a>
      }

```

```

66a  <Compute  $S$   $\gamma$ -unprojections and sum the results 66a>≡
      rs = &rx5[s];
      for (c = 0; c < Nc; c++) {
        k = 6; <Unproject  $(1 + \gamma_3)$  link 6f>
        k = 7; <Unproject and accumulate  $(1 - \gamma_3)$  link 6h>
        k = 2; <Unproject and accumulate  $(1 + \gamma_1)$  link 5f>
        k = 3; <Unproject and accumulate  $(1 - \gamma_1)$  link 5h>
        k = 1; <Unproject and accumulate  $(1 - \gamma_0)$  link 5d>
        k = 0; <Unproject and accumulate  $(1 + \gamma_0)$  link 5b>
        k = 5; <Unproject and accumulate  $(1 - \gamma_2)$  link 6d>
        k = 4; <Unproject and accumulate  $(1 + \gamma_2)$  link 6b>
      }

```

5.8.36 Parts of $\eta - Q_{xx}^{-1}Q_{xy}\psi$

```

66b  <Compute inside part for  $1 - Q_{xx}^{-1}Q_{xy}$  66b>≡
      for (i = 0; i < nb->inside_size; i++) {
        const vFermion *ex5, *es;

        xyzt = nb->inside[i];
        xyzt5 = xyzt * Sv;
        <Extract 1-d addresses 71a>
        ex5 = &eta[xyzt5];
        <Build SSE  $SU(3)$  objects 69d>
        <Compute  $Q_{xy}$  part on the inside s-chain 60f>
        <Compute  $1 - Q_{xx}^{-1}$  part on the s-chain 66d>
      }

66c  <Compute boundary part for  $1 - Q_{xx}^{-1}Q_{xy}$  66c>≡
      for (i = 0; i < nb->boundary_size; i++) {
        const vFermion *ex5, *es;
        int m = nb->boundary[i].mask;

        xyzt = nb->boundary[i].index;
        xyzt5 = xyzt * Sv;
        <Extract 1-d addresses 71a>
        ex5 = &eta[xyzt5];
        <Build SSE  $SU(3)$  objects 69d>
        <Compute  $Q_{xy}$  part on the boundary s-chain 61a>
        <Compute  $1 - Q_{xx}^{-1}$  part on the s-chain 66d>
      }

66d  <Compute  $1 - Q_{xx}^{-1}$  part on the s-chain 66d>≡
      <Compute  $Q_{xx}^{-1}$  part on the s-chain 45a>
      for (s = 0; s < Sv; s++) {
        rs = &rx5[s];
        es = &ex5[s];
        nv = vmk1(0.0);
        for (c = 0; c < Nc; c++) {
          <Compute  $(*rs) \leftarrow \eta - (*rs)$  and collect  $\langle r, r \rangle$  67a>
        }
        *norm += vsum(nv);
      }

```

67a $\langle \text{Compute } (*rs) \leftarrow \eta - (*rs) \text{ and collect } \langle r, r \rangle \text{ 67a} \rangle \equiv$

```

vv = es->f[0][c].re - rs->f[0][c].re; rs->f[0][c].re = vv; nv += vv * vv;
vv = es->f[0][c].im - rs->f[0][c].im; rs->f[0][c].im = vv; nv += vv * vv;
vv = es->f[1][c].re - rs->f[1][c].re; rs->f[1][c].re = vv; nv += vv * vv;
vv = es->f[1][c].im - rs->f[1][c].im; rs->f[1][c].im = vv; nv += vv * vv;
vv = es->f[2][c].re - rs->f[2][c].re; rs->f[2][c].re = vv; nv += vv * vv;
vv = es->f[2][c].im - rs->f[2][c].im; rs->f[2][c].im = vv; nv += vv * vv;
vv = es->f[3][c].re - rs->f[3][c].re; rs->f[3][c].re = vv; nv += vv * vv;
vv = es->f[3][c].im - rs->f[3][c].im; rs->f[3][c].im = vv; nv += vv * vv;

```

5.8.37 Parts of $Q_{xx}\eta + Q_{xy}\psi$

67b $\langle \text{Compute inside part for } Q_{xx}\eta + Q_{xy}\psi \text{ 67b} \rangle \equiv$

```

for (i = 0; i < nb->inside_size; i++) {
    const vFermion *ex5, *es;

    xyzt = nb->inside[i];
    xyzt5 = xyzt * Sv;
     $\langle \text{Extract 1-d addresses 71a} \rangle$ 
    ex5 = &eta[xyzt5];
     $\langle \text{Build SSE SU(3) objects 69d} \rangle$ 
     $\langle \text{Compute } Q_{xy} \text{ part on the inside s-chain 60f} \rangle$ 
     $\langle \text{Compute } Q_{xx}\eta + \chi \text{ part on the s-chain 67d} \rangle$ 
}

```

67c $\langle \text{Compute boundary part for } Q_{xx}\eta + Q_{xy}\psi \text{ 67c} \rangle \equiv$

```

for (i = 0; i < nb->boundary_size; i++) {
    const vFermion *ex5, *es;
    int m = nb->boundary[i].mask;

    xyzt = nb->boundary[i].index;
    xyzt5 = xyzt * Sv;
     $\langle \text{Extract 1-d addresses 71a} \rangle$ 
    ex5 = &eta[xyzt5];
     $\langle \text{Build SSE SU(3) objects 69d} \rangle$ 
     $\langle \text{Compute } Q_{xy} \text{ part on the boundary s-chain 61a} \rangle$ 
     $\langle \text{Compute } Q_{xx}\eta + \chi \text{ part on the s-chain 67d} \rangle$ 
}

```

67d $\langle \text{Compute } Q_{xx}\eta + \chi \text{ part on the s-chain 67d} \rangle \equiv$

```

 $\langle \text{Compute } \chi + A\eta \text{ on the upper components 68c} \rangle$ 
 $\langle \text{Compute } \chi + B\eta \text{ on the lower components 69b} \rangle$ 

```

5.8.38 Parts of $S_{xx}\eta + S_{xy}\psi$

67e $\langle \text{Compute inside part for } S_{xx}\eta + S_{xy}\psi \text{ 67e} \rangle \equiv$

```

for (i = 0; i < nb->inside_size; i++) {
    const vFermion *ex5, *es;

    xyzt = nb->inside[i];
    xyzt5 = xyzt * Sv;
     $\langle \text{Extract 1-d addresses 71a} \rangle$ 
    ex5 = &eta[xyzt5];
     $\langle \text{Build SSE SU(3) objects 69d} \rangle$ 
     $\langle \text{Compute } S_{xy} \text{ part on the inside s-chain 65e} \rangle$ 
     $\langle \text{Compute } S_{xx}\eta + \chi \text{ part on the s-chain 68b} \rangle$ 
}

```

68a $\langle \text{Compute boundary part for } S_{xx}\eta + S_{xy}\psi \text{ 68a} \rangle \equiv$
 for (i = 0; i < nb->boundary_size; i++) {
 const vFermion *ex5, *es;
 int m = nb->boundary[i].mask;

 xyzt = nb->boundary[i].index;
 xyzt5 = xyzt * Sv;
 $\langle \text{Extract 1-d addresses 71a} \rangle$
 ex5 = &eta[xyzt5];
 $\langle \text{Build SSE SU(3) objects 69d} \rangle$
 $\langle \text{Compute } S_{xy} \text{ part on the boundary s-chain 65f} \rangle$
 $\langle \text{Compute } S_{xx}\eta + \chi \text{ part on the s-chain 68b} \rangle$
 }

68b $\langle \text{Compute } S_{xx}\eta + \chi \text{ part on the s-chain 68b} \rangle \equiv$
 $\langle \text{Compute } \chi + B\eta \text{ on the upper components 69a} \rangle$
 $\langle \text{Compute } \chi + A\eta \text{ on the lower components 68d} \rangle$

5.8.39 Computing A and B

68c $\langle \text{Compute } \chi + A\eta \text{ on the upper components 68c} \rangle \equiv$
 for (s = Sv, vbc = vb3c, es1 = &ex5[0]; s--; vbc = vb4) {
 es = &ex5[s];
 rs = &rx5[s];

 #define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
 + vbc * shift_up1(es->f[d][c].r, es1->f[d][c].r)
 QXX(0,0,re); QXX(0,0,im);
 QXX(0,1,re); QXX(0,1,im);
 QXX(0,2,re); QXX(0,2,im);
 QXX(1,0,re); QXX(1,0,im);
 QXX(1,1,re); QXX(1,1,im);
 QXX(1,2,re); QXX(1,2,im);
 #undef QXX
 es1 = es;
 }

68d $\langle \text{Compute } \chi + A\eta \text{ on the lower components 68d} \rangle \equiv$
 for (s = Sv, vbc = vb3c, es1 = &ex5[0]; s--; vbc = vb4) {
 es = &ex5[s];
 rs = &rx5[s];

 #define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
 + vbc * shift_up1(es->f[d][c].r, es1->f[d][c].r)
 QXX(2,0,re); QXX(2,0,im);
 QXX(2,1,re); QXX(2,1,im);
 QXX(2,2,re); QXX(2,2,im);
 QXX(3,0,re); QXX(3,0,im);
 QXX(3,1,re); QXX(3,1,im);
 QXX(3,2,re); QXX(3,2,im);
 #undef QXX
 es1 = es;
 }

```

69a  <Compute  $\chi + B\eta$  on the upper components 69a>≡
      for (s = 0, vbc = vcb3, es1 = &ex5[Sv_1]; s < Sv; s++, vbc = vb4) {
          es = &ex5[s];
          rs = &rx5[s];

          #define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
                          + vbc * shift_up3(es1->f[d][c].r, es->f[d][c].r)
          QXX(0,0,re); QXX(0,0,im);
          QXX(0,1,re); QXX(0,1,im);
          QXX(0,2,re); QXX(0,2,im);
          QXX(1,0,re); QXX(1,0,im);
          QXX(1,1,re); QXX(1,1,im);
          QXX(1,2,re); QXX(1,2,im);
          #undef QXX
          es1 = es;
      }

```

```

69b  <Compute  $\chi + B\eta$  on the lower components 69b>≡
      for (s = 0, vbc = vcb3, es1 = &ex5[Sv_1]; s < Sv; s++, vbc = vb4) {
          es = &ex5[s];
          rs = &rx5[s];

          #define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
                          + vbc * shift_up3(es1->f[d][c].r, es->f[d][c].r)
          QXX(2,0,re); QXX(2,0,im);
          QXX(2,1,re); QXX(2,1,im);
          QXX(2,2,re); QXX(2,2,im);
          QXX(3,0,re); QXX(3,0,im);
          QXX(3,1,re); QXX(3,1,im);
          QXX(3,2,re); QXX(3,2,im);
          #undef QXX
          es1 = es;
      }

```

```

69c  <Dxx locals 69c>≡
      const vFermion *es1;
      vReal vbc;

```

5.8.40 Miscallenious

We also need to uplift the gauge fields

```

69d  <Build SSE SU(3) objects 69d>≡
      Uup = &U[nb->site[xyzt].Uup];
      for (d = 0; d < DIM; d++, Uup++) {
          Udown = &U[nb->site[xyzt].Udown[d]];
          for (c1 = 0; c1 < Nc; c1++) {
              for (c2 = 0; c2 < Nc; c2++) {
                  /* conjugate down-link */
                  V[d*2+0].v[c1][c2].re = vmk1(Udown->v[c2][c1].re);
                  V[d*2+0].v[c1][c2].im = vmk1(-Udown->v[c2][c1].im);
                  /* normal up-link */
                  V[d*2+1].v[c1][c2].re = vmk1(Uup->v[c1][c2].re);
                  V[d*2+1].v[c1][c2].im = vmk1(Uup->v[c1][c2].im);
              }
          }
      }

```

We want to keep code small, so computing the neighbors is done in a loop:

```
70a  ⟨Construct neighbor pointers 70a⟩≡
      for (d = 0; d < 2*DIM; d++) {
        ps[d] = p5[d] + s;
      }
```

5.8.41 Combined pieces

In these cases, Q_{xx}^{-1} is applied to the result of Q_{xy}

```
70b  ⟨Static function prototypes 20b⟩+≡
      static void compute_Qxx1Qxy(vFermion *d,
                                   const vFermion *s,
                                   struct neighbor *nb);
      static void inline compute_Qee1Qeo(vEvenFermion *d, const vOddFermion *s)
      {
        compute_Qxx1Qxy(&d->f, &s->f, &even_odd);
      }

      static void compute_Sxx1Sxy(vFermion *d,
                                   const vFermion *s,
                                   struct neighbor *nb);
      static void inline compute_See1Seo(vEvenFermion *d, const vOddFermion *s)
      {
        compute_Sxx1Sxy(&d->f, &s->f, &even_odd);
      }

      static void compute_1Qxx1Qxy(vFermion *d,
                                   double *norm,
                                   const vFermion *q,
                                   const vFermion *s,
                                   struct neighbor *nb);
      static void inline compute_1Qoo1Qoe(vOddFermion *d,
                                   double *norm,
                                   const vOddFermion *q,
                                   const vEvenFermion *s)
      {
        compute_1Qxx1Qxy(&d->f, norm, &q->f, &s->f, &odd_even);
      }
```

5.8.42 Common locals

Some local bindings are used by all parts above. Let us collect them together.

```
70c  ⟨Q common locals 70c⟩≡
      int i, xyzt5, s, c;
      vFermion * __restrict__ rx5, * __restrict__ rs;
```

Others are used only in Z_{xy} parts:

```
70d  ⟨Qxy locals 70d⟩≡
      int xyzt, k, d;
      const vFermion *f;
      vHalfFermion *g;
      vHalfFermion gg[2*DIM], hh[2*DIM];
      vSU3 V[2*DIM];
      int ps[2*DIM], p5[2*DIM];
```

```
70e  ⟨Qxy locals 70d⟩+≡
      const SU3 *Uup, *Udown;
      int c1, c2;
```

For the inside sites, compute the s -chain address of the neighbor. For the boundary sites, the address of the s -chain in the receive buffer is used instead:

```

71a  ⟨Extract 1-d addresses 71a⟩≡
      for (d = 0; d < 2*DIM; d++)
          p5[d] = nb->site[xyzt].F[d];
      ⟨Compute rx5 71b⟩
71b  ⟨Compute rx5 71b⟩≡
      rx5 = &chi[xyzt5];
71c  ⟨Compute qx5 71c⟩≡
      qx5 = &psi[xyzt5];

```

5.8.43 Common globals

Some of these values depend of $m0$ and M . Here we compute their values:

```

71d  ⟨Compute constant values for  $Q_{xx}^{-1}$  and  $S_{xx}^{-1}$  71d⟩≡
      {
          double a = M;
          double b = 2.;
          double c = -2*m0;

          ⟨Compute values from a, b and c 45h⟩
      }

```

5.9 QMP Pieces

Here are miscellaneous piece of QMP:

We are ready to use the result of the global sum. Check that it has been computed.

```

71e  ⟨Finalize ⟨r,r⟩ computation 71e⟩≡
      /* relax, QMP does not support split reductions yet. */

```

Because there is no confirming QMP implementation, we have to deal with esoteric restrictions that are imposed rhyme or reason and are not documented either. Otherwise, they would be not esoteric, ain't they?

Rrestriction one says that deadlocks happen if there no than one active handle at a time. We had already packed all communication into a single superhandle, now it is a time to use it. To be extra paranoid, we check if there is a handle before using it.

```

71f  ⟨Start sends and receives 71f⟩≡
      if (nb->qmp_smask) {
          QMP_start(nb->qmp_handle);
          DEBUG_QMP1("(): start sends and receives (0x%x)\n", (int)nb->qmp_handle)
      }

```

When it is a time to use the received data, we wait for the sends as well. It is a waste, but we have to work with software that we have.

```

71g  ⟨Finish sends and receives 71g⟩≡
      if (nb->qmp_smask) {
          QMP_wait(nb->qmp_handle);
          DEBUG_QMP1("(): waiting for sends and receives (0x%x)\n",
                      (int)nb->qmp_handle)
      }

```

5.9.1 Global sums

Until the split global sums are implemented in QMP, everything is done at the beginning, when ***norm** contains the local part of the sum. Start the global operation which will distribute the pieces, compute the sum, and provide the result to each node.

```

71h  ⟨Start ⟨r,r⟩ computation 71h⟩≡
      QMP_sum_double(norm);
      DEBUG_QMP1("(): sum_double(%p)\n", norm)

```

5.10 SSE Types and Operations

It is convenient to place all SSE specific matter into a separate file which we include into the solver source:

```
72a  <Include files 34b>+≡
      <Macro definitions 19b>
      #define Vs 4 /* Length of SSE vector */
      #define REAL float /* floating point type compatible with vReal */
      #include <sse.h>
```

Here we define the top level structure of `sse.h`:

```
72b  <sse.h 72b>≡
      #ifndef _SSE_H
      #define _SSE_H
      <SSE types 72c>
      <SSE inline functions 73b>
      #endif
```

5.10.1 SSE types

Let us start with the floating point scalar type. The C standard does not provide proper type encapsulation in `typedef`, but we do not need a fool-proof solution here.

First is our floating point vector type. The fact that its length is four is used heavily in the code above and below. The attribute `aligned(16)` helps gcc to keep variables properly aligned.

```
72c  <SSE types 72c>≡
      typedef REAL vReal __attribute__((mode(V4SF),aligned(16)));
```

Now, let us declare complex types. They come in two kinds: scalar and vector, as usual.

```
72d  <SSE types 72c>+≡
      typedef struct {
          REAL re, im;
      } complex;

      typedef struct {
          vReal re, im;
      } vcomplex;
```

We handled enough general cases to express lattice specific data types. The gauge field needs two kind of types (yes, they are scalar and vector, what else?):

```
72e  <SSE types 72c>+≡
      typedef struct SU3 {
          complex v[Nc][Nc];
      } SU3;

      typedef struct {
          vcomplex v[Nc][Nc];
      } vSU3;
```

But we only use vector fermions. However, two component spinors come handy (note that we have committed to the color index varying faster than the spinor index. Is it a good choice?—That is unclear. Changes throughout the code are needed, however, to flip the order of indices.)

```
72f  <SSE types 72c>+≡
      typedef struct {
          vcomplex f[Fd][Nc];
      } vFermion;

      typedef struct {
          vcomplex f[Fd/2][Nc];
      } vHalfFermion;
```


Strictly speaking, there is no need to have separate types for even/odd sublattices. But, while writing the CG, the compiler caught quite a few logic errors because of these two tiny structures.

```
73a  <SSE types 72c>+≡
      typedef struct {
          vFermion f;
      } vEvenFermion;

      typedef struct {
          vFermion f;
      } vOddFermion;
```

5.10.2 SSE inline functions

For efficiency, all functions dealing with SSE data are inlined. The code below requires `gcc 3.3.x`. By the good grace of `gcc` we already have arithmetic operations and assignments on SSE vectors. A few more functions will complete the needed set. All functions below are defined `inline`, so that `gcc` can eliminate the standard function call dance. [NB: In fact, `gcc` does a reasonably good job in dissolving inline function calls in C, but sometimes a residue is left. If you want to use `inline`, frequent consultations with `gcc -S` are advantageous]. First, propagate a scalar value to all four components of the SSE vector.

```
73b  <SSE inline functions 73b>≡
      static inline vReal vmk1(REAL a) {
          vReal v = __builtin_ia32_loadss((float *)&a);
          asm("shufps\t$0,%0,%0" : "+x" (v));
          return v;
      }
```

Packaging four values into an SSE vector is next. This defines the numbering conventions: which element is zeroth etc.

```
73c  <SSE inline functions 73b>+≡
      static inline vReal vmk4(REAL a0, REAL a1, REAL a2, REAL a3) {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a0;
          r[1] = a1;
          r[2] = a2;
          r[3] = a3;
          return v;
      }
```

Next, sum all four components of the SSE vector. Maybe there is a craftier way to do it, but let us leave it as an exercise for now:

```
73d  <SSE inline functions 73b>+≡
      static inline REAL vsum(vReal v)
      {
          REAL *vv = (REAL *)&v;
          return vv[0] + vv[1] + vv[2] + vv[3];
      }
```

Mutators for vector numbers. We only need access to the 0^{th} and the 3^{rd} elements of the vector:

```
73e  <SSE inline functions 73b>+≡
      static inline void vput_3(vReal *v, REAL a3)
      {
          ((REAL *)v)[3] = a3;
      }

      static inline void vput_0(vReal *v, REAL a0)
      {
          ((REAL *)v)[0] = a0;
      }
```

Given

$$\begin{aligned} a &= (a_0, a_1, a_2, a_3), \\ b &= (b_0, b_1, b_2, b_3), \end{aligned}$$

compute various shifts as follows:

$$\text{shift_up1} \leftarrow (a_1, a_2, a_3, b_0)$$

74a $\langle \text{SSE inline functions 73b} \rangle + \equiv$

```
static inline vReal shift_up1(vReal a, vReal b)
{
    vReal x = a;
    vReal y = b;
    asm("shufps\t$0x30,%0,%1\n\t"
        "shufps\t$0x29,%1,%0"
        : "+x" (x), "+x" (y));
    return x;
}
```

$$\text{shift_up2} \leftarrow (a_2, a_3, b_0, b_1)$$

74b $\langle \text{SSE inline functions 73b} \rangle + \equiv$

```
static inline vReal shift_up2(vReal a, vReal b)
{
    vReal x = a;
    asm("shufps\t$0x4e,%1,%0"
        : "+x" (x): "x" (b));
    return x;
}
```

$$\text{shift_up3} \leftarrow (a_3, b_0, b_1, b_2)$$

74c $\langle \text{SSE inline functions 73b} \rangle + \equiv$

```
static inline vReal shift_up3(vReal a, vReal b)
{
    vReal x = a;
    asm("shufps\t$0x03,%1,%0\n\t"
        "shufps\t$0x9c,%1,%0"
        : "+x" (x): "x" (b));
    return x;
}
```

$$\text{shift_down1} \leftarrow (a_3, b_0, b_1, b_2)$$

74d $\langle \text{SSE inline functions 73b} \rangle + \equiv$

```
static inline vReal shift_down1(vReal a, vReal b)
{
    return shift_up3(a, b);
}
```

$$\text{shift_down2} \leftarrow (a_2, a_3, b_0, b_1)$$

74e $\langle \text{SSE inline functions 73b} \rangle + \equiv$

```
static inline vReal shift_down2(vReal a, vReal b)
{
    return shift_up2(a, b);
}
```

$\text{shift_down3} \leftarrow (a_1, a_2, a_3, b_0)$

```
75a  <SSE inline functions 73b>+≡
      static inline vReal shift_down3(vReal a, vReal b)
      {
          return shift_up1(a, b);
      }
```

The very last of the SSE functions: clear a half fermion:

```
75b  <SSE inline functions 73b>+≡
      static inline void vhfzero(vHalfFermion *v)
      {
          vReal z = vmk1(0.0);

          v->f[0][0].re = v->f[0][0].im =
          v->f[0][1].re = v->f[0][1].im =
          v->f[0][2].re = v->f[0][2].im =
          v->f[1][0].re = v->f[1][0].im =
          v->f[1][1].re = v->f[1][1].im =
          v->f[1][2].re = v->f[1][2].im = z;
      }
```

5.11 Generally Useful Functions

Here is a collection of simple functions that are useful throughout the code:

```
75c  <Static function prototypes 20b>+≡
      static inline int
      parity(const int x[DIM])
      {
          int i, v;
          for (i = v = 0; i < DIM; i++)
              v += x[i];
          return v & 1;
      }
```

5.12 Debug Aids

Here are macros for debugging QMP.

```
75d  <Definitions 75d>≡
      #ifdef DEBUG_QMP
      #define DEBUG_QMP0(msg)          do_debug(__FILE__, __LINE__, __FUNCTION__, msg);
      #define DEBUG_QMP1(msg,a)        do_debug(__FILE__, __LINE__, __FUNCTION__, msg,a);
      #define DEBUG_QMP2(msg,a,b)      do_debug(__FILE__, __LINE__, __FUNCTION__, msg,a,b);
      #define DEBUG_QMP3(msg,a,b,c)    do_debug(__FILE__, __LINE__, __FUNCTION__, msg,a,b,c);
      #define DEBUG_QMP4(msg,a,b,c,d)  do_debug(__FILE__, __LINE__, __FUNCTION__, msg,a,b,c,d);

      static void do_debug(const char *fname, int line, const char *fun,
                          const char *fmt, ...);

      #else
      #define DEBUG_QMP0(msg)
      #define DEBUG_QMP1(msg,a)
      #define DEBUG_QMP2(msg,a,b)
      #define DEBUG_QMP3(msg,a,b,c)
      #define DEBUG_QMP4(msg,a,b,c,d)
      #endif
```

It is convenient to do debug prints in a procedure.

```
76a  <Static functions 19a>+≡
      #ifdef DEBUG_QMP
      static void
      do_debug(const char *fname, int line, const char *fun,
                const char *fmt, ...)
      {
          va_list va;

          printf("%s:%d: %s", fname, line, fun);
          va_start(va, fmt);
          vprintf(fmt, va);
          va_end(va);
          fflush(stdout);
      }
      #endif
```

5.13 Handy Constants

For some constants it is better to have symbolic names even if one can not easily change their values.

```
76b  <Macro definitions 19b>+≡
      #define Nc   3      /* Number of colors */
      #define DIM  4      /* number of dimensions */
      #define Fd   4      /* Fermion representation dimension */
```

5.14 Source File

Finally, let us put together all the pieces:

```
76c  <dwf.c 76c>≡
      #include <stdlib.h>
      #include <stdarg.h>
      #include "sse-dwf-cg.h"
      <Include files 34b>

      <Definitions 75d>

      <Data types 12f>
      <Global variables 10>
      <Static function prototypes 20b>
      <Static functions 19a>
      <Interface functions 11b>
```

6 CHUNKS

<Advance DIM-d index for DIM-1-d scan 22f>
 <Advance DIM-d index for full sublattice scan 22d>
 <Advance DIM-d index for full sublattice scan locally 22e>
 <Advance **x** at **i** 22g>
 <Advance **x** at **i** locally 22h>
 <Allocate **boundary** table 27d>
 <Allocate down buffers 35c>
 <Allocate fields 38c>
 <Allocate **inside** table 27c>
 <Allocate up buffers 35b>
 <Boundary multiply by V s 62c>
 <Build SSE $SU(3)$ objects 69d>
 <Build $(1 + \gamma_0)$ projection of ***f** in ***g** 5a>
 <Build $(1 + \gamma_1)$ projection of ***f** in ***g** 5e>
 <Build $(1 + \gamma_2)$ projection of ***f** in ***g** 6a>
 <Build $(1 + \gamma_3)$ projection of ***f** in ***g** 6e>
 <Build $(1 - \gamma_0)$ projection of ***f** in ***g** 5c>
 <Build $(1 - \gamma_1)$ projection of ***f** in ***g** 5g>
 <Build $(1 - \gamma_2)$ projection of ***f** in ***g** 6c>
 <Build $(1 - \gamma_3)$ projection of ***f** in ***g** 6g>
 <Build local neighbors 30b>
 <Build outside indices 30c>
 <Check floating point size 12a>
 <Check lattice size 12b>
 <Check xx -aliasing of q 47a>
 <Cleanup QMP 37b>
 <Clump up and down directions 35a>
 <Compute $A^{-1}\psi$ on the lower two components 45d>
 <Compute $A^{-1}\psi$ on the upper two components 45c>
 <Compute $B^{-1}\psi$ on the lower two components 45f>
 <Compute $B^{-1}\psi$ on the upper two components 45e>
 <Compute L_A^{-1} on the lower components 48b>
 <Compute L_A^{-1} on the upper components 47d>
 <Compute L_B^{-1} on the lower components 49b>
 <Compute L_B^{-1} on the upper components 49a>
 <Compute Q boundary γ -projections 61c>
 <Compute Q γ -unprojections and sum the results 62a>
 <Compute Q inside γ -projections 61b>
 <Compute $1 - Q_{xx}^{-1}$ part on the s -chain 66d>
 <Compute Q_{xx}^{-1} part on the s -chain 45a>
 <Compute $Q_{xx}\eta + \chi$ part on the s -chain 67d>
 <Compute Q_{xy} part on the boundary s -chain 61a>
 <Compute Q_{xy} part on the inside s -chain 60f>
 <Compute R_A^{-1} on the lower components 50c>
 <Compute R_A^{-1} on the upper components 50b>
 <Compute R_B^{-1} on the lower components 50e>
 <Compute R_B^{-1} on the upper components 50d>
 <Compute S boundary γ -projections 64a>
 <Compute $1 - S$ γ -unprojections and sum the results 64b>
 <Compute S γ -unprojections and sum the results 66a>
 <Compute S inside γ -projections 63d>
 <Compute S_{xx}^{-1} part on the s -chain 45b>
 <Compute $S_{xx}\eta + \chi$ part on the s -chain 68b>
 <Compute $1 - S_{xy}$ part on the boundary s -chain 63c>
 <Compute S_{xy} part on the boundary s -chain 65f>
 <Compute $1 - S_{xy}$ part on the inside s -chain 63b>

⟨Compute S_{xy} part on the inside s -chain 65e⟩
 ⟨Compute boundary part for $1 - Q_{xx}^{-1}Q_{xy}$ 66c⟩
 ⟨Compute boundary part for $Q_{xx}^{-1}Q_{xy}$ 65b⟩
 ⟨Compute boundary part for $Q_{xx}\eta + Q_{xy}\psi$ 67c⟩
 ⟨Compute boundary part for Q_{xy} 60e⟩
 ⟨Compute boundary part for $S_{xx}^{-1}S_{xy}$ 65d⟩
 ⟨Compute boundary part for $S_{xx}\eta + S_{xy}\psi$ 68a⟩
 ⟨Compute boundary part for $1 - S_{xy}$ 63a⟩
 ⟨Compute $\chi + A\eta$ on the lower components 68d⟩
 ⟨Compute $\chi + A\eta$ on the upper components 68c⟩
 ⟨Compute $\chi + B\eta$ on the lower components 69b⟩
 ⟨Compute $\chi + B\eta$ on the upper components 69a⟩
 ⟨Compute constant values for Q_{xx} and S_{xx} 9⟩
 ⟨Compute constant values for Q_{xx}^{-1} and S_{xx}^{-1} 71d⟩
 ⟨Compute init sizes 26a⟩
 ⟨Compute inside part for $1 - Q_{xx}^{-1}Q_{xy}$ 66b⟩
 ⟨Compute inside part for $Q_{xx}^{-1}Q_{xy}$ 65a⟩
 ⟨Compute inside part for $Q_{xx}\eta + Q_{xy}\psi$ 67b⟩
 ⟨Compute inside part for Q_{xy} 60d⟩
 ⟨Compute inside part for $S_{xx}^{-1}S_{xy}$ 65c⟩
 ⟨Compute inside part for $S_{xx}\eta + S_{xy}\psi$ 67e⟩
 ⟨Compute inside part for $1 - S_{xy}$ 62e⟩
 ⟨Compute **inside_size** and **boundary_size** 27b⟩
 ⟨Compute **p** and **m** 29d⟩
 ⟨Compute projections for Q send 58c⟩
 ⟨Compute projections for S send 59a⟩
 ⟨Compute ψ_e 40a⟩
 ⟨Compute **qx5** 71c⟩
 ⟨Compute $(*rs) \leftarrow \eta - (*rs)$ and collect $\langle r, r \rangle$ 67a⟩
 ⟨Compute $(*rs) \leftarrow \eta - (*rs)$ for color c 64c⟩
 ⟨Compute **rx5** 71b⟩
 ⟨Compute send sizes and allocate index tables 27e⟩
 ⟨Compute values from a , b and c 45h⟩
 ⟨Compute φ_o 38a⟩
 ⟨Compute wall value in **zX[c]** 50a⟩
 ⟨Compute $y_{k,[0]}^{(A)}$ 51b⟩
 ⟨Compute $y_{k,[1]}^{(A)}$ 51c⟩
 ⟨Compute $y_{k,[2]}^{(A)}$ 51d⟩
 ⟨Compute $y_{k,[3]}^{(A)}$ 52a⟩
 ⟨Compute $y_{k,[0]}^{(B)}$ 52b⟩
 ⟨Compute $y_{k,[1]}^{(B)}$ 52c⟩
 ⟨Compute $y_{k,[2]}^{(B)}$ 53a⟩
 ⟨Compute $y_{k,[3]}^{(B)}$ 53b⟩
 ⟨Compute $zV \leftarrow zV + fx * qs^{down}$ 48c⟩
 ⟨Compute $zV \leftarrow zV + fx * qs^{up}$ 48a⟩
 ⟨Construct $(1 + \gamma_0)$ send k -buffer 59b⟩
 ⟨Construct $(1 + \gamma_1)$ send k -buffer 59d⟩
 ⟨Construct $(1 + \gamma_2)$ send k -buffer 59f⟩
 ⟨Construct $(1 + \gamma_3)$ send k -buffer 60b⟩
 ⟨Construct $(1 - \gamma_0)$ send k -buffer 59c⟩
 ⟨Construct $(1 - \gamma_1)$ send k -buffer 59e⟩
 ⟨Construct $(1 - \gamma_2)$ send k -buffer 60a⟩
 ⟨Construct $(1 - \gamma_3)$ send k -buffer 60c⟩
 ⟨Construct neighbor pointers 70a⟩
 ⟨Construct the collective handle 36d⟩

<Construct the initial point of the hypersurface 31c>
 <Construct the neighbor's network coordinates **xc** and bounds **xb** 31b>
 <Data types 12f>
 <Definitions 75d>
 <Dxx locals 69c>
 <End xx-aliasing of q 47b>
 <Extract 1-d addresses 71a>
 <Finalize $\langle r, r \rangle$ computation 71e>
 <Find index of a borrowed gauge link 34a>
 <Find index of a regular gauge link 33c>
 <Finish sends and receives 71g>
 <Finish xy-aliasing of q 57c>
 <Free QMP buffers 37f>
 <Free common handle 37e>
 <Free fields 38d>
 <Free tables 32d>
 <Get network topology 21a>
 <Global variables 10>
 <Handle init errors 11c>
 <Include files 34b>
 <Init out of bound y 51a>
 <Initialize QMP 34c>
 <Initialize **out** and **p** 29b>
 <Initialize tables 25b>
 <Insert **k** into **site[p].F[dx]** 32c>
 <Inside multiply by V s 62b>
 <Interface functions 11b>
 <Load DIM gauge links from U at **x** 22a>
 <Load a **d** gauge link from V at **x** 23b>
 <Load an s -line of fermion at **x** 23d>
 <Load gauge boundary in direction **d** 23a>
 <Macro definitions 19b>
 <Multiply ***u** by ***g** and store the result in ***h** 62d>
 <Q common locals 70c>
 <Qxx locals 47c>
 <Qxy locals 70d>
 <Read fermion 23c>
 <Read gauge field 21c>
 <SSE inline functions 73b>
 <SSE types 72c>
 <Save an s -line of fermion at **x** 24c>
 <Setup **boundary** 30a>
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